

5 Unconfined aquifers

Figure 5.1 shows a pumped unconfined aquifer underlain by an aquiclude. The pumping causes a dewatering of the aquifer and creates a cone of depression in the watertable. As pumping continues, the cone expands and deepens, and the flow towards the well has clear vertical components.

There are thus some basic differences between unconfined and confined aquifers when they are pumped:

- First, a confined aquifer is not dewatered during pumping; it remains fully saturated and the pumping creates a drawdown in the piezometric surface;
- Second, the water produced by a well in a confined aquifer comes from the expansion of the water in the aquifer due to a reduction of the water pressure, and from the compaction of the aquifer due to increased effective stresses;
- Third, the flow towards the well in a confined aquifer is and remains horizontal, provided, of course, that the well is a fully penetrating one; there are no vertical flow components in such an aquifer.

In unconfined aquifers, the water levels in piezometers near the well often tend to decline at a slower rate than that described by the Theis equation. Time-drawdown curves on log-log paper therefore usually show a typical S-shape, from which we can recognize three distinct segments: a steep early-time segment, a flat intermediate-time segment, and a relatively steep late-time segment (Figure 5.2). Nowadays, the widely used explanation of this S-shaped time-drawdown curve is based on the concept of 'delayed watertable response'. Boulton (1954, 1963) was the first to introduce this concept, which he called 'delayed yield'. He developed a semi-empirical solution that

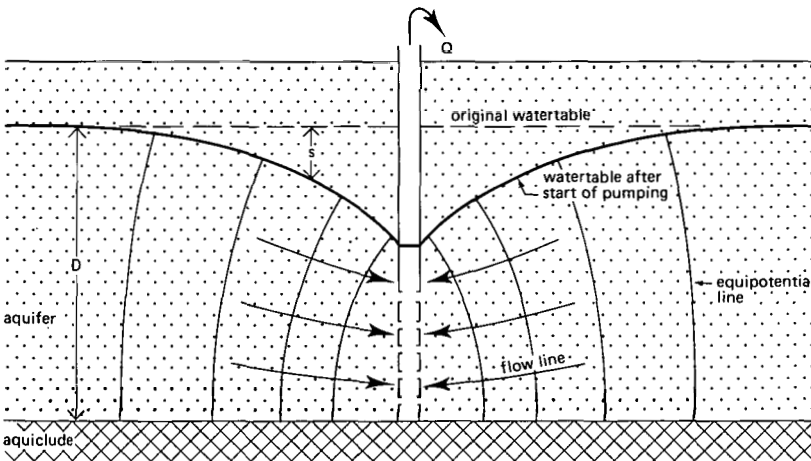


Figure 5.1 Cross-section of a pumped unconfined aquifer

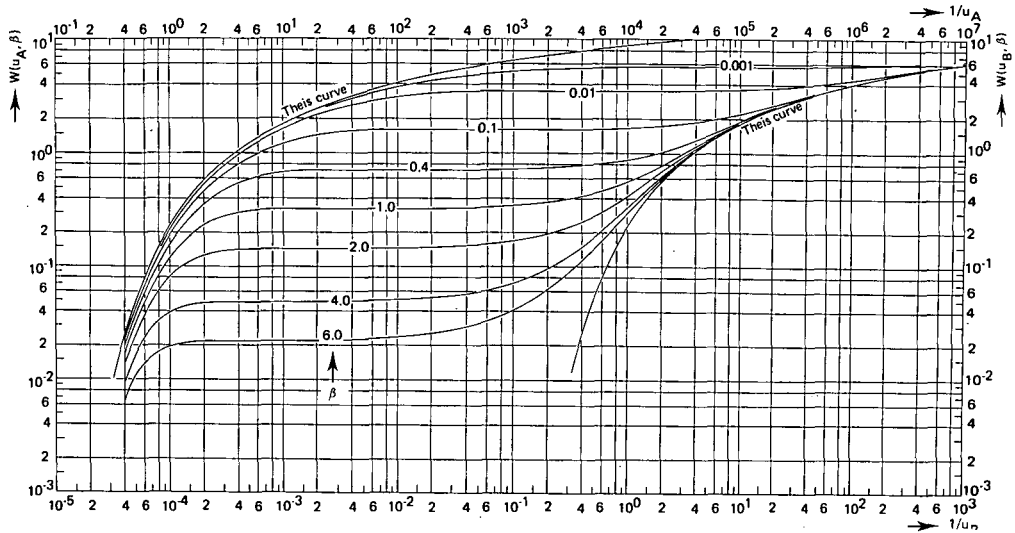


Figure 5.2 Family of Neuman type curves: $W(u_A, \beta)$ versus $1/u_A$ and $W(u_B, \beta)$ versus $1/u_B$ for different values of β

reproduced all three segments of this curve. Although useful in practice, Boulton's solution has one drawback: it requires the definition of an empirical constant, known as the Boulton's delay index, which is not clearly related to any physical phenomenon. The concept of delayed watertable response was further developed by Neuman (1972, 1973, 1979); Streltsova (1972a and b, 1973, 1976); and Gambolati (1976). According to these authors, the three time segments of the curve should be understood as follows:

- The steep early-time segment covers only a brief period after the start of pumping (often only the first few minutes). At early pumping times, an unconfined aquifer reacts in the same way as a confined aquifer: the water produced by the well is released instantaneously from storage by the expansion of the water and the compaction of the aquifer. The shape of the early-time segment is similar to the Theis type curve;
- The flat intermediate-time segment reflects the effect of the dewatering that accompanies the falling watertable. The effect of the dewatering on the drawdown is comparable to that of leakage: the increase of the drawdown slows down with time and thus deviates from the Theis curve. After a few minutes to a few hours of pumping, the time-drawdown curve may approach the horizontal;
- The relatively steep late-time segment reflects the situations where the flow in the aquifer is essentially horizontal again and the time-drawdown curve once again tends to conform to the Theis curve.

Section 5.1 presents Neuman's curve-fitting method, which is based on the concept of delayed watertable response. Neuman's method allows the determination of the horizontal and vertical hydraulic conductivities, the storativity S_A , and the specific yield S_Y .

It must be noted, however, that unreasonably low S_Y values are often obtained,

because flow in the (saturated) capillary fringe above the watertable is neglected (Van der Kamp 1985).

Under favourable conditions, the early and late-time drawdown data can also be analyzed by the methods given in Section 3.2. For example, the Theis method can be applied to the early-time segment of the time-drawdown curve, provided that data from piezometers near the well are used because the drawdown in distant piezometers during this period will often be too small to be measured. The storativity S_A computed from this segment of the curve, however, cannot be used to predict long-term drawdowns. The late-time segment of the curve may again conform closely to the Theis type curve, thus enabling the late-time drawdown data to be analyzed by the Theis equation and yielding the transmissivity and the specific yield S_Y of the aquifer. The Theis method yields a fairly realistic value of S_Y (Van der Kamp 1985).

If a pumped-unconfined aquifer does not show phenomena of delayed watertable response, the time-drawdown curve only follows the late-time segment of the S-shaped curve. Because the flow pattern around the well is identical to that in a confined aquifer, the methods in Section 3.2 can be used.

True steady-state flow cannot be reached in a pumped unconfined aquifer of infinite areal extent. Nevertheless, the drawdown differences will gradually diminish with time and will eventually become negligibly small. Under these transient steady-state conditions we can use the Thiem-Dupuit method (Section 5.2).

The methods presented in this chapter are all based on the following assumptions and conditions:

- The aquifer is unconfined;
- The aquifer has a seemingly infinite areal extent;
- The aquifer is homogeneous and of uniform thickness over the area influenced by the test;
- Prior to pumping, the watertable is horizontal over the area that will be influenced by the test;
- The aquifer is pumped at a constant discharge rate;
- The well penetrates the entire aquifer and thus receives water from the entire saturated thickness of the aquifer.

In practice, the effect of flow in the unsaturated zone on the delayed watertable response can be neglected (Cooley and Case 1973; Kroszynski and Dagan 1975). According to Bouwer and Rice (1978), air entry phenomena may influence the drawdown.

Although the aquifer is assumed to be of uniform thickness, this condition is not met if the drawdown is large compared with the aquifer's original saturated thickness. A corrected value for the observed drawdown s then has to be applied. Jacob (1944) proposed the following correction

$$s' = s - (s^2/2D)$$

where

s' = corrected drawdown

s = observed drawdown

D = original saturated aquifer thickness

According to Neuman (1975), Jacob's correction is strictly applicable only to the late-time drawdown data, which fall on the Theis curve.

5.1 Unsteady-state flow

5.1.1 Neuman's curve-fitting method

Neuman (1972) developed a theory of delayed watertable response which is based on well-defined physical parameters of the unconfined aquifer. Neuman treats the aquifer as a compressible system and the watertable as a moving material boundary. He recognizes the existence of vertical flow components and his general solution of the drawdown is a function of both the distance from the well r and the elevation head. When considering an average drawdown, he is able to reduce his general solution to one that is a function of r alone. Mathematically, Neuman simulated the delayed watertable response by treating the elastic storativity S_A and the specific yield S_Y as constants.

Neuman's drawdown equation (Neuman 1975) reads

$$s = \frac{Q}{4\pi KD} W(u_A, u_B, \beta) \quad (5.1)$$

Under early-time conditions, this equation describes the first segment of the time-drawdown curve (Figure 5.2) and reduces to

$$s = \frac{Q}{4\pi KD} W(u_A, \beta) \quad (5.2)$$

where

$$u_A = \frac{r^2 S_A}{4KDt} \quad (5.3)$$

S_A = volume of water instantaneously released from storage per unit surface area per unit decline in head (= elastic early-time storativity).

Under late-time conditions, Equation 5.1 describes the third segment of the time-drawdown curve and reduces to

$$s = \frac{Q}{4\pi KD} W(u_B, \beta) \quad (5.4)$$

where

$$u_B = \frac{r^2 S_Y}{4KDt} \quad (5.5)$$

S_Y = volume of water released from storage per unit surface area per unit decline of the watertable, i.e. released by dewatering of the aquifer (= specific yield)

Neuman's parameter β is defined as

$$\beta = \frac{r^2 K_v}{D^2 K_h} \quad (5.6)$$

where

K_v = hydraulic conductivity for vertical flow, in m/d

K_h = hydraulic conductivity for horizontal flow, in m/d

For isotropic aquifers, $K_v = K_h$, and $\beta = r^2/D^2$.

Neuman's curve-fitting method can be used if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of this chapter;
- The aquifer is isotropic or anisotropic;
- The flow to the well is in an unsteady state;
- The influence of the unsaturated zone upon the drawdown in the aquifer is negligible;
- $S_v/S_A > 10$;
- An observation well screened over its entire length penetrates the full thickness of the aquifer;
- The diameters of the pumped and observation wells are small, i.e. storage in them can be neglected.

As stated by Rushton and Howard (1982), fully-penetrating observation wells allow the 'short-circuiting' of vertical flow. Consequently, the water levels observed in them will not always be equivalent to the average of groundwater heads in a vertical section of the aquifer, as assumed in Neuman's theory. The theory should still be valid, however, for piezometers with short screened sections, provided that the drawdowns are averaged over the full thickness of the aquifer (Van der Kamp 1985).

Procedure 5.1

- Construct the family of Neuman type curves by plotting $W(u_A, u_B, \beta)$ versus $1/u_A$ and $1/u_B$ for a practical range of values of β on log-log paper, using Annex 5.1. The left-hand portion of Figure 5.2 shows the type A curves [$W(u_A, \beta)$ versus $1/u_A$] and the right-hand portion the type B curves [$W(u_B, \beta)$ versus $1/u_B$];
- Prepare the observed data curve on another sheet of log-log paper of the same scale by plotting the values of the drawdown s against the corresponding time t for a single observation well at a distance r from the pumped well;
- Match the early-time observed data plot with one of the type A curves. Note the β value of the selected type A curve;
- Select an arbitrary point A on the overlapping portion of the two sheets and note the values of s , t , $1/u_A$, and $W(u_A, \beta)$ for this point;
- Substitute these values into Equations 5.2 and 5.3 and, knowing Q and r , calculate $K_h D$ and S_A ;
- Move the observed data curve until as many as possible of the late-time observed data fall on the type B curve with the same β value as the selected type A curve;
- Select an arbitrary point B on the superimposed sheets and note the values of s , t , $1/u_B$, and $W(u_B, \beta)$ for this point;
- Substitute these values into Equations 5.4 and 5.5 and, knowing Q and r , calculate

- $K_h D$ and S_Y . The two calculations should give approximately the same value for $K_h D$;
- From the $K_h D$ value and the known initial saturated thickness of the aquifer D , calculate the value of K_h ;
 - Substitute the numerical values of K_h , β , D , and r into Equation 5.6 and calculate K_v ;
 - Repeat the procedure with the observed drawdown data from any other observation well that may be available. The calculated results should be approximately the same.

Remarks

- To check whether the condition $S_Y/S_A > 10$ is fulfilled, the value of this ratio should be determined;
- Gambolati (1976) (see also Neuman 1979) pointed out that, theoretically, the effects of elastic storage and dewatering become additive at large t , the final storativity being equal to $S_A + S_Y$. However, in situations where the effect of delayed watertable response is clearly evident, $S_A \ll S_Y$ and the influence of S_A at larger times can safely be neglected.

Example 5.1

To illustrate the Neuman curve-fitting method, we shall use data from the pumping test ‘Vennebulten’, The Netherlands (De Ridder 1966). Figure 5.3 shows a lithostratigraphical section of the pumping test area as derived from the drilling data. The impermeable base consists of Middle Miocene marine clays. The aquifer is made up of very coarse fluvio-glacial sands and coarse fluvial deposits, which grade upward into very fine sand and locally into loamy cover sand. The finer part of the aquifer is about 10 m thick. A well screen was placed between 10 and 21 m below ground surface, and piezometers were placed at distances of 10, 30, 90, and 280 m from the well at

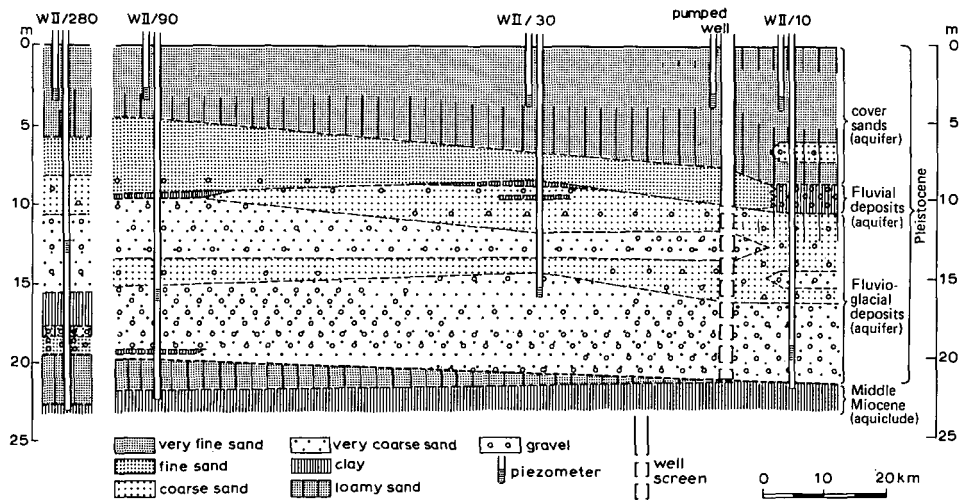


Figure 5.3 Lithostratigraphical cross-section of the pumping-test site ‘Vennebulten’, The Netherlands (after De Ridder 1966)

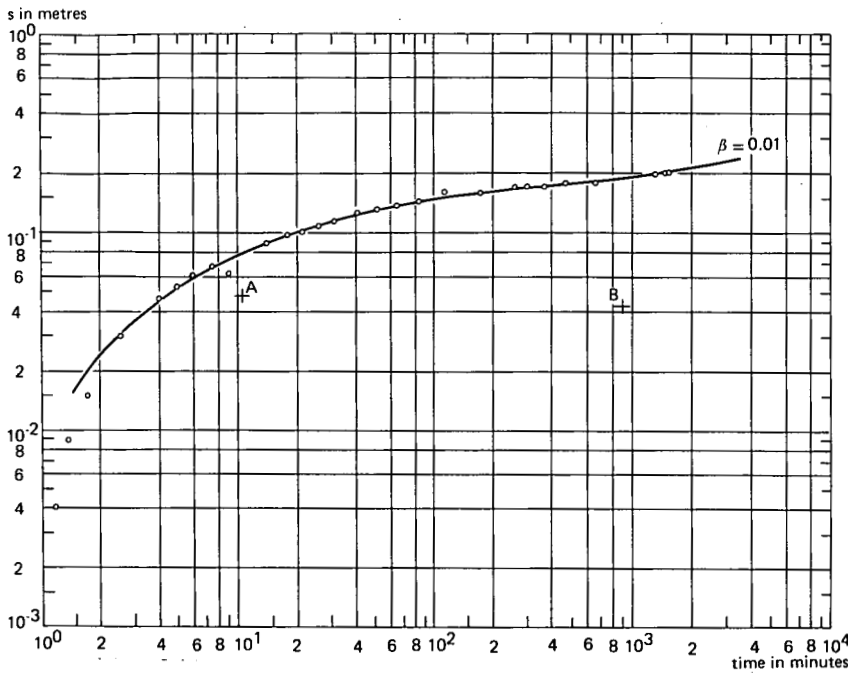


Figure 5.4 Analysis of data from pumping test 'Vennebulten', The Netherlands ($r = 90$ m) with the Neuman curve-fitting method

depths ranging from 12 to 19 m. Shallow piezometers (at a depth of about 3 m) were placed at the same distances. The aquifer was pumped for 25 hours at a constant discharge of $36.37 \text{ m}^3/\text{hr}$ (or $873 \text{ m}^3/\text{d}$). Table 5.1 summarizes the drawdown observations in the piezometer at 90 m.

The observed time-drawdown data of Table 5.1 are plotted on log-log paper (Figure 5.4). The early-time segment of the plot gives the best match with the Neuman type A curve for $\beta = 0.01$. The match point A has the coordinates $1/u_A = 10$, $W(u_A, \beta) = 1$, $s = 4.8 \times 10^{-2} \text{ m}$, and $t = 10.5 \text{ min} = 7.3 \times 10^{-3} \text{ d}$.

The values of $K_h D$ and S_A are obtained from Equations 5.2 and 5.3

$$K_h D = \frac{Q}{4\pi s} W(u_A, \beta) = \frac{873}{4\pi \times 4.8 \times 10^{-2}} \times 1 = 1447 \text{ m}^2/\text{d}$$

$$S_A = \frac{4K_h D t u_A}{r^2} = \frac{4 \times 1447 \times 7.3 \times 10^{-3} \times 10^{-1}}{90^2} = 5.2 \times 10^{-4}$$

The coordinates for match point B of the observed data plot and the type B curve for $\beta = 0.01$ are $1/u_B = 10^2$, $W(u_B, \beta) = 1$, $s = 4.3 \times 10^{-2} \text{ m}$ and $t = 880 \text{ min} = 6.1 \times 10^{-1} \text{ d}$.

Calculating the values of $K_h D$ and S_Y from Equations 5.4 and 5.5, we obtain

$$K_h D = \frac{Q}{4\pi s} W(u_B, \beta) = \frac{873}{4\pi \times 4.3 \times 10^{-2}} \times 1 = 1616 \text{ m}^2/\text{d}$$

$$S_Y = \frac{4K_h D t u_B}{r^2} = \frac{4 \times 1616 \times 6.1 \times 10^{-1} \times 10^{-2}}{90^2} = 4.9 \times 10^{-3}$$

Knowing the thickness of the aquifer $D = 21$ m, we can calculate the hydraulic conductivity for horizontal flow

$$K_h = \frac{K_h D}{D} = \left(\frac{1447 + 1616}{2} \right) / 21 = 73 \text{ m/d}$$

Table 5.1 Summary of data from piezometer WII/90; pumping test 'Vennebulten', The Netherlands (after De Ridder 1966)

Time (min)	Drawdown deep piezometer (m)	Drawdown shallow piezometer (m)	Time (min)	Drawdown deep piezometer (m)	Drawdown shallow piezometer (m)
0	0	0	41	0.128	0.018
1.17	0.004		51	0.133	0.022
1.34	0.009		65	0.141	0.026
1.7	0.015		85	0.146	0.028
2.5	0.030		115	0.161	0.033
4.0	0.047		175	0.161	0.044
5.0	0.054		260	0.172	0.050
6.0	0.061	0.005	300	0.173	0.055
7.5	0.068		370	0.173	
9	0.064	0.006	430	0.179	
14	0.090	0.008	485	0.183	0.061
18	0.098	0.010	665	0.182	0.071
21	0.103		1.340	0.200	0.096
26	0.110	0.011	1.490	0.203	0.099
31	0.115	0.014	1.520	0.204	0.099

From Equation 5.6, the hydraulic conductivity for vertical flow can be calculated

$$K_v = \frac{\beta D^2 K_h}{r^2} = \frac{0.01 \times 21^2 \times 73}{90^2} = 4 \times 10^{-2} \text{ m/d}$$

The value of the ratio S_Y/S_A is

$$\frac{S_Y}{S_A} = \frac{4.9 \times 10^{-3}}{5.2 \times 10^{-4}} = 9.4$$

The condition of $S_Y/S_A > 10$ is therefore nearly satisfied. Note that the value of S_Y calculated by means of the 'B' curves is unreasonably low. This is in agreement with earlier observations that the determination of S_Y from 'B' curves remains a dubious procedure (Van der Kamp 1985).

5.2 Steady-state flow

When the drawdown differences have become negligibly small with time, the Thiem-

Dupuit method can be used to calculate the transmissivity of an unconfined aquifer.

5.2.1 Thiem-Dupuit's method

The Thiem-Dupuit method can be used if the following assumptions and conditions are satisfied:

- The assumptions listed in the beginning of this chapter;
- The aquifer is isotropic;
- The flow to the well is in steady state;
- The Dupuit (1863) assumptions are satisfied, i.e.:
 - The velocity of flow is proportional to the tangent of the hydraulic gradient instead of the sine as it is in reality;
 - The flow is horizontal and uniform everywhere in a vertical section through the axis of the well.

If these assumptions are met, the well discharge for steady horizontal flow to a well pumping an unconfined aquifer (Figure 5.5) can be described by

$$Q = 2\pi rKh \frac{dh}{dr}$$

After integration between r_1 and r_2 (with $r_2 > r_1$), this yields

$$Q = \pi K \frac{h_2^2 - h_1^2}{\ln(r_2/r_1)} \quad (5.7)$$

which is known as the formula of Dupuit.

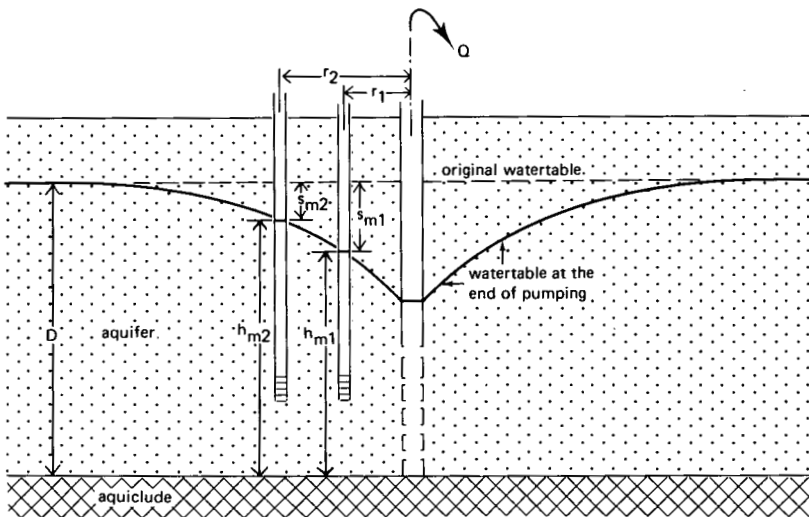


Figure 5.5 Cross-section of a pumped unconfined aquifer (steady-state flow)

Since $h = D - s$, Equation 5.7 can be transformed into

$$Q = \frac{\pi K [(D - s_{m2})^2 - (D - s_{m1})^2] 2D / 2D}{\ln(r_2/r_1)} = \frac{2\pi K D [(s_{m1} - s_{m1}^2/2D) - (s_{m2} - s_{m2}^2/2D)]}{\ln(r_2/r_1)}$$

Replacing $s - s^2/2D$ with s' = the corrected drawdown, yields

$$Q = \frac{2\pi K D (s'_{m1} - s'_{m2})}{\ln(r_2/r_1)} = \frac{2\pi K D (s'_{m1} - s'_{m2})}{2.30 \log (r_2/r_1)} \quad (5.8)$$

This formula is identical to the Thiem formula (Equation 3.2) for a confined aquifer, so the methods in Section 3.1.1 can also be used for an unconfined aquifer.

Remarks

- The Dupuit formula (Equation 5.7) fails to give an accurate description of the draw-down curve near the well, where the strong curvature of the watertable contradicts the Dupuit assumptions. These assumptions ignore the existence of a seepage face at the well and the influence of the vertical velocity components, which reach their maximum in the vicinity of the well;
- An approximate steady-state flow condition in an unconfined aquifer will only be reached after long pumping times, i.e. when the flow in the aquifer is essentially horizontal again and the drawdown curve has followed the late-time segment of the S-shaped curve that coincides with the Theis curve for sufficiently long time.