7 Wedge-shaped and sloping aquifers

The standard methods of analysis are all based on the assumption that the thickness of the aquifer is constant over the area influenced by the pumping test. In wedge-shaped aquifers this assumption is not fullfilled and other methods of analysis should be used (Section 7.1). Standard methods also assume a horizontal watertable prior to a test. In some cases the watertable in unconfined aquifers is sloping and these methods cannot be used. Sections 7.2 and 7.3 present methods of analysis for unconfined aquifers with a sloping watertable.

7.1 Wedge-shaped confined aquifers, unsteady-state flow

7.1.1 Hantush's method

According to Hantush (1962), if the thickness of a confined aquifer varies exponentially in the flow direction (x-direction) while remaining constant in the y-direction (Figure 7.1), the drawdown equation for unsteady-state flow takes the form

$$s = \left[\frac{Q}{4\pi KD_w} \exp\left(\frac{r}{a}\cos\Theta\right)\right] W\left(u, \left|\frac{r}{a}\right|\right) \tag{7.1}$$

where

D_w = thickness of the aquifer at the location of the well
Θ = the angle between the x-direction and a line through the well and a piezometer, in radians
a = constant defining the exponential variation of the aquifer thickness
u = $\dfrac{r^2S}{4KD_wt}$

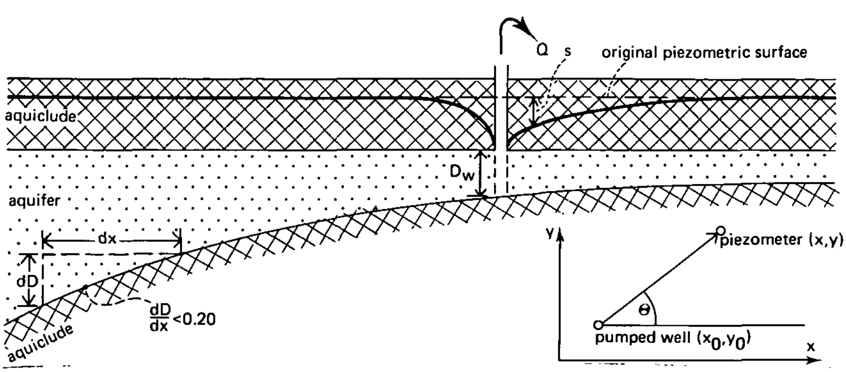

Figure 7.1 Cross-section and plan view of a pumped wedge-shaped confined aquifer

This equation has the same form as Equation 4.6, which describes the drawdown for unsteady state in a leaky aquifer of constant thickness. So, to determine the values of KD_w, S, and a of a wedge-shaped confined aquifer, we can use a method analogous to the Hantush inflection-point method for leaky aquifers of constant thickness (Procedure 4.4) (Hantush 1964).

At the inflection point P of the time-drawdown curve for a pumped confined aquifer of non-uniform thickness, Equations 4.8, 4.9, 4.10, and 4.12 become

$$s_p = \frac{1}{2} s_m = \left[\frac{Q}{4\pi KD_w} \exp\left(\frac{r}{a}\cos\Theta\right)\right] K_0\left(\left|\frac{r}{a}\right|\right) \quad (7.2)$$

$$u_p = \frac{r^2 S}{4KD_w t_p} = \frac{r}{2a} \quad (7.3)$$

The slope of the curve at the inflection point is

$$\Delta s_p = \left[\frac{2.30Q}{4\pi KD_w} \exp\left(\frac{r}{a}\cos\Theta\right)\right] e^{-r/a} \quad (7.4)$$

The relation between the drawdown and the slope of the curve is

$$2.30 \frac{s_p}{\Delta s_p} = e^{r/a} K_0\left(\left|\frac{r}{a}\right|\right) \quad (7.5)$$

Hantush's inflection-point method (Procedure 4.4) can be applied if the following assumptions and conditions are fulfilled:

- The assumptions listed at the beginning of Chapter 3, with the exception of the third assumption, which is replaced by:
  - The aquifer is homogeneous and isotropic over the area influenced by the pumping test;
  - The thickness of the aquifer varies exponentially in the direction of flow;
  - $\frac{dD}{dx} < 0.20$, i.e. $t < \frac{r_o^2 S}{20KD_w}$ with $r_o = \frac{a}{2}\ln\left(\frac{a}{10D_w}\right)$.

The following condition is added:

- The flow to the well is in an unsteady state, but the steady-state drawdown should be approximately known.

Procedure 7.1

- For one of the piezometers, plot s versus t on semi-log paper (t on the logarithmic scale) and draw the curve that fits best through the plotted points;
- Determine the value of s_m by extrapolation;
- Calculate s_p from Equation 7.2. The value of s_p on the curve locates the inflection point P;
- From the time axis, read the value of t_p at the inflection point;
- Determine the slope Δs_p of the curve at the inflection point by reading the drawdown difference per log cycle of time over the tangent to the curve at the inflection point;
- Substitute the values of s_p and Δs_p into Equation 7.5 and find r/a by interpolation from the table of the function $e^x K_o(x)$ in Annex 4.1;
- Knowing r/a and r, calculate a;

- Knowing Q, s_p, Δs_p, r/a, and cos θ, and using Annex 4.1, calculate KD_w from Equation 7.4 or Equation 7.2;
- Knowing KD_w, t_p, r, and r/a, calculate S from Equation 7.3.

Remarks
- To check whether the time condition is fulfilled, calculate the value of $(r_o^2S)/20KD_w$;
- If the well and all the piezometers are located on a single straight line, i.e. θ is the same for all piezometers, we can use a method analogous to the Hantush inflection-point method for leaky aquifers (Procedure 4.5).

7.2 Sloping unconfined aquifers, steady-state flow

7.2.1 Culmination-point method

If an unconfined aquifer with a constant saturated thickness slopes uniformly in the direction of flow (x-axis) (Figure 7.2), the slope of the watertable i is equal to the slope of the impermeable base α and the flow rate per unit width is

$$q = \frac{Q}{F} = KD\alpha \tag{7.6}$$

or

$$\alpha = \frac{q}{KD}$$

When such an aquifer is pumped at a constant discharge Q, the slope of the cone of depression along the x-axis downstream of the well is given for steady-state flow as

$$-\frac{dh}{dx} = \frac{Q}{2\pi rKD} \tag{7.7}$$

On the x-axis, there is a point where the slopes α and dh/dx are numerically the same but have opposite signs; hence the combined slope is zero. In this culmination point of the depression cone, which lies on the x-axis, the distance to the well r is designated by x_o. Consequently, a combination of Equations 7.6 and 7.7 (Huisman 1972) yields

$$\alpha = \frac{Q}{2\pi KDx_o} \tag{7.8}$$

The width of the zone from which the water is derived is $F = 2\pi x_o$.

The transmissivity can be calculated if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of Chapter 3, with the exception of the first and fourth assumptions, which are replaced by:
  - The aquifer is unconfined;
  - Prior to pumping, the watertable slopes in the direction of flow.

The following condition is added:
- The flow to the well is in steady state.

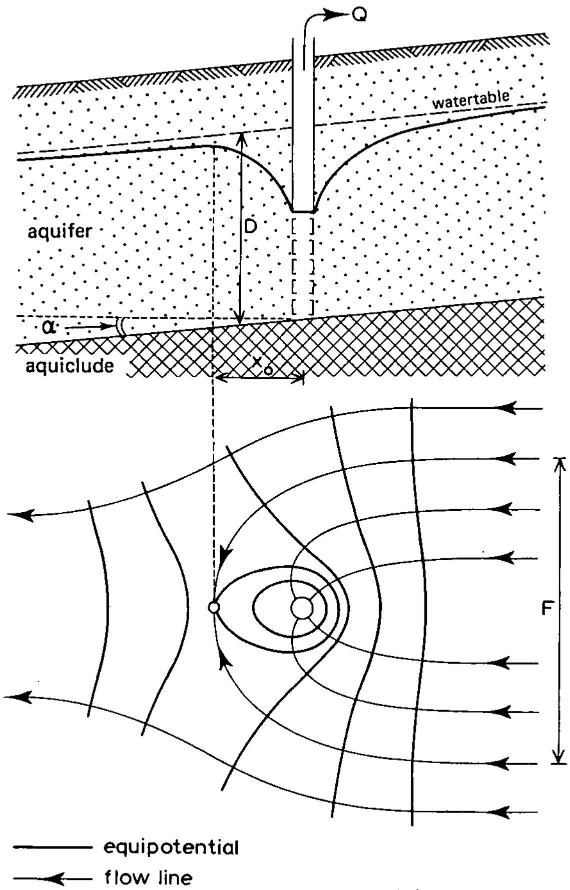

Figure 7.2 Cross-section and plan view of a pumped sloping unconfined aquifer

Procedure 7.2

- Instead of plotting the drawdown, plot the water-level elevations with reference to a horizontal datum plane versus r on arithmetic paper;
- Determine the distance x_0 from the well to the point where the slope of the depression cone is zero;
- Introduce the values of Q, α, and x_0 into Equation 7.8 and calculate KD.

7.3 Sloping unconfined aquifers, unsteady-state flow

7.3.1 Hantush's method

According to Hantush (1964), the unsteady-state drawdown in a sloping unconfined aquifer of constant thickness (Figure 7.2) is

$$s' = s - \frac{s^2}{2D} = \left\{\frac{Q}{4\pi KD} \exp\left(-\frac{r}{\gamma}\cos\theta\right)\right\} W\left(u,\frac{r}{\gamma}\right) \qquad (7.9)$$

where

s' = corrected drawdown

s = observed drawdown

θ = the angle between the line through the well and a piezometer, and the direction of flow, in radians

$\gamma = \frac{2D}{i}$

$u = \frac{r^2S}{4KDt}$

i = slope of the watertable

This equation has the same form as Equation 4.6, which describes the drawdown for unsteady state in a leaky horizontal aquifer of constant thickness.
According to Hantush (1964), Equation 7.9 can be written alternatively as

$$s - \frac{s^2}{2D} = \left[\frac{Q}{4\pi KD} \exp\left(-\frac{r}{\gamma}\cos\theta\right)\right] \left[2K_o\left(\frac{r}{\gamma}\right) - W\left(q,\frac{r}{\gamma}\right)\right] \qquad (7.10)$$

where

$$q = \frac{r^2}{4\gamma^2}\frac{1}{u} = \frac{KDt}{S\gamma^2} \qquad (7.11)$$

If $q > 2\frac{r}{\gamma}$, Equation 7.10 can be approximated by

$$s'_m - s' = \frac{Q}{4\pi KD} \exp\left(-\frac{r}{\gamma}\cos\theta\right) W(q) \qquad (7.12)$$

where

$$s'_m = s_m - \frac{{s_m}^2}{2D} = \frac{Q}{2\pi KD} \exp\left(-\frac{r}{\gamma}\cos\theta\right) K_o\left(\frac{r}{\gamma}\right) \qquad (7.13)$$

s'_m = corrected maximum or steady-state drawdown

If s'_m in a piezometer at distance r from the well can be extrapolated from a plot of s' versus t on semi-log paper (t on logarithmic scale), the drawdown at the inflection point P can be calculated ($s'_p = 0.5\ s'_m$) and t_p (the time corresponding to s'_p) can be read from the graph.

If a sufficient number of data fall within the period $t > 4t_p$, the Hantush method can be used, provided that the following assumptions and conditions are also satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and fourth assumptions, which are replaced by:
  - The aquifer is unconfined;
  - Prior to pumping, the watertable slopes in the direction of flow with a hydraulic gradient $i < 0.20$.

The following conditions are added:

- The flow to the well is in unsteady state;

$$q > 2\frac{r}{\gamma}$$

$$t > 4t_p.$$

Procedure 7.3

- For one of the piezometers, plot s' versus t on semi-log paper (t on logarithmic scale) and find the maximum drawdown s'_m by extrapolation;
- Using Annex 3.1, prepare a type curve by plotting W(q) versus q on log-log paper. This curve is identical with a plot of W(u) versus u;
- On another sheet of log-log paper of the same scale, plot the observed data curve $(s'_m - s')$ versus t. Obviously, one can only use the data of one piezometer at a time because, although q is independent of r, this is not so with $(Q/4\pi KD) \exp \left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$;
- Match the observed data curve with the type curve. It will be seen that the observed data in the period $t < 4t_p$ fall below the type curve because, in this period, Equation 7.12 does not apply;
- Choose a match point A on the superimposed sheets and note for A the values of $(s'_m - s')$, t, q, and W(q);
- Substitute the values of $(s'_m - s')$ and W(q) into Equation 7.12 and calculate $(Q/4\pi KD) \exp \left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$;
- Multiply this value by 2, which gives $\frac{Q}{2\pi KD} \exp \left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$. Substitute this value and that of s'_m into Equation 7.13, which gives a value of $K_o\left(\frac{r}{\gamma}\right)$. The value of $\frac{r}{\gamma}$ can be found from Annex 4.1 and, because r is known, γ can be calculated. With the values of $\frac{r}{\gamma}$ and θ known, $\left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$ can be found, and $\exp \left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$ can be obtained from Annex 4.1;
- Substitute the values of $\exp \left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$, Q, and D into $\frac{Q}{2\pi KD} \exp \left[-\left(\frac{r}{\gamma}\right)\cos\theta\right]$ and calculate K;
- Substitute the values t and q of point A and those of KD and γ into Equation 7.11 and calculate S;
- Repeat this procedure for all available piezometers.

Remarks

- When delayed watertable response phenomena are apparent (Chapter 5), the condition 'The water removed from storage is discharged instantaneously with decline of head' is not met and this Hantush method is not applicable;
- Because of the analogy between Equations 4.6 and 7.9, we can also use a method analogous to the Hantush method for horizontal leaky aquifers of constant thick-

ness (Procedure 4.4). If the well and all the piezometers are located on a single straight line, i.e. θ is the same for all piezometers, we can use a method analogous to the Hantush method for leaky aquifers (Procedure 4.5).