8 Anisotropic aquifers

The standard methods of analysis are all based on the assumption that the aquifer is isotropic, i.e. that the hydraulic conductivity is the same in all directions. Many aquifers, however, are anisotropic. In such aquifers, it is not unusual to find hydraulic conductivities that differ by a factor of between two and twenty when measured in one or another direction. Anisotropy is a common feature in water-laid sedimentary deposits (e.g. fluvial, clastic lake, deltaic and glacial outwash deposits). Aquifers that are composed of water-laid deposits may exhibit anisotropy on the horizontal plane. The hydraulic conductivity in the direction of flow tends to be greater than that perpendicular to flow. Because of the differences in hydraulic conductivity, lines of equal drawdown around a pumped well in these aquifers will form ellipses rather than concentric circles.

In addition such aquifers are often stratified, i.e. they are made up of alternating layers of coarse and fine sands, gravels, and occasional clays, with each layer possessing a unique value of K. Any layer with a low K will retard vertical flow, but horizontal flow can occur easily through any layer with relatively high K. Obviously, K_h, i.e. parallel to the bedding planes, will be much higher than K_v, and the aquifer is said to be *anisotropic on the vertical plane*.

Aquifers that are anisotropic on both the horizontal and vertical planes, are said to exhibit three-dimensional anisotropy, with principal axes of K in the vertical direction, the horizontal direction parallel to stream flows that prevailed in the past, and the horizontal direction at a right angle to those flows.

It will be clear that, in the analysis of pumping tests, anisotropy poses a special problem. Methods of analysis that take anisotropy on the horizontal plane into account are presented in Section 8.1 for confined aquifers and in Section 8.2 for leaky aquifers. Sections 8.3, 8.4 and 8.5 discuss anisotropy on the vertical plane in confined aquifers, leaky aquifers, and unconfined aquifers.

8.1 Confined aquifers, anisotropic on the horizontal plane

8.1.1 Hantush's method

The unsteady-state drawdown in a confined isotropic aquifer is given by the Theis equation (Equation 3.5)

$$s = \frac{Q}{4\pi KD} W(u)$$

where

$$u = \frac{r^2 S}{4KDt}$$

In a confined aquifer that is anisotropic on the horizontal plane, with the principal axes of anisotropy X and Y, the above equations, according to Hantush (1966), are replaced by

$$s = \frac{Q}{4\pi(KD)_e} W(u_{XY}) \tag{8.1}$$

where

$$u_{XY} = \frac{r^2 S}{4t(KD)_n} \tag{8.2}$$

$$(KD)_e = \sqrt{(KD)_X \times (KD)_Y} = \text{the effective transmissivity} \tag{8.3}$$

$(KD)_X$ = transmissivity in the major direction of anisotropy
$(KD)_Y$ = transmissivity in the minor direction of anisotropy
$(KD)_n$ = transmissivity in a direction that makes an angle $(\theta + \alpha)$ with the X axis (θ and α will be defined below)

If we have one or more piezometers on a ray that forms an angle $(\theta + \alpha)$ with the X axis, we can apply the methods for isotropic aquifers and obtain values for $(KD)_e$ and $S/(KD)_n$. Consequently, to calculate S and $(KD)_n$, we need data from more than one ray of piezometers.

Hantush (1966) showed that if θ is defined as the angle between the first ray of piezometers (n = 1) and the X axis and α_n as the angle between the nth ray of piezometers and the first ray of piezometers (Figures 8.1A and B), $(KD)_n$ is given by

$$(KD)_n = \frac{(KD)_X}{\cos^2(\theta + \alpha_n) + m \sin^2(\Theta + \alpha_n)} \tag{8.4}$$

where

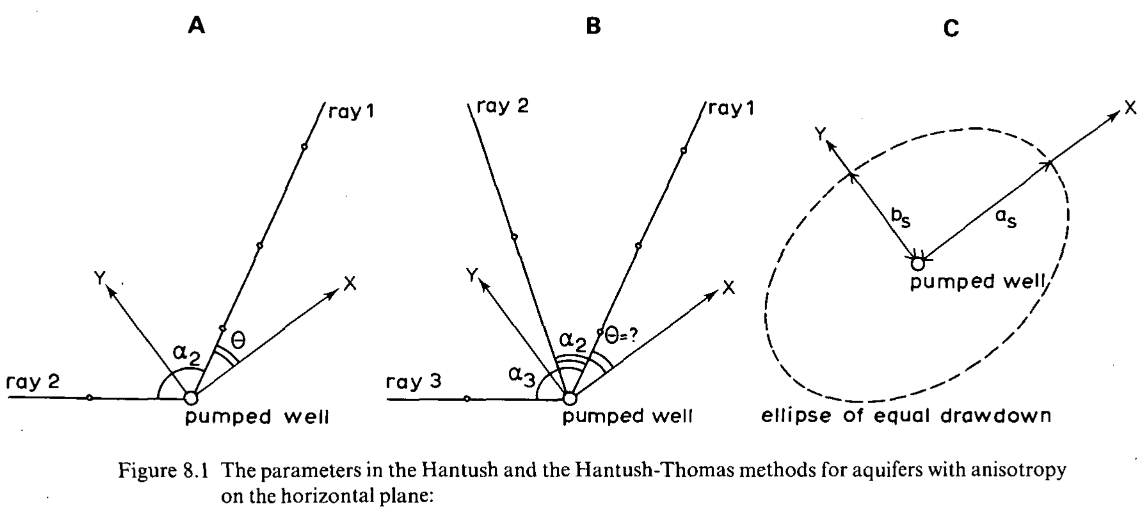

Figure 8.1 The parameters in the Hantush and the Hantush-Thomas methods for aquifers with anisotropy on the horizontal plane:
A. Principal directions of anisotropy known
B. Principal directions of anisotropy not known
C. Ellipse of equal drawdown

$$m = \frac{(KD)_X}{(KD)_Y} = \left[\frac{(KD)_e}{(KD)_Y}\right]^2 \qquad (8.5)$$

Because $\alpha_1 = 0$ for the first ray of piezometers, Equation 8.4 reduces to

$$(KD)_1 = \frac{(KD)_X}{\cos^2\Theta + m\sin^2\Theta} \qquad (8.6)$$

and consequently

$$a_n = \frac{(KD)_1}{(KD)_n} = \frac{\cos^2(\theta + \alpha_n) + m\sin^2(\theta + \alpha_n)}{\cos^2\theta + m\sin^2\theta} \qquad (8.7)$$

It goes without saying that $a_1 = 1$.
A combination of Equations 8.5 and 8.7 yields

$$m = \left[\frac{(KD)_e}{(KD)_Y}\right]^2 = \frac{a_n\cos^2\theta - \cos^2(\theta + \alpha_n)}{\sin^2(\theta + \alpha_n) - a_n\sin^2\theta} \qquad (8.8)$$

If the principal directions of anisotropy are not known, one needs at least three piezometers on different rays from the pumped well to solve Equation 8.7 for θ, using

$$\tan(2\theta) = -2\frac{(a_3 - 1)\sin^2\alpha_2 - (a_2 - 1)\sin^2\alpha_3}{(a_3 - 1)\sin 2\alpha_2 - (a_2 - 1)\sin 2\alpha_3} \qquad (8.9)$$

Equation 8.9 has two roots for the angle (2θ) in the range 0 to 2π of the XY plane. If one of the roots is δ, the other will be $\pi + \delta$. Consequently, θ has two values: $\delta/2$ and $(\pi + \delta)/2$. One of the values of θ yields $m > 1$ and the other $m < 1$. Since the X axis is assumed to be along the major axis of anisotropy, the value of θ that will make $m = (KD)_X/(KD)_Y > 1$ locates the major axis of anisotropy, X; the other value locates the minor axis of anisotropy, Y. (It should be noted that a negative value of θ indicates that the positive X axis lies to the left of the first ray of piezometers.)
The Hantush method can be applied if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the third assumption, which is replaced by:
  - The aquifer is homogeneous, anisotropic on the horizontal plane, and of uniform thickness over the area influenced by the pumping test.

The following conditions are added:

- The flow to the well is in unsteady state;
- If the principal directions of anisotropy are known, drawdown data from two piezometers on different rays from the pumped well will be sufficient. If the principal directions of anisotropy are not known, drawdown data must be available from at least three rays of piezometers.

Procedure 8.1 (principal directions of anisotropy known)

- Apply the methods for isotropic confined aquifers (Sections 3.2.1 and 3.2.2) to the data of each of the two rays of piezometers. This results in values for $(KD)_e$, $S/(KD)_1$, and $S/(KD)_2$;
- A combination of the last two values gives a_2 (cf. Equation 8.7). Because θ and α_2 are known, substitute the values of θ, α, a, and $(KD)_e$ into Equation 8.8 and calculate m;

- Knowing $(KD)_e$ and m, calculate $(KD)_X$ and $(KD)_Y$ from Equation 8.5;
- Substitute the values of $(KD)_X$, m, θ, and α_2 into Equations 8.6 and 8.7 and solve for $(KD)_1$ and $(KD)_2$;
- A combination of the last two values with those for $S/(KD)_1$ and $S/(KD)_2$, respectively, yields values for S, which should be essentially the same.

Procedure 8.2 (principal directions of anisotropy unknown)
- Apply the methods for isotropic confined aquifers (Sections 3.2.1 and 3.2.2) to the data from each of the three rays of piezometers. This results in values for $(KD)_e$, $S/(KD)_1$, $S/(KD)_2$, and $S/(KD)_3$;
- A combination of $S/(KD)_1$ with $S/(KD)_2$ and $S/(KD)_3$, respectively, yields values for a_2 and a_3. Because α_2 and α_3 are known, θ can be calculated from Equation 8.9;
- Substitute the values of θ, $(KD)_e$, α_2, and a_2 (or α_3 and a_3) into Equation 8.8 and calculate m;
- Knowing $(KD)_e$ and m, calculate $(KD)_X$ and $(KD)_Y$ from Equation 8.5;
- Substitute the values of $(KD)_X$, m, and θ and the values of $\alpha_1 = 0$, α_2, and α_3 into Equation 8.4 and solve for $(KD)_1$, $(KD)_2$, and $(KD)_3$;
- A combination of these values with those of $S/(KD)_1$, $S/(KD)_2$, and $S/(KD)_3$, respectively, yields values for S, which should be essentially the same.

Remarks
- The observed data should permit the use of those methods for isotropic confined aquifers that give a value for $S/(KD)_n$. Hence, the methods for steady-state flow in isotropic confined aquifers (Section 3.1) are not applicable;
- The analysis of the data from each ray of piezometers yields a value of $(KD)_e$. These values should all be essentially the same.

Example 8.1
Using Procedure 8.2, we shall analyse the drawdown data presented by Papadopulos (1965). The data are from a pumping test conducted in an anisotropic confined aquifer. During the test, the well PW was pumped at a discharge rate of 1086 m^3/d. The drawdown was observed in three observation wells OW-1, OW-2, and OW-3, located as shown in Figure 8.2.

For each observation well, we plot the drawdown data on semi-log paper (Figure 8.3). The data allow the application of Jacob's straight line method (Chapter 3) to determine the values of $(KD)_e$ and $S/(KD)_1$, $S/(KD)_2$, and $S/(KD)_3$

$$(KD)_e = \frac{2.30Q}{4\pi\Delta s} = \frac{2.30 \times 1086}{4 \times 3.14 \times 1.15} = 173 \text{ m}^2/\text{d}$$

$$\frac{S}{(KD)_1} = \frac{2.25\, t_{01}}{r^2} = \frac{2.25 \times 0.37}{(28.3)^2 \times 1440} = 7.22 \times 10^{-7} \text{ d/m}^2$$

$$\frac{S}{(KD)_2} = \frac{2.25\, t_{02}}{r^2} = \frac{2.25 \times 0.72}{(9^2 + 33.5^2) \times 1440} = 9.35 \times 10^{-7} \text{ d/m}^2$$

$$\frac{S}{(KD)_3} = \frac{2.25\, t_{03}}{r^2} = \frac{2.25 \times 0.24}{(19.3^2 + 5.2^2) \times 1440} = 9.39 \times 10^{-7} \text{ d/m}^2$$

Subsequently, we calculate the values of a_2 and a_3: $a_2 = 1.295$ and $a_3 = 1.300$. The value of Θ can now be derived from Equation 8.9

$$\tan(2\Theta) = -2\left\{\frac{(1.300-1)\sin^2 75° - (1.295-1)\sin^2 196°}{1.300\sin(2\times 75°) - (1.295-1)\sin(2\times 196°)}\right\} = 82$$

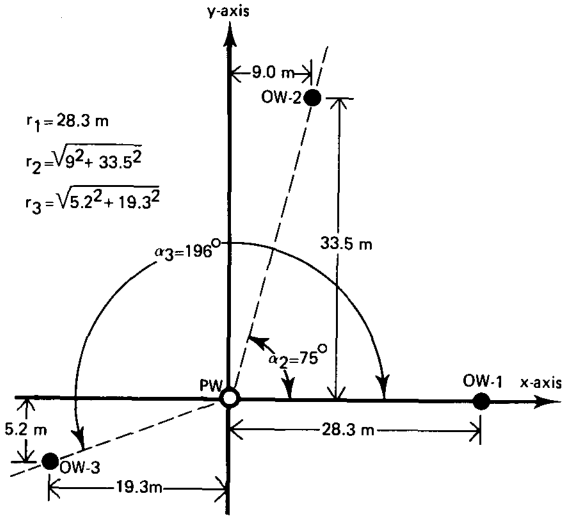

Figure 8.2 Location of the pumped well and observation wells (Papadopulos pumping test, Example 8.1)

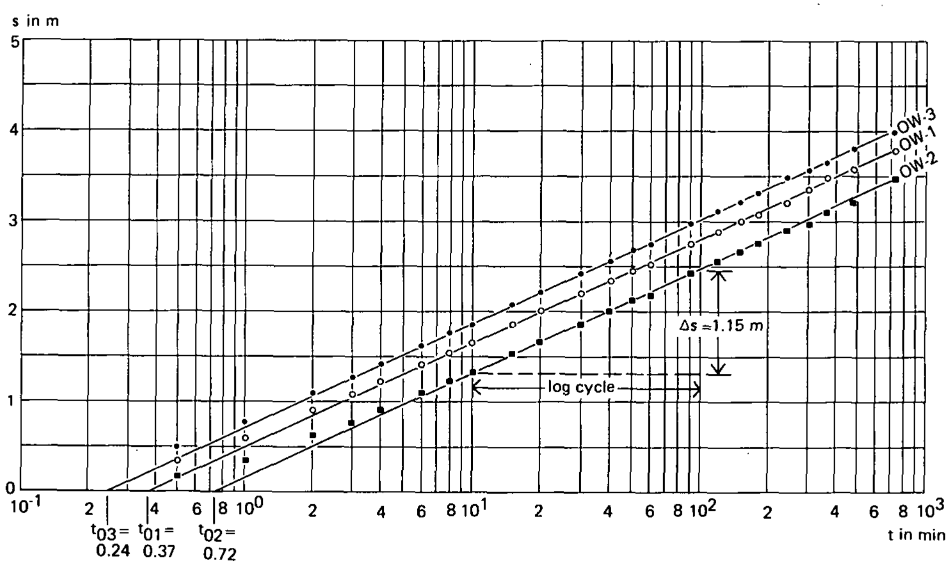

Figure 8.3 Analysis of data from the Papadopulos pumping test with the Jacob method

The two possible values of Θ are 45° and 135°.
Using $\Theta = 45°$, and subsequently $\Theta = 135°$, and the appropriate values of $(KD)_e$, α_3, and a_3 in Equation 8.8 gives the following values for m

$$\text{for } \Theta = 45°: m = \frac{1.3 \cos^2 45° - \cos^2(45° + 196°)}{\sin^2(45° + 196°) - 1.3 \sin^2 45°} = 3.6 \text{ (i.e. } m > 1)$$

$$\text{for } \Theta = 135°: m = 0.2771 \text{ (i.e. } m < 1)$$

We use $m = 3.6$ to solve $(KD)_X$ and $(KD)_Y$ from Equation 8.5. The transmissivity in the major direction of anisotropy is $(KD)_X = 328$ m²/d, and that in the minor direction of anisotropy is $(KD)_Y = 91$ m²/d.
We determine the transmissivity in the direction of each observation well from Equation 8.4

$$(KD)_1 = \frac{328}{\{\cos^2(45° + 0°) + 3.6 \sin^2(45° + 0°)\}} = 143 \text{ m}^2/\text{d}$$

and calculate in the same way $(KD)_2 = 111$ m²/d and $(KD)_3 = 110$ m²/d.
Finally, we calculate the storativity of the anisotropic confined aquifer.

$$\frac{S}{(KD)_1} = \frac{S}{143} = 7.22 \times 10^{-7}.$$

Solved for S, the equation yields $S = 1 \times 10^{-4}$.

Table 8.1 Drawdown data from the Papadopulos pumping test (from Papadopulos 1965)

Time t since pumping started (minutes)	Drawdown s (metres)		
	OW-1	OW-2	OW-3
0.5	0.335	0.153	0.492
1	0.591	0.343	0.762
2	0.911	0.611	1.089
3	1.082	0.762	1.284
4	1.215	0.911	1.419
6	1.405	1.089	1.609
8	1.549	1.225	1.757
10	1.653	1.329	1.853
15	1.853	1.531	2.071
20	2.019	1.677	2.210
30	2.203	1.853	2.416
40	2.344	2.019	2.555
50	2.450	2.123	2.670
60	2.541	2.210	2.750
90	2.750	2.416	2.963
120	2.901	2.555	3.118
150	2.998	2.670	3.218
180	3.075	2.750	3.310
240	3.235	2.901	3.455
300	3.351	2.998	3.565
360	3.438	3.118	3.649
480	3.587	3.247	3.802
720	3.784	3.455	3.996

8.1.2 Hantush-Thomas's method

In an isotropic aquifer, the lines of equal drawdown around a pumped well form concentric circles, whereas in an aquifer that is anisotropic on the horizontal plane, those lines form ellipses, which satisfy the equation

$$\frac{x^2}{a_s^2} + \frac{y^2}{b_s^2} = 1 \tag{8.10}$$

where a_s and b_s are the lengths of the principal axes of the ellipse of equal drawdown s at the time t_s (Figure 8.1C).
It can be shown that

$$(KD)_n = (r_n^2/a_s b_s)(KD)_e \tag{8.11}$$

$$(KD)_X = (a_s/b_s)(KD)_e \tag{8.12}$$

$$(KD)_Y = (b_s/a_s)(KD)_e \tag{8.13}$$

$$\frac{4\pi s(KD)_e}{Q} = W(u_{XY}) \tag{8.14}$$

where

$$u_{XY} = \frac{r_n^2 S}{4(KD)_n t} = \frac{a_s b_s S}{4(KD)_e t_s} \tag{8.15}$$

Hantush and Thomas (1966) stated that when $(KD)_e$, a_s, and b_s are known the other hydraulic characteristics can be calculated. Hence, it is not necessary to have values of $S/(KD)_n$, provided that one has sufficient observations to draw the ellipses of equal drawdown.

The Hantush-Thomas method can be applied if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of Chapter 3, with the exception of the third assumption, which is replaced by:
  - The aquifer is homogeneous, anisotropic on the horizontal plane, and of uniform thickness over the area influenced by the pumping test.

The following condition is added:
- The flow to the well is in unsteady state.

Procedure 8.3
- Apply the methods for isotropic confined aquifers (Sections 3.1 and 3.2) to the data from each ray of piezometers; this yields values for $(KD)_e$ and sometimes $S/(KD)_n$. The factor $(KD)_e$ is constant for the whole flow system, and $S/(KD)_n$ is constant along each ray;
- Substitute the values of $(KD)_e$ and $S/(KD)_n$ into Equations 8.1 and 8.2 and calculate the drawdown at any desired time and at any distance along each ray of piezometers;
- Construct one or more ellipses of equal drawdown (Figure 8.1C), using observed (or calculated) data, and calculate for each ellipse a_s and b_s;
- Calculate $(KD)_n$, $(KD)_X$, and $(KD)_Y$ from Equations 8.11 to 8.13;

- Calculate the value of $W(u_{XY})$ from Equation 8.14 and find the corresponding value of u_{XY} from Annex 3.1;
  With the value of u_{XY} known, calculate S from Equation 8.15;
- Repeat this procedure for several values of s. This should produce approximately the same values for $(KD)_n$, $(KD)_X$, $(KD)_Y$, and S.

8.1.3 Neuman's extension of the Papadopulos method

In aquifers that are anisotropic on the horizontal plane, the orientation of the hydraulic-head gradients and the flow velocity seldom coincide; the flow tends to follow the direction of the highest permeability. This leads us to regard the hydraulic conductivity as a tensorial property, which is simply the mathematical translation of our observation of the non-coincidence. Regarding the hydraulic conductivity in this way, we must define the tensor K, which is a matrix of nine coefficients, symmetrical to the diagonal. This allows us to transform the components of the hydraulic gradient into components of velocity. Along the principal axes of such a tensor (X,Y), the velocity and hydraulic gradients have the same directions.

By making use of the tensor properties, Papadopulos (1965) developed an equation for the unsteady-state drawdown induced in a confined aquifer that is anisotropic on the horizontal plane

$$s = \frac{Q}{4\pi(KD)_e} W(u_{xy}) \tag{8.16}$$

where

$$(KD)_e = \sqrt{(KD)_{xx}(KD)_{yy} - (KD)^2_{xy}}$$

$$u_{xy} = \frac{S}{4t}\left(\frac{(KD)_{xx}y^2 + (KD)_{yy}x^2 - 2(KD)_{xy}xy}{(KD)_{xx}(KD)_{yy} - (KD)^2_{xy}}\right)$$

$$= \frac{S}{4t}\left(\frac{(KD)_{xx}y^2 + (KD)_{yy}x^2 - 2(KD)_{xy}xy}{(KD)^2_e}\right) \tag{8.17}$$

where x and y are local coordinates (Figure 8.4) and $(KD)_{xx}$, $(KD)_{yy}$, and $(KD)_{xy}$ are components of the transmissivity tensor.

For $u < 0.01$, Equation 8.16 reduces to

$$s = \frac{2.30Q}{4\pi(KD)_e} \log\frac{2.25\,t}{S}\left\{\frac{(KD)_{xx}(KD)_{yy} - (KD)^2_{xy}}{(KD)_{xx}y^2 + (KD)_{xx}x^2 - 2(KD)_{xy}xy}\right\} \tag{8.18}$$

The following relations between the principal transmissivity and the transmissivity tensors hold

$$(KD)_X = \frac{1}{2}\{(KD)_{xx} + (KD)_{yy} + \sqrt{[(KD)_{xx} - (KD)_{yy}]^2 + 4(KD)^2_{xy}}\} \tag{8.19}$$

$$(KD)_Y = \frac{1}{2}\{(KD)_{xx} + (KD)_{yy} - \sqrt{[(KD)_{xx} - (KD)_{yy}]^2 + 4(KD)^2_{xy}}\} \tag{8.20}$$

where X and Y are global coordinates of the transmissivity tensor (Figure 8.4).

The X axis is parallel to the major direction of anisotropy; the Y axis is parallel

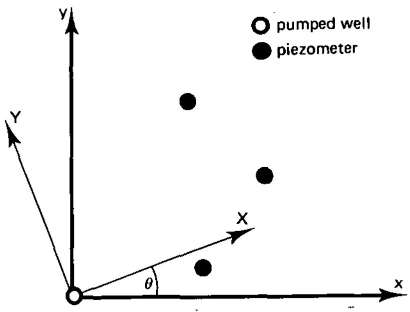

Figure 8.4 Relationship between the global coordinates (X and Y) and the local coordinates (x and y)

to the minor direction. The orientation of the X and Y axes is given by

$$\Theta = \arctan\frac{(KD)_X - (KD)_{xx}}{(KD)_{xy}} \tag{8.21}$$

where Θ is the angle between the x and the X axis ($0 \leq Z\ \Theta < \pi$). The angle of Θ is positive to the left of the axis.

If the principal directions of anisotropy are known, Equations 8.16 and 8.17 reduce to

$$s = \frac{Q}{4\pi\sqrt{(KD)_X(KD)_Y}} W(u_{XY}) \tag{8.22}$$

$$u_{XY} = \frac{S}{4t}\left(\frac{(KD)_X Y^2 + (KD)_Y X^2}{(KD)_X(KD)_Y}\right) \tag{8.23}$$

Taking the above equations as his basis, Papadopulos (1965) developed a method of determining the principal directions of anisotropy and the corresponding minimum and maximum transmissivities. This method requires drawdown data from at least three wells, other than the pumped well, all three located on different rays from the pumped well.

Neuman et al. (1984) showed that the Papadopulos method can be used with drawdown data from only three wells, provided that two pumping tests are conducted in sequence in two of those wells. When water is pumped from Well 1 at a constant rate Q_1, two sets of drawdown data, s_{12} and s_{13}, are available from Wells 2 and 3 (Figure 8.5). This is not sufficient to allow the use of the Papadopulos equations. But, if at least one other pumping test is conducted, say in Well 2, at a constant rate Q_2, and the resulting drawdown is observed at least in Well 3, these drawdown data, s_{23}, provide the third set of data needed to complete the analysis. Equation 8.17 as used in the Papadopulos method can now be replaced by

$$u_{12} = \frac{S}{4t_{12}(KD)_e^2}\left[(KD)_{xx}y_{12}^2 + (KD)_{yy}x_{12}^2 - 2(KD)_{xy}x_{12}y_{12}\right] \tag{8.24}$$

$$u_{13} = \frac{S}{4t_{13}(KD)_e^2}\left[(KD)_{xx}y_{13}^2 + (KD)_{yy}x_{13}^2 - 2(KD)_{xy}x_{13}y_{13}\right] \tag{8.25}$$

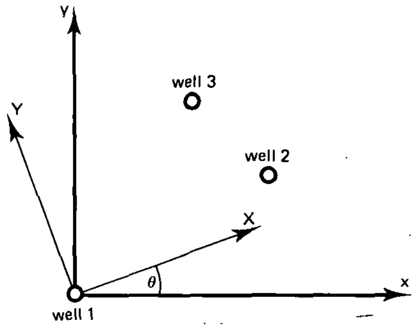

Figure 8.5 The three-well arrangement used in Neuman's extension of the Papadopulos method

$$u_{23} = \frac{S}{4t_{23}(KD)_e^2}\left[(KD)_{xx}y_{23}^2 + (KD)_{yy}x_{23}^2 - 2(KD)_{xy}x_{23}y_{23}\right] \qquad (8.26)$$

Neuman's three-well method is applicable if the following assumptions and conditions are fulfilled:

- The assumptions listed at the beginning of Chapter 3, with the exception of the third assumption, which is replaced by:
  - The aquifer is homogeneous, anisotropic on the horizontal plane, and of uniform thickness over the area influenced by the pumping test.

The following conditions are added:

- The flow to the well is in an unsteady state;
- The aquifer is penetrated by three wells, which are not on one ray. Two of them are pumped in sequence.

Procedure 8.4

- Apply one of the methods for confined isotropic aquifers (Section 3.2) to the drawdown data from each well, using Equations 8.16, 8.24, 8.25, and 8.26. This results in values for $(KD)_e$, $S(KD)_{xx}$, $S(KD)_{yy}$, and $S(KD)_{xy}$;
- Knowing $(KD)_e$, $S(KD)_{xx}$, $S(KD)_{yy}$, and $S(KD)_{xy}$, calculate S from $S = \sqrt{S(KD)_{xx}S(KD)_{yy} - \{S(KD)_{xy}\}^2/(KD)_e}$
- Knowing S, $S(KD)_{xx}$, $S(KD)_{yy}$, and $S(KD)_{xy}$, calculate $(KD)_{xx}$, $(KD)_{yy}$, and $(KD)_{xy}$;
- Calculate $(KD)_X$ by substituting the known values of $(KD)_{xx}$, $(KD)_{yy}$, and $(KD)_{xy}$ into Equation 8.19;
- Calculate $(KD)_Y$ by substituting the known values of $(KD)_{xx}$, $(KD)_{yy}$, and $(KD)_{xy}$ into Equation 8.20;
- Determine the angle Θ by substituting the known values of $(KD)_X$, $(KD)_{xx}$, and $(KD)_{xy}$ into Equation 8.21.

Remarks

- The drawdown induced by the pumping test in Well 2 should be observed in Well 3 and not in the previously pumped Well 1, because s_{21} will be proportional to s_{12} under ideal conditions. Hence Equation 8.26 will not be linearly independent of

Equation 8.24 and no unique solutions can be found for the Equations 8.24, 8.25, and 8.26;

- According to Neuman et al. (1984), more reliable results can be obtained by conducting three pumping tests, pumping one well at a time and observing the drawdown in the other two wells. Equation 8.17 should then be replaced in the calculations by up to six equations of the form

$$u_{ij} = \frac{S}{4t_{ij}(KD)_e^2}\{(KD)_{xx}y_{ij}^2 + (KD)_{yy}x_{ij}^2 - 2(KD)_{xy}x_{ij}y_{ij}\}$$

  where $i, j = 1, 2, 3$.

  A least-squares procedure can be used to solve these equations and determine $S(KD)_{xx}$, $S(KD)_{yy}$, and $S(KD)_{xy}$. (For more information, see Neuman et al. 1984);
- If drawdown data are available from at least three piezometers or observation wells on different rays from the pumped well, the Papadopulos method can be used. The procedure is the same as Procedure 8.4, except that in the first step of Procedure 8.4, Equation 8.18 should be used instead of Equations 8.24, 8.25, and 8.26 to determine the values of $S(KD)_{xx}$, $S(KD)_{yy}$, and $S(KD)_{xy}$.

Example 8.2

We shall use the data from the Papadopulos pumping test (Example 8.1, Table 8.1, Figures 8.2 and 8.3) to illustrate the Papadopulos method, Procedure 8.4.

From Example 8.1 we know the value of the effective transmissivity: $(KD)_e = 173$ m²/d. Figure 8.3 shows the semi-log plot of the drawdown data for each observation well. The three straight lines through the plotted points intercept the t axis at $t_{01} = 0.37$ min., $t_{02} = 0.72$ min., and $t_{03} = 0.24$ min. These straight lines are described by Equation 8.18. For $s = 0$, Equation 8.18 reduces to

$$t_0 = \frac{S}{2.25}\left\{\frac{(KD)_{xx}y^2 + (KD)_{yy}x^2 - 2(KD)_{xy}xy}{(KD)_{xx}\,(KD)_{yy} - (KD)_{xy}^2}\right\}$$

$$= \frac{S}{2.25}\left\{\frac{(KD)_{xx}y^2 + (KD)_{yy}x^2 - 2(KD)_{xy}xy}{(KD)_e^2}\right\}$$

Hence, $2.25\,(KD)_e^2 \times t_0 = S(KD)_{xx}y^2 + S(KD)_{yy}x^2 - 2\,S(KD)_{xy}xy$.

Using this expression, we can determine $S(KD)_{xx}$, $S(KD)_{yy}$, and $S(KD)_{xy}$.

For observation well OW-1:

$$2.25 \times (KD)_e^2 \times t_{01} = 2.25 \times 173^2 \times \frac{0.37}{1440} = S(KD)_{xx} \times 0 + S(KD)_{yy} \times 28.3^2 - 2S(KD)_{xy} \times 0$$

For observation well OW-2:

$$2.25\,(KD)_e^2 \times t_{02} = 2.25 \times 173^2 \times \frac{0.72}{1440} = S(KD)_{xx} \times 33.5^2 + S(KD)_{yy} \times 9^2 - 2\,S(KD)_{xy} \times 33.5 \times 9$$

For observation well OW-3:

$$2.25\,(KD)_e^2 \times t_{03} = 2.25 \times 173^2 \times \frac{0.24}{1440}$$

$$= S(KD)_{xx} \times 5.2^2 + S(KD)_{yy} \times 19.3^2 - 2\,S(KD)_{xy} \times 19.3 \times 5.2$$

Solving these three equations gives

$$S(KD)_{xx} = 0.0215\ m^2/d$$
$$S(KD)_{yy} = 0.0216\ m^2/d$$
$$S(KD)_{xy} = -0.0219\ m^2/d$$

Substituting these values together with the value of $(KD)_e$ into

$$S = \frac{\sqrt{S(KD)_{xx}\,S(KD)_{yy} - \{S(KD)_{xy}\}^2}}{(KD)_e} \text{ yields } S = 1 \times 10^{-4}$$

The values of $(KD)_{xx}$, $(KD)_{yy}$, and $(KD)_{xy}$ can now be calculated

$$(KD)_{xx} = 215\ m^2/d$$
$$(KD)_{yy} = 216\ m^2/d$$
$$(KD)_{xy} = -129\ m^2/d$$

The transmissivity $(KD)_X$ in the principal direction of anisotropy is calculated from Equation 8.19

$$(KD)_X = \frac{1}{2}\{215 + 216 + \sqrt{(215-216)^2 + 4(-129)^2}\} = 345\ m^2/d$$

The transmissivity $(KD)_Y$ in the minor direction of anisotropy is calculated from Equation 8.20

$$(KD)_Y = \frac{1}{2}\{215 + 216 - \sqrt{(215-216)^2 + 4(-129)^2}\} = 86\ m^2/d$$

The orientation of the X and Y axes is determined from Equation 8.21

$$\Theta = \arctan\left\{\frac{(KD)_X - (KD)_{xx}}{(KD)_{xy}}\right\} = \arctan\left\{\frac{345-215}{-129}\right\} = \arctan(-1) = 135°$$

The X axis is 135° to the left of the x axis (or 45° to the right of the x axis, see Example 8.1).

8.2 Leaky aquifers, anisotropic on the horizontal plane

8.2.1 Hantush's method

The flow to a well in a leaky aquifer which is anisotropic on the horizontal plane can be analyzed with a method that is essentially the same as the Hantush method for confined aquifers with anisotropy on the horizontal plane. There is, however, one more unknown parameter involved, the leakage factor L, which is given by Hantush (1966) as

$$L_n = \sqrt{(KD)_n c} \tag{8.27}$$

Because c is a constant, Equation 8.7 also gives the relationship between L_n and L_1

$$a_n = \frac{(KD)_l}{(KD)_n} = \left[\frac{L_l}{L_n}\right]^2 = \frac{\cos^2(\Theta + \alpha_n) + m \sin^2(\Theta + \alpha_n)}{\cos^2\Theta + m \sin^2\Theta} \qquad (8.28)$$

The Hantush method can be applied if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and third assumptions, which are replaced by:
  - The aquifer is leaky;
  - The aquifer is homogeneous, anisotropic on the horizontal plane, and of uniform thickness over the area influenced by the pumping test.

The following condition is added:

- The flow to the well is in an unsteady state.

Procedure 8.5

This procedure is the same as Procedures 8.1 and 8.2 (the Hantush method for confined aquifers with anisotropy on the horizontal plane), except that, in the first step of Procedure 8.5, the methods for leaky isotropic aquifers (Section 4.2) are used to determine values for $(KD)_e$, $S/(KD)_n$, and L_n. Further, Equation 8.28 is used instead of Equation 8.7.

8.3 Confined aquifers, anisotropic on the vertical plane

The flow towards a well that completely penetrates a confined, horizontally stratified aquifer takes place essentially in planes parallel to the aquifer's bedding planes. Even if the hydraulic conductivities vary appreciably in horizontal and vertical directions, the effect of any anisotropy on the vertical plane may not be of any great significance.

In thick aquifers, however, wells usually penetrate only a portion of the aquifer. The flow to such partially penetrating wells is not horizontal, but three-dimensional, i.e. the flow has significant vertical components, at least in the vicinity of the well, where most observations of the drawdown are made. In aquifers with very pronounced anisotropy on the vertical plane, the yield of partially penetrating wells may be appreciably smaller than that of similar wells in isotropic aquifers.

8.3.1 Weeks's method

For large values of pumping time ($t > DS/2K_v$) in a well that partially penetrates a confined aquifer, Hantush (1961a) developed a solution for the drawdown. After modification for the influence of anisotropy on the vertical plane, this equation becomes (Hantush 1964; Weeks 1969)

$$s = \frac{Q}{4\pi KD}\left\{W(u) + f_s\left(\beta', \frac{b}{D}, \frac{d}{D}, \frac{a}{D}\right)\right\} = \frac{Q}{4\pi KD} W(u) + \delta s \qquad (8.29)$$

where

$W(u)$ = Theis well function

b, d, a = geometric parameters (Figure 8.6)

$$\beta' = \frac{r}{D}\sqrt{K_v/K_h} \tag{8.30}$$

K_v = hydraulic conductivity in vertical direction

K_h = hydraulic conductivity in horizontal direction

$$f_s = \frac{4D}{\pi(b-d)} \sum_{n=1}^{\infty} \frac{1}{n} K_0(n\pi\beta') \left\{\cos \frac{n\pi a}{D}\right\} \left\{\sin \frac{n\pi b}{D} - \sin \frac{n\pi d}{D}\right\} \tag{8.31}$$

δs = difference in drawdown between the observed drawdowns and the drawdowns predicted by the Theis equation (Equation 3.5). This difference in drawdown is given by

$$\partial s = \frac{Q}{4\pi KD} f_s \tag{8.32}$$

Values of f_s for different values of β', b/D, d/D, and a/D as tabulated by Weeks (1969) are presented in Annex 8.1.

The assumptions and conditions underlying the Weeks method are:

- The assumptions listed at the beginning of Chapter 3, with the exception of the third and sixth assumptions, which are replaced by:
  - The aquifer is homogeneous, anisotropic in the vertical plane, and of uniform thickness over the area influenced by the pumping test;
  - The pumped well does not penetrate the entire thickness of the aquifer.

The following conditions are added:

- The flow to the well is in an unsteady state;
- $t > SD/2K_v$;
- Drawdown data from at least two piezometers are available; one piezometer at a distance $r > 2D\sqrt{K_h/K_v}$.

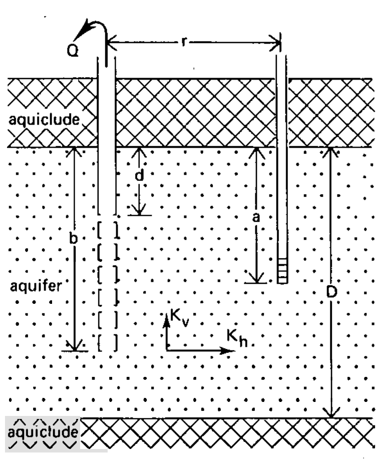

Figure 8.6 The parameters used in Weeks's method

Procedure 8.6

- Apply one of the methods for confined, fully penetrated, isotropic aquifers (Section 3.2) to the observed drawdown data of Piezometer 1 at $r > 2D\sqrt{K_h/K_v}$, and determine the values of K_hD and S;
- For Piezometer 2 at $r < 2D\sqrt{K_h/K_v}$, plot the observed drawdown s versus t on semi-log paper (t on logarithmic scale). Draw a straight line through the late-time data;
- Knowing Q, K_hD, S, and r, calculate, for different values of t, the values of s that would have occurred in Piezometer 2 if the pumped well had been fully penetrating; use Equation 3.5, $s = \frac{Q}{4\pi KD} W(u)$, and Annex 3.1;
- Plot these calculated values of s versus t on the same sheet of semi-log paper as used for the observed time-drawdown plot. Draw a straight line through the late-time data. The straight lines of the two data plots should be parallel;
- Determine the vertical distance δs between the two straight lines;
- Knowing δs, Q, and K_hD, calculate f_s from Equation 8.32;
- Knowing f_s, use Annex 8.1 to determine the value of β' for the values of b/D, d/D, and a/D nearest to the observed values for Piezometer 2;
- Knowing β' and r/D for Piezometer 2, calculate K_v/K_h from Equation 8.30;
- Knowing K_v/K_h, K_hD, and D, calculate K_h and K_v.

Remarks

- Instead of determining K_hD and S with data from a piezometer at $r > 2D\sqrt{K_h/K_v}$ from the partially penetrating well, one can, of course, also obtain these values from the data of a separate pumping test conducted in the same aquifer with a fully penetrating well;
- Whether ∂s will have a positive or a negative value depends on the location of Piezometer 2 relative to that of the screen of the partially penetrating well. When both are located at the same depth in the aquifer, the observed drawdown in Piezometer 2 will be greater than the theoretical drawdown for a fully penetrating well and consequently, ∂s will have a positive value.

8.4 Leaky aquifers, anisotropic on the vertical plane

8.4.1 Weeks's method

For large values of pumping time ($t > DS/2K_v$) in a well that partially penetrates a leaky aquifer with anisotropy on the vertical plane, the drawdown response is given by (Hantush 1964; Weeks 1969)

$$s = \frac{Q}{4\pi K_hD}\left\{W(u,r/L) + f_s\left(\beta',\frac{b}{D},\frac{d}{D},\frac{a}{D}\right)\right\} = \frac{Q}{4\pi K_hD} W(u,r/L) + \delta s \qquad (8.33)$$

where

$W(u,r/L)$ = Walton's well function

f_s, β', b, d, a, and δs are as defined in Section 8.3.1.
A procedure similar to Procedure 8.6 can be applied to leaky aquifers.

The following assumptions and conditions should be satisfied:
- The assumptions listed at the beginning of Chapter 3, with the exception of the first, third, and sixth assumptions, which are replaced by:
  - The aquifer is leaky;
  - The aquifer is homogeneous, anisotropic on the vertical plane, and of uniform thickness over the area influenced by the pumping test;
  - The pumped well does not penetrate the entire thickness of the aquifer.

The following conditions are added:
- The aquitard is incompressible;
- The flow to the well is in unsteady state;
- $t > SD/2K_v$;
- Drawdown data from at least two piezometers are available; one piezometer at a distance $r > 2D\sqrt{K_h/K_v}$.

Procedure 8.7
- Apply one of the methods for leaky, fully penetrated, isotropic aquifers (Sections 4.2.1, 4.2.2, or 4.2.3) to the observed drawdown data of Piezometer 1 at $r > 2D\sqrt{K_h/K_v}$, and determine the values of K_hD, S, and L;
- For Piezometer 2 at $r < 2D\sqrt{K_h/K_v}$, plot the observed drawdown s versus t on log-log paper;
- Knowing Q, K_hD, S, L, and r, calculate for different values of t the values of s that would have occurred in Piezometer 2 if the pumped well had been fully penetrating; use Equation 4.6

  $$s = \frac{Q}{4\pi K_h D} W(u, r/L)$$

  and Annex 4.2;
- Plot these calculated values of s versus t on the same sheet of log-log paper as used for the observed time-drawdown plot. The late-time parts of the data curves should be parallel;
- Determine the vertical distance δs between the late-time parallel parts of the data curves;
- Knowing δs, Q, and K_hD, calculate f_s from Equation 8.32;
- Knowing f_s, use Annex 8.1 to determine the value of β' for the values of b/D, d/D and a/D nearest to the observed values for Piezometer 2;
- Knowing β' and r/D for Piezometer 2, calculate K_v/K_h from Equation 8.30;
- Knowing K_v/K_h, K_hD, and D, calculate K_h and K_v.

8.5 Unconfined aquifers, anisotropic on the vertical plane

The flow to a well that pumps an unconfined aquifer is considered to be three-dimensional during the time that the delayed watertable response prevails (see Chapter 5). As three-dimensional flow is affected by anisotropy on the vertical plane, one of the standard methods for unconfined aquifers already takes this anisotropy into account: Neuman's curve-fitting method (Section 5.1.1).

Apart from that standard method, there are other methods that take anisotropy on the vertical plane into account. They can be used when the well is partially penetrating. They are Streltsova's curve-fitting method (Section 10.4.1), Neuman's curve-fitting method (Section 10.4.2), and Boulton-Streltsova's curve-fitting method (Section 11.2.1).