9 Multi-layered aquifer systems

Multi-layered aquifer systems may be one of three kinds. The first consists of two or more aquifer layers, separated by aquicludes. If data on the transmissivity and storativity of the individual aquifer layers are needed, a pumping test can be conducted in each layer, and each test can then be analyzed by the appropriate method for a single-layered aquifer.

If a well fully penetrates the aquifer system and thus pumps more than one of the aquifer layers at a time, single-layered methods are not applicable. For an aquifer system that consists of two confined aquifers, Papadopulos (1966) derived asymptotic solutions for unsteady-state flow to a well that fully penetrates the system and thus pumps both aquifers at the same time.

For an aquifer system that consists of an unconfined aquifer overlying a confined aquifer, Abdul Khader and Veerankutty (1975) derived a solution for unsteady-state flow to a fully penetrating well.

Either of these solutions allows the hydraulic characteristics of the individual aquifers to be calculated. Both, however, require the use of a computer.

The second multi-layered aquifer system consists of two or more aquifers, each with its own hydraulic characteristics, and separated by interfaces that allow unrestricted crossflow (Figure 9.1). This system's response to pumping will be analogous to that of a single-layered aquifer whose transmissivity and storativity are equal to the sum of the transmissivity and storativity of the individual layers. Hence, in an aquifer with unrestricted crossflow, the same methods as used for single-layered aquifers can be applied. One has to keep in mind, however, that only the hydraulic characteristics

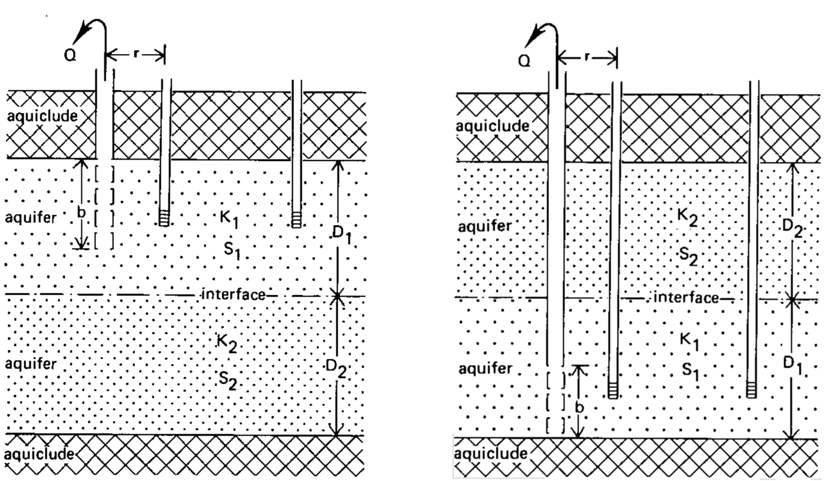

Figure 9.1 Confined two-layered aquifer system, partially penetrating well, either in the upper layer from the top downwards or in the lower layer from the bottom upwards

of the equivalent aquifer system can be determined in this way.

In a confined two-layered aquifer system with unrestricted crossflow, the hydraulic characteristics of the individual aquifers can be determined with the Javandel-Witherspoon method presented in Section 9.1.1.

The third multi-layered aquifer system consists of two or more aquifer layers, separated by aquitards. Pumping one layer of this leaky system has measurable effects in layers other than the pumped layer. The resulting drawdown in each layer is a function of several parameters, which depend on the hydraulic characteristics of the aquifer layers and those of the aquitards. Only for small values of pumping time can the drawdown in the unpumped layers be assumed to be negligible, and only then can methods for leaky single-layered aquifers (Chapter 4) be used to estimate the hydraulic characteristics of the pumped layer.

For longer pumping times, Bruggeman (1966) has developed a method for the analysis of data from leaky two-layered aquifer systems in which steady-state flow prevails. This method is presented in Section 9.2.1.

Various analytical solutions have been derived for steady and unsteady-state flow to a well pumping a leaky multi-layered aquifer system, e.g. Hantush (1967), Neuman and Whitherspoon (1969a, 1969b), and Hemker (1984, 1985). Because of the large number of unknown parameters involved, these methods require the use of a computer.

9.1 Confined two-layered aquifer systems with unrestricted crossflow, unsteady-state flow

9.1.1 Javandel-Witherspoon's method

Javandel and Witherspoon (1983) developed analytical solutions for the drawdown in both layers of a confined two-layered aquifer system pumped by a well that is partially screened, either in the upper layer from the top downwards, or in the underlying layer from the bottom upwards (Figure 9.1). Asymptotic solutions for small and large values of pumping time are derived from the general solution.

For small values of pumping time ($t \leq (D_1 - b)^2/\{(10K_1D_1)/S_1\}$), the drawdown equation for the pumped layer is identical with the equation for unsteady-state flow in a confined single-layered aquifer that is pumped by a partially penetrating well (see Section 10.2.1).

For large values of pumping time and at distances from the pumped well beyond $r \geq 1.5\,\{D_1 + (K_2D_2)/K_1\}$, the partial penetration effects of the well can be ignored and the drawdown in the pumped layer approaches the following expression

$$s_1 = \frac{Q}{4\pi(K_1D_1 + K_2D_2)} W(u) \tag{9.1}$$

where

$$u = \frac{r^2(S_1 + S_2)}{4t(K_1D_1 + K_2D_2)} \tag{9.2}$$

This drawdown equation has the form of the Theis equation for unsteady flow in a confined single-layered aquifer pumped by a fully penetrating well (Section 3.2.1). The response of the two-layered system reflects the hydraulic characteristics of the equivalent single-layered system:

$$KD_{eq} = K_1 D_1 + K_2 D_2$$

and

$$S_{eq} = S_1 + S_2$$

Since t is assumed to be large, u will be small. Hence, in analogy to Equation 3.7 (Jacob's method, Section 3.2.2), Equation 9.1 can be written as

$$s_1 = \frac{2.30Q}{4\pi(K_1D_1 + K_2D_2)} \log \frac{25\,(K_1D_1 + K_2D_2)t}{r^2(S_1 + S_2)} \qquad (9.3)$$

A plot on semi-log paper of s versus t will show a straight line for large values of t. The slope of this straight line is given by

$$\Delta s = \frac{2.30Q}{4\pi(K_1D_1 + K_2D_2)} \qquad (9.4)$$

The intercept t_0 of the straight line with the t axis where $s = 0$ is given by

$$t_0 = \frac{r^2\,(S_1 + S_2)}{2.25\,(K_1D_1 + K_2D_2)} \qquad (9.5)$$

The Javandel-Witherspoon method is applicable if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of Chapter 3, with the exception of the third and sixth assumptions, which are replaced by:
  - The system consists of two aquifer layers. Each layer has its own hydraulic characteristics, is of apparent infinite areal extent, is homogeneous, isotropic, and of uniform thickness over the area influenced by the test. The interface between the two layers is an open boundary, i.e. no discontinuity of potential or its gradient is allowed across the interface;
  - The pumped well does not penetrate the entire thickness of the aquifer system, but is partially screened, either in the upper layer from the top downwards, or in the lower layer from the bottom upwards.

The following conditions are added:
- The flow to the well is in unsteady state;
- The piezometers are placed at a depth that coincides with the middle of the well screen;
- Drawdown data are available for small values of pumping time $t \leq (D_1 - b)^2/(10K_1 D_1/S_1)$ and for large values of pumping time. The late-time drawdown data are measured at $r \geq 1.5\,\{D_1 + (K_2D_2)/K_1\}$.

Procedure 9.1
- Apply the Hantush modification of the Theis method (see Section 10.2.1) to the early-time drawdown data $\{t \leq (D_1\text{-}b)^2/(10K_1D_1/S_1)\}$ and determine K_1D_1 and S_1 of the pumped layer;
- Determine K_2D_2 and S_2 of the unpumped layer with the procedure outlined for the Jacob method (Section 3.2.2):

- Plot for one of the piezometers, $r \geq 1.5\{D_1 + (K_2D_2)/K_1\}$, the observed drawdown s versus the corresponding time t on semi-log paper (t on logarithmic scale);
- Draw the best-fitting straight line through the late-time portion of the plotted points;
- Extend the straight line until it intercepts the time axis where $s = 0$, and read the value of t_0;
- Determine the slope of the straight line, i.e. the drawdown difference Δs per log cycle of time;
- Substitute the known values of Q, Δs, and K_1D_1 into Equation 9.4

$$K_2D_2 = \frac{2.30Q}{4\pi\Delta s} - K_1D_1$$

  and calculate K_2D_2 of the unpumped layer;
- Substitute the known values of t_0, K_1D_1, K_2D_2, r^2, and S_1 into Equation 9.5

$$S_2 = \frac{2.25t_0(K_1D_1 + K_2D_2)}{r^2} - S_1$$

  and calculate S_2.

Remarks

- To analyze the late-time drawdown data, the Theis curve-fitting method (Section 3.2.1) can be used instead of the Jacob method;
- Javandel and Witherspoon (1983) observed that the condition $r \geq 1.5\{D_1 + (K_2D_2)/K_1\}$ is on the conservative side;
- If only one piezometer at $r \geq 1.5\{D_1 + (K_2D_2)/K_1\}$ from the well is available, there may not be sufficient early-time drawdown data to determine the hydraulic characteristics of the pumped layer. Hence, only the combined hydraulic characteristics KD_{eq} ($= K_1D_1 + K_2D_2$) and S_{eq}($= S_1 + S_2$) of the equivalent aquifer system can be determined;
- Javandel and Witherspoon (1980) also developed a semi-analytical solution for the drawdown distribution in both layers of a slightly different type of two-layered aquifer system with unrestricted crossflow. The upper layer of this system is bounded by an aquiclude. The lower layer is considered to be very thick compared with the upper layer. The system is pumped by a well that partially penetrates the upper layer. For more information, see the original literature.

9.2 Leaky two-layered aquifer systems with crossflow through aquitards, steady-state flow

Figure 9.2 shows a cross-section of a pumped leaky two-layered aquifer system, overlain by an aquitard, and with another aquitard separating the two aquifer layers. If the hydraulic resistance of the aquitard separating the layers is high compared with that of the overlying aquitard, and if the base layer is an aquiclude, the upper and lower parts of the system can be treated as two separate single-layered leaky aquifers.

Matters become more complicated if the hydraulic resistance of the separating aquitard is appreciably lower than that of the overlying aquitard. If the upper part of

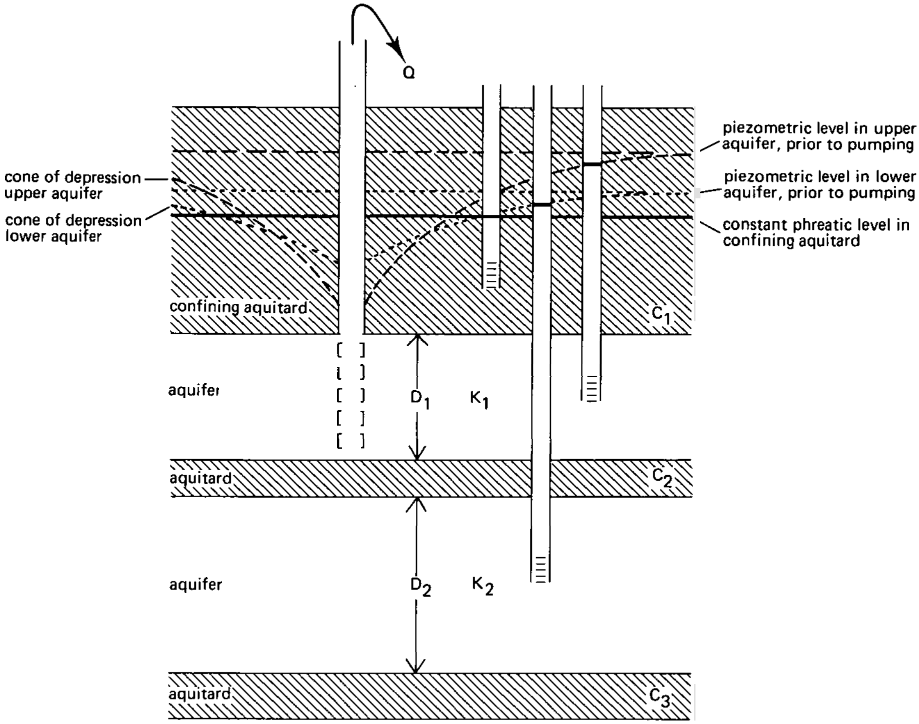

Figure 9.2 Pumped leaky two-layered aquifer system, overlain by an aquitard, and with another aquitard separating the two aquifer layers

that system is pumped, the discharged water would come from the pumped upper layer, the lower aquifer layer (through the separating aquitard), and the overlying aquitard. Bruggeman (1966) has developed a method of analysis for such a system.

9.2.1 Bruggeman's method

The Bruggeman method calls for a double pumping test in which the lower layer is pumped until a steady state is reached, and then, after complete recovery, the upper layer is pumped, again until a steady state is reached. Bruggeman (1966) does not stipulate that the aquifer system be underlain by an aquiclude; it may also be an aquitard. Bruggeman showed that the following relations are valid

$$s'_{1,1} + P_1 s'_{2,1} = \frac{Q'}{2\pi K_1 D_1} K_0(r/\lambda_1) \tag{9.6}$$

$$s'_{1,1} + P_2 s'_{2,1} = \frac{Q'}{2\pi K_1 D_1} K_0(r/\lambda_2) \tag{9.7}$$

$$s'_{1,2} + P_1 s'_{2,2} = P_1 \frac{Q'}{2\pi K_2 D_2} K_0(r/\lambda_1) \tag{9.8}$$

$$s'_{1,2} + P_2 s'_{2,2} = P_2 \frac{Q'}{2\pi K_2 D_2} K_0(r/\lambda_2) \tag{9.9}$$

where

$$s' = \frac{Q'}{Q} s \tag{9.10}$$

Q' = standardized discharge rate

The first index to s indicates the aquifer layer in which the piezometer is installed. The second index indicates which layer is being pumped. For example, $s'_{2,1}$ is the drawdown observed in the lower layer when the upper layer is pumped at a standardized discharge rate Q'.
Moreover

$$P_1 + P_2 = \frac{(K_2D_2/K_1D_1)(s'_{2,2} - s'_{1,1})}{s'_{1,2}} \tag{9.11}$$

$$P_1P_2 = -(K_2D_2/K_1D_1) \tag{9.12}$$

where P_1, P_2, λ_1, and λ_2 are constants which are related to one another by

$$\frac{1}{\lambda_1^2} = a_1 + b_1 - a_2P_1 \tag{9.13}$$

$$\frac{1}{\lambda_2^2} = a_1 + b_1 - a_2P_2 \tag{9.14}$$

$$\frac{P_1}{\lambda_1^2} = -b_1 + b_2P_1 + a_2P_1 \tag{9.15}$$

$$\frac{P_2}{\lambda_2^2} = -b_1 + b_2P_2 + a_2P_2 \tag{9.16}$$

where a_1, a_2, b_1, and b_2 are also constants dependent on K_1D_1, K_2D_2, c_1, and c_2, according to the following equations

$$a_1 = \frac{1}{K_1D_1c_1} \tag{9.17}$$

$$b_1 = \frac{1}{K_1D_1c_2} \tag{9.18}$$

$$a_2 = \frac{1}{K_2D_2c_2} \tag{9.19}$$

and

$$b_2 = \frac{1}{K_2D_2c_3} \tag{9.20}$$

The Bruggeman method is based on the following assumptions and conditions:
- The assumptions listed at the beginning of Chapter 3, with the exception of the first, third and sixth assumptions, which are replaced by:

- The aquifer system consists of two aquifer layers separated by an aquitard. Each layer is homogeneous, isotropic, and of uniform thickness over the area influenced by the test. The aquifer system is overlain by an aquitard;
- The well receives water by horizontal flow from the entire thickness of the pumped layer.

The following conditions are added:

- The flow to the well is in steady state;
- r/L is small ($r/L < 0.05$);
- $c_1 > c_2$;
- $K_2D_2 > K_1D_1$;
- $c_3 \leq \infty$;
- A pumping test is first conducted in the lower layer until a steady state is reached; then after complete recovery, a pumping test is conducted in the upper layer, again until steady state is reached.

Procedure 9.2

- With Equation 9.10, transform the observed drawdown data to corrected drawdown data for an arbitrarily chosen standard discharge rate Q'. Check whether $s'_{1,2} = s'_{2,1}$ because this should be so for the application of this method;
- Plot $s'_{1,1}$ versus r on semi-log paper and calculate K_1D_1 with

$$\Delta s'_{1,1} = \frac{2.30Q'}{2\pi K_1D_1}$$

where $\Delta s'_{1,1}$ is the difference in $s'_{1,1}$ per log cycle of r;

- In the same way, calculate K_2D_2 from a plot of $s'_{2,2}$ versus r;
- Calculate P_1P_2 with Equation 9.12;
- Calculate $P_1 + P_2$ by introducing into Equation 9.11, for a given value of r, the corresponding values of $s'_{2,2}$ and $s'_{1,1}$ and the values of K_2D_2 and K_1D_1. When this is repeated for several values of r, it provides a check on the values of K_2D_2 and K_1D_1 already calculated, because $P_1 + P_2$ should be independent of r. Calculate P_1 and P_2 by combining the values of $P_1 + P_2$ and P_1P_2.
  A comparison of Equations 9.6 to 9.9 with Equation 4.1 shows the analogy between the Bruggeman equations and the De Glee equation;
- Therefore plot the curve $s'_{1,1} + P_1s'_{2,1}$ versus r on log-log paper and, using De Glee's method (Section 4.1.1, Procedure 4.1), calculate the values of λ_1. In the same way, calculate λ_2 from a plot of $s'_{1,1} + P_2s'_{2,1}$ versus r. Check the values of λ_1 and λ_2 by calculating λ_1 and λ_2 from plots on log-log paper of $(1/P_2)\, s'_{1,2} + s'_{2,2}$ versus r and $(1/P_2)\, s'_{1,2} + s'_{2,2}$ versus r with the De Glee method;
- Using Equations 9.13 to 9.16, calculate a_1, a_2, b_1, and b_2 from the known values of λ_1, λ_2, P_1, and P_2;
- Finally, calculate c_1, c_2, K_1D_1, and K_2D_2 from Equations 9.17 to 9.20. Calculating K_1D_1 and K_2D_2 in this way provides a check on the earlier calculations of K_1D_1 and K_2D_2.