6 Bounded aquifers

Pumping tests sometimes have to be performed near the boundary of an aquifer. A boundary may be either a recharging boundary (e.g. a river or a canal) or a barrier boundary (e.g. an impermeable valley wall). When an aquifer boundary is located within the area influenced by a pumping test, the general assumption that the aquifer is of infinite areal extent is no longer valid.

Presented in Sections 6.1 and 6.2 are methods of analysis developed for confined or unconfined aquifers with various boundaries and boundary configurations. Section 6.3 presents a method for leaky or confined aquifers bounded laterally by two parallel barrier boundaries.

To analyze the flow in bounded aquifers, we apply the principle of superposition. According to this principle, the drawdown caused by two or more wells is the sum of the drawdown caused by each separate well. So, by introducing imaginary wells, or image wells, we can transform an aquifer of finite extent into one of seemingly infinite extent, which allows us to use the methods presented in earlier chapters.

Figure 6.1A shows a fully penetrating straight canal which forms a recharging boundary with an assumed constant head. In Figure 6.1B, we replace this bounded system with an equivalent system, i.e. an imaginary system of infinite areal extent. In this system, there are two wells: the real discharging well on the left and an image recharging well on the right. The image well recharges the aquifer at a constant rate Q equal to the constant discharge of the real well. Both the real well and the image well are located on a line normal to the boundary and are equidistant from the boundary (Figure 6.1C). If we now sum the cone of depression from the real well and the cone of impression from the image well, we obtain an imaginary zero drawdown in the infinite system at the real constant-head boundary of the real bounded system.

Figure 6.1D shows a system with a straight impermeable valley wall which forms a barrier boundary. Figure 6.1E shows the real bounded system replaced by an equivalent system of infinite areal extent. The imaginary system has two wells discharging at the same constant rate: the real well on the left and an image well on the right. The image well induces a hydraulic gradient from the boundary towards the image well, which is equal to the hydraulic gradient from the boundary towards the real well. A groundwater divide thus exists at the boundary and there is no flow across the boundary. The resultant real cone of depression is the algebraic sum of the depression cones of both the real and the image well. Note that between the real well and the boundary, the real depression cone is flatter than it would be if no boundary were present, and is steeper on the opposite side away from the boundary.

If there is more than one boundary, more image wells are needed. For instance, if two boundaries are at right angles to each other, the imaginary system includes two primary image wells, both reflections of the real well, and one secondary image well, which is a reflection of the primary image wells. If the boundaries are parallel to one another, the number of image wells is theoretically infinite, but in practice it is only necessary to add pairs of image wells until the next pair would have a negligible

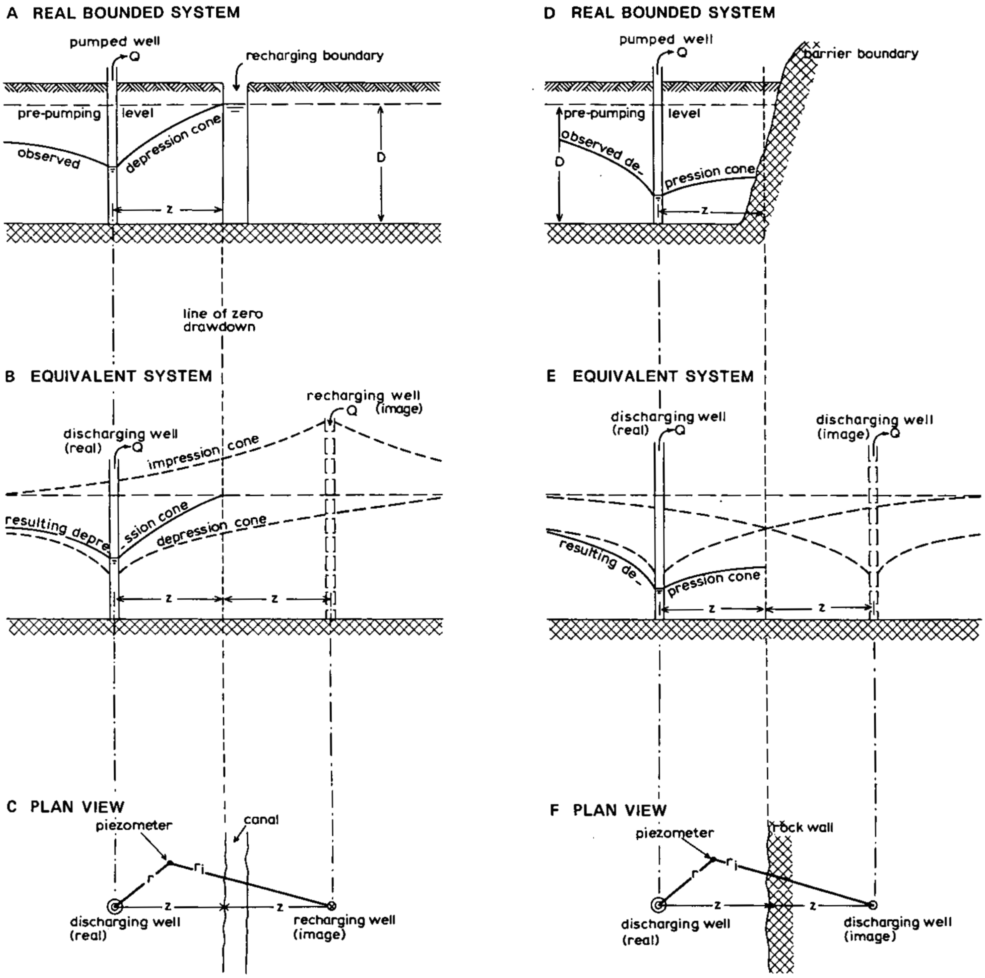

Figure 6.1 Drawdowns in the watertable of an aquifer bounded by:
A) A recharging boundary;
D) A barrier boundary.
B) and E) Equivalent systems of infinite areal extent.
C) and F) Plan views

influence on the sum of all image-well effects. Some of these boundary configurations will be discussed below.

6.1 Bounded confined or unconfined aquifers, steady-state flow

6.1.1 Dietz's method, one or more recharge boundaries

Dietz (1943) published a method of analyzing tests conducted in the vicinity of straight recharge boundaries under conditions of steady-state flow. Dietz's method, which is based on the work of Muskat (1937), uses Green's functions to describe the influence of the boundaries: in a piezometer with coordinates x_l and y_l, the steady-state drawdown caused by a well with coordinates x_w and y_w is given by

$$s_m = \frac{Q}{2\pi KD} G(x,y) \tag{6.1}$$

where $G(x,y)$ = Green's function for a certain boundary configuration.

For one straight recharge boundary (Figure 6.2A), the function reads

$$G(x,y) = \frac{1}{2} \ln \frac{(x_l + x_w)^2 + (y_l - y_w)^2}{(x_l - x_w)^2 + (y_l - y_w)^2} \tag{6.2}$$

For two straight recharge boundaries at right angles to each other (Figure 6.2B), the function reads

$$G(x,y) = \frac{1}{2} \ln \frac{[(x_l - x_w)^2 + (y_l + y_w)^2]\,[(x_l + x_w)^2 + (y_l - y_w)^2]}{[(x_l - x_w)^2 + (y_l - y_w)^2]\,[(x_l + x_w)^2 + (y_l + y_w)^2]} \tag{6.3.}$$

For two straight parallel recharge boundaries (Figure 6.2C), the function reads

$$G(x,y) = \frac{1}{2} \ln \frac{\cosh \frac{\pi(y_l - y_w)}{2a} + \cos \frac{\pi(x_l + x_w)}{2a}}{\cosh \frac{\pi(y_l - y_w)}{2a} - \cos \frac{\pi(x_l - x_w)}{2a}} \tag{6.4}$$

For a U-shaped recharge boundary (Figure 6.2D), the function reads

$$G(x,y) = \frac{1}{2} \ln \frac{\left[\cosh \frac{\pi(y_l - y_w)}{2a} + \cos \frac{\pi(x_l + x_w)}{2a}\right]}{\left[\cosh \frac{\pi(y_l - y_w)}{2a} - \cos \frac{\pi(x_l - x_w)}{2a}\right]}$$

$$\times \frac{\left[\cosh \frac{\pi(y_l + y_w)}{2a} - \cos \frac{\pi(x_l - x_w)}{2a}\right]}{\left[\cosh \frac{\pi(y_l + y_w)}{2a} + \cos \frac{\pi(x_l + x_w)}{2a}\right]} \tag{6.5}$$

The assumptions and conditions underlying the Dietz method are:
- The assumptions listed at the beginning of Chapter 3, except for the first and second assumptions, which are replaced by:
  - The aquifer is confined or unconfined;
  - Within the zone influenced by the pumping test, the aquifer is crossed by one or more straight, fully penetrating recharge boundaries with a constant water level;
  - The hydraulic contact between the recharge boundaries and the aquifer is as permeable as the aquifer.

The following condition is added:
- The flow to the well is in a steady state.

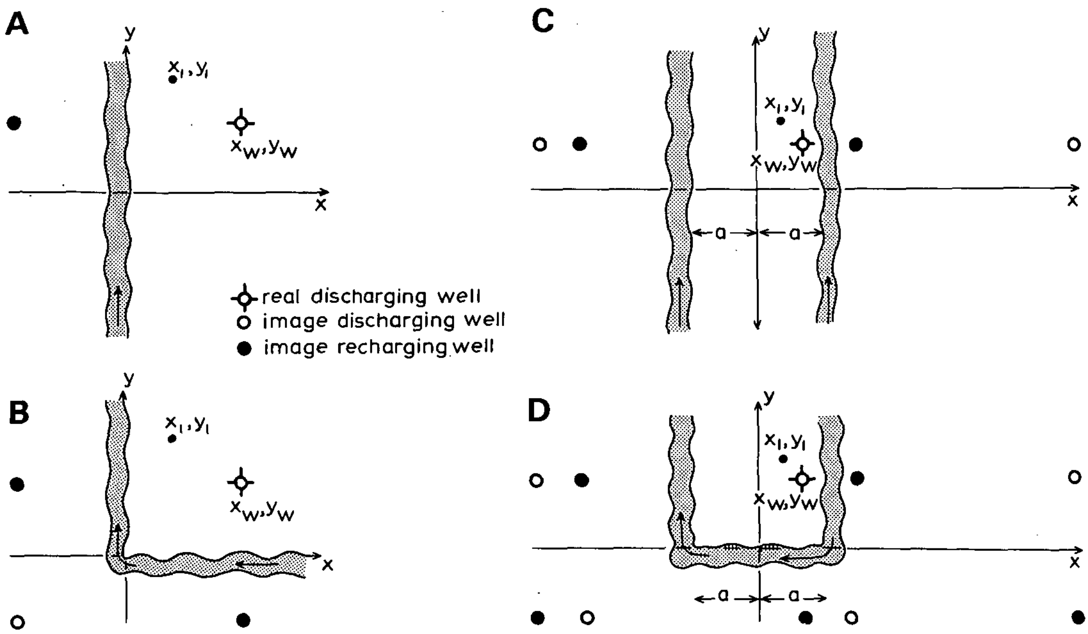

Figure 6.2 Image well systems for bounded aquifers (Dietz method)
A) One straight recharge boundary
B) Two straight recharge boundaries at right angles
C) Two straight parallel recharge boundaries
D) U-shaped recharge boundary

Procedure 6.1

- Determine the boundary configuration and substitute the appropriate Green function into Equation 6.1;
- Measure the values of x_w, y_w, x_l, and y_l on the map of the pumping site;
- Substitute the values of Q, x_w, y_w, x_l, y_l, and s_{ml} into Equation 6.1 and calculate KD;
- Repeat this procedure for all available piezometers. The results should show a close agreement.

Remarks

- The angles in Equations 6.4 and 6.5 are expressed in radians;
- For unconfined aquifers, the maximum drawdown s_m should be replaced by $s'_m = s_m - (s^2_m/2D)$.

6.2 Bounded confined or unconfined aquifer, unsteady-state flow

6.2.1 Stallman's method, one or more boundaries

Stallman (as quoted by Ferris et al. 1962) developed a curve-fitting method for aquifers that have one or more straight recharge or barrier boundaries.

The distance between the real well and a piezometer is r; the distance between an image well and the piezometer is r_i, and their ratio is $r_i/r = r_r$.
If

$$u = \frac{r^2 S}{4KDt} \tag{6.6}$$

and

$$u_i = \frac{r_i^2 S}{4KDt} = \frac{r_r^2 r^2 S}{4KDt} = r_r^2 u \tag{6.7}$$

the drawdown in the piezometer is described by

$$s = \frac{Q}{4\pi KD}[W(u) \pm W(r_{r1}^2 u) \pm W(r_{r2}^2 u) \pm \ldots \pm W(r_{rn}^2 u)] \tag{6.8}$$

or

$$s = \frac{Q}{4\pi KD} W(u, r_{r1 \rightarrow n}) \tag{6.9}$$

Numerical values of $W(r_r^2 u)$ are given in Annex 6.1. In Equation 6.8, the number of terms between brackets depends on the number of image wells. If there is only one image well, there are two terms between brackets: the term $(Q/4\pi KD)\, W(u)$ describing the influence of the real well and the term $(Q/4\pi KD)\, W(r_r^2 u)$ describing the influence of the image well. If there are two straight boundaries intersecting at right angles, three image wells are required, and there are consequently four terms between brackets. With parallel boundaries, the number of image wells becomes infinite, but those where $r_r > 100$ can be neglected.

A discharge well – real or image – gives terms with a positive sign; a recharge well gives terms with a negative sign. Consequently, the drawdown in a piezometer caused by a well near a boundary can be described by the following equations.

One straight boundary

One recharge boundary (Figure 6.1A-C)

$$s = \frac{Q}{4\pi KD}[W(u) - W(r_r^2 u)] \tag{6.10}$$

or

$$s = \frac{Q}{4\pi KD} W_R(u, r_r) \tag{6.11}$$

One barrier boundary (Figure 6.1D-F)

$$s = \frac{Q}{4\pi KD}[W(u) + W(r_r^2 u)] \tag{6.12}$$

or

$$s = \frac{Q}{4\pi KD} W_B(u, r_r) \tag{6.13}$$

Two straight boundaries at right angles to each other
One barrier boundary and one recharge boundary (Figure 6.3A)

$$s = \frac{Q}{4\pi KD}[W(u) + W(r_{r1}^2u) - W(r_{r2}^2u) - W(r_{r3}^2u)] \tag{6.14}$$

Two barrier boundaries (Figure 6.3B)

$$s = \frac{Q}{4\pi KD}[W(u) + W(r_{r1}^2u) + W(r_{r2}^2u) + W(r_{r3}^2u)] \tag{6.15}$$

Two recharge boundaries (Figure 6.3C)

$$s = \frac{Q}{4\pi KD}[W(u) - W(r_{r1}^2u) - W(r_{r2}^2u) + W(r_{r3}^2u)] \tag{6.16}$$

Two parallel boundaries
One barrier and one recharge boundary (Figure 6.4A)

$$s = \frac{Q}{4\pi KD}[W(u) + W(r_{r1}^2u) - W(r_{r2}^2u) - W(r_{r3}^2u) - ... \pm W(r_{rn}^2u)] \tag{6.17}$$

Two barrier boundaries (Figure 6.4B)

$$s = \frac{Q}{4\pi KD}[W(u) + W(r_{r1}^2u) + W(r_{r2}^2u) + W(r_{r3}^2u) + ... + W(r_{rn}^2u)] \tag{6.18}$$

Two recharge boundaries (Figure 6.4C)

$$s = \frac{Q}{4\pi KD}[W(u) - W(r_{r1}^2u) - W(r_{r2}^2u) + W(r_{r3}^2u) + ... \pm W(r_{rn}^2u)] \tag{6.19}$$

For three and four straight boundaries (Figures 6.5 and 6.6), the drawdown equations can be composed in the same way.

Stallman's method can be applied if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and second assumptions, which are replaced by:
  - The aquifer is confined or unconfined;
  - Within the zone influenced by the pumping test, the aquifer is crossed by one or more straight, fully penetrating recharge or barrier boundaries;
  - Recharge boundaries have a constant water level and the hydraulic contacts between the recharge boundaries and the aquifer are as permeable as the aquifer.

The following condition is added:

- The flow to the well is in unsteady state.

Procedure 6.2

- Determine the boundary configuration and prepare a plan of the equivalent system of image wells;
- Determine for one of the piezometers the value of r and the value or values of r_i;
- Calculate $r_r = r_i/r$ for each of the image wells and determine the sign for each of the terms between brackets in Equation 6.8;

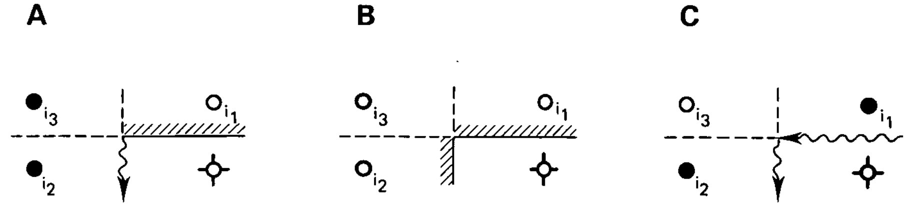

Figure 6.3 Two straight boundaries intersecting at right angles

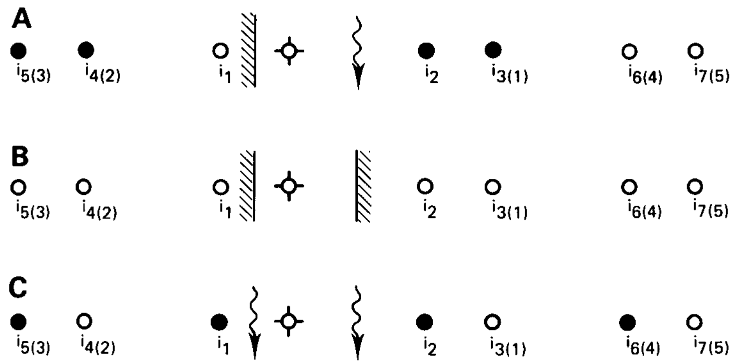

Figure 6.4 Two straight parallel boundaries

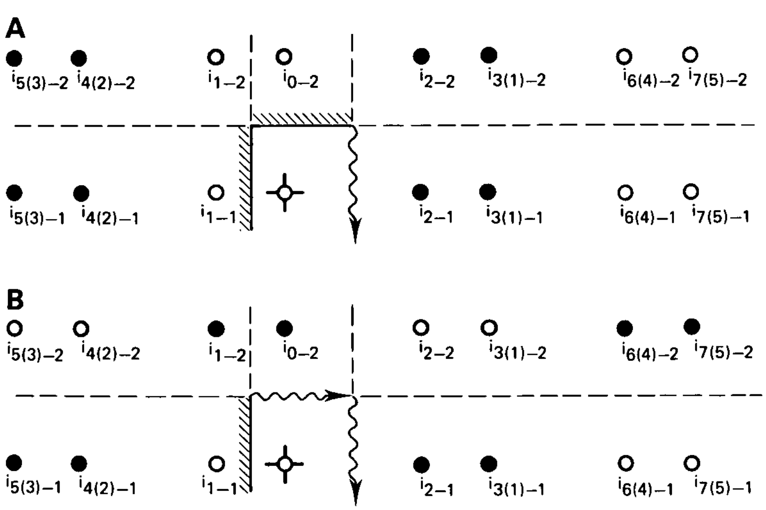

Figure 6.5 Two straight parallel boundaries intersected at right angles by a third boundary

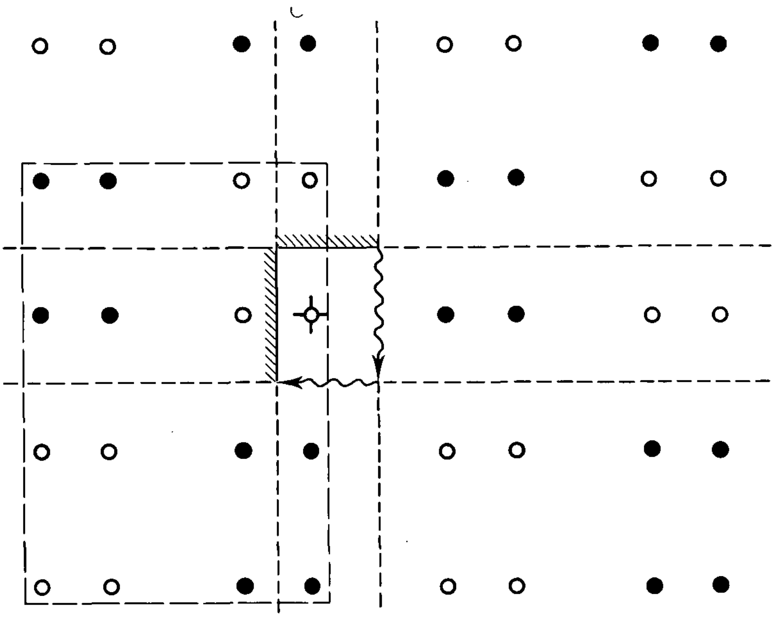

Figure 6.6 Four straight boundaries, i.e. two pairs of straight parallel boundaries intersecting at right angles

- Using Annex 6.1, calculate the numerical values of $W(u,r_{r1\rightarrow n})$ with respect to u according to the appropriate form of Equation 6.8, and plot the type curve $W(u,r_{r1\rightarrow n})$ versus u on log-log paper;
  (For one-boundary systems, the values of $W_R(u,r_r)$ and $W_B(u,r_r)$ can be read directly from Annexes 6.2 and 6.3);
- On another sheet of log-log paper, plot s as observed in the piezometer versus 1/t; this is the observed data curve;
- Match the observed data curve with the type curve;
- Select a matchpoint A and note its coordinate values u, $W(u,r_{r1\rightarrow n})$, s, and 1/t;
- Substitute these values of s and $W(u,r_{r1\rightarrow n})$ and the known value of Q into Equation 6.9 and calculate KD;
- Substitute the values of Q, r, u, KD, and 1/t into Equation 6.6 and calculate S;
- Repeat this procedure for all available piezometers. It will be noted that each piezometer has its own type curve because the value of $W(u,r_{r1\rightarrow n})$ depends on the value of the ratio $r_i/r = r_r$, which is different for each piezometer.

Remarks
- This method can also be used to analyze the drawdown data from an aquifer pumped

by more than one real well, or from an aquifer that is both pumped and recharged by real wells, provided all wells operate at the same constant rate Q;

- Equation 6.8 is based on the Theis well function for confined aquifers. Stallman's method, however, is also applicable to data from unconfined aquifers as long as Assumption 7 (Chapter 3) is met, i.e. no delayed watertable response is apparent.

6.2.2 Hantush's method, one recharge boundary

The Hantush image method is useful when the effective line of recharge does not correspond with the bank or the streamline of the river or canal. This may be due to the slope of the bank, to partial penetration effects of the river or canal, or to an entrance resistance at the boundary contact. When the effects of these conditions are small but not negligible, they can be compensated for by making the distance between the pumped well and the hydraulic boundary in the equivalent system (line of zero drawdown in Figure 6.1B) greater than the distance between the pumped well and the actual boundary (Figure 6.7).

As was shown by the Stallman method, the drawdown in an aquifer limited at one side by a recharge boundary can be expressed by Equation 6.10

$$s = \frac{Q}{4\pi KD}[W(u) - W(r_r^2 u)]$$

where, according to Equation 6.6,

$$u = \frac{r^2 S}{4KDt}$$

and

$$r_r = \frac{r_i}{r}$$

$r = \sqrt{(x^2 + y^2)}$ is the distance between the piezometer and the real discharging well

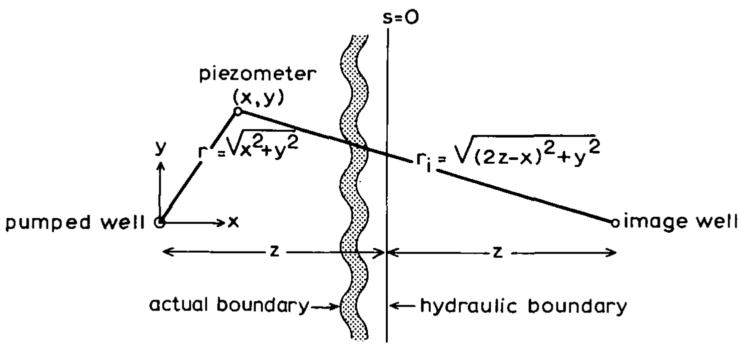

Figure 6.7 The parameters in the Hantush image method

$r_i = \sqrt{\{(2z-x)^2 + y^2\}}$ is the distance between the piezometer and the recharging well; x, y are the coordinates of the piezometer with respect to the real discharging well (see Figure 6.7)

The distance between the real discharging well and the recharging image well is 2z. The hydraulic boundary, i.e. the effective line of recharge, intersects the connecting line midway between the real well and the image well. The lines are at right angles to each other. It should be kept in mind that, especially with recharge boundaries, the hydraulic boundary does not always coincide with the bank of the river or its streamline. It is not necessary to know z beforehand, nor the location of the image well, nor the distance r_i dependent on it; neither need the relation $r_i/r = r_r$ be known beforehand.
The relation between r_r, x, r, and z is given by

$$4z^2 - 4xz - r^2(r_r^2 - 1) = 0 \tag{6.20}$$

Hantush (1959b) observed that if the drawdown s is plotted on semi-log paper versus t (with t on logarithmic scale), there is an inflection point P on the curve (Figure 6.8). At this point, the value of u is given by

$$u_p = \frac{r^2S}{4KDt_p} = \frac{2 \ln r_r}{r_r^2 - 1} \tag{6.21}$$

The slope of the curve at this point is

$$\Delta s_p = \frac{2.30Q}{4\pi KD}\left(e^{-u_p} - r^{-r_r^2 u_p}\right) \tag{6.22}$$

and the drawdown at this point is

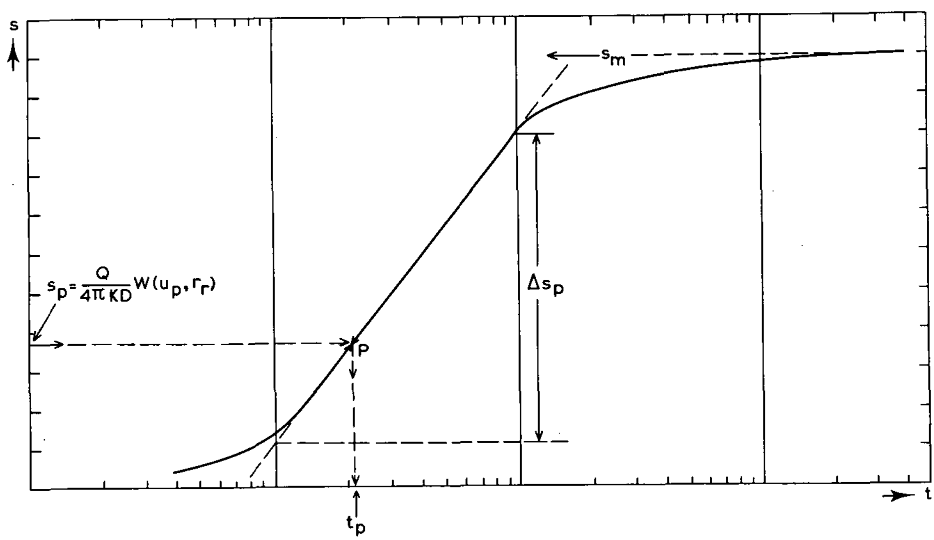

Figure 6.8 The application of the Hantush image method

$$s_p = \frac{Q}{4\pi KD} W(u_p,r_r) \qquad (6.23)$$

For values of $t > 4t_p$, the drawdown s approaches the maximum drawdown

$$s_m = \frac{Q}{2\pi KD} \ln r_r \qquad (6.24)$$

It will be noted that the ratio of s_m, as given by Equation 6.24, and Δs_p, as given by Equation 6.22, depends solely on the value of r_r. So

$$\frac{s_m}{\Delta s_p} = \frac{2 \log r_r}{e^{-u_p} - e^{-r_r^2 u_p}} = f(r_r) \qquad (6.25)$$

where u_p is given by Equation 6.21.

The Hantush image method is based on the following assumptions and conditions:
- The assumptions listed at the beginning of Chapter 3, with the exception of the first and second assumptions, which are replaced by:
  - The aquifer is confined or unconfined;
  - The aquifer is crossed by a straight recharge boundary within the zone influenced by the pumping test;
  - The recharge boundary has a constant water level, but the effective line of recharge need not necessarily be known beforehand. Entrance resistances, however, should be small, although not negligible.

The following conditions are added:
- The flow to the well is in unsteady state;
- It should be possible to extrapolate the steady-state drawdown for each of the piezometers.

Procedure 6.3
- On semi-log paper, plot s versus t for one of the piezometers (t on logarithmic scale), and draw the time-drawdown curve through the plotted points (Figure 6.8);
- Extrapolate the curve to determine the value of the maximum drawdown s_m;
- Calculate the slope Δs_p of the straight portion of the curve; this is an approximation of the slope at the inflection point P;
- Calculate the ratio $s_m/\Delta s_p$ according to Equation 6.25; this is equal to $f(r_r)$. Use Annex 6.4 to find the value of r_r from $f(r_r)$;
- Substitute the values of s_m, Q, and r_r into Equation 6.24 and calculate KD;
- Obtain the values of u_p and $W(u_p,r_r)$ from Annex 6.4;
- Substitute the values of Q, KD, and $W(u_p,r_r)$ into Equation 6.23 and calculate s_p;
- Knowing s_p, locate the inflection point on the curve and read t_p;
- Substitute the values of KD, t_p, u_p, and r into Equation 6.21 and calculate S;
- Using Equation 6.20, calculate z;
- Apply this procedure to the data from all available piezometers. The calculated values of KD and S should show a close agreement.

Remarks
- To check whether any errors have been made in the approximation of s_m and Δs_p,

the theoretical time-drawdown curve should be calculated with Equations 6.6 and 6.10, Annex 6.2, and the calculated values of r_r, KD, and S. This theoretical curve should show a close agreement with the observed time-drawdown curve. If not, the procedure should be repeated with corrected approximations of s_m and Δs_p.

- Procedure 6.3 can be applied to analyze data from unconfined aquifers when Assumption 7 (Chapter 3) is met.

6.3 Bounded leaky or confined aquifers, unsteady-state flow

6.3.1 Vandenberg's method (strip aquifer)

Leaky aquifers bounded laterally by two parallel barrier boundaries form an 'infinite strip aquifer', or a 'parallel channel aquifer'. In the analysis of such aquifers, we have to consider not only boundary effects, but also leakage effects. Vandenberg (1976; 1977) proposed a method by which the values of KD, S, and L of such aquifers can be determined.

If the distance, x, measured along the axis of the channel between the pumped well and the piezometer (Figure 6.9), is greater than the width of the channel, w, (i.e. x/w > 1), Vandenberg showed that for parallel unsteady-state flow the following drawdown function is applicable

$$s = \frac{Qx}{(2KDw)} F(u,x/L) \qquad (6.26)$$

where

$$F(u,x/L) = \frac{1}{2\sqrt{\pi}} \int_u^\infty y^{-3/2} \exp(-y-x^2/4L^2y)dy \qquad (6.27)$$

$$u = \frac{x^2S}{4KDt} \qquad (6.28)$$

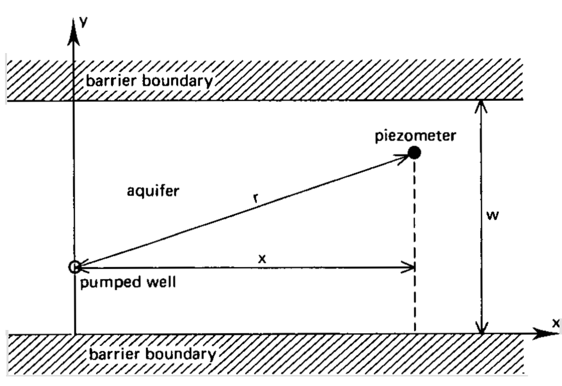

Figure 6.9 Plan view of a parallel channel aquifer

L $= \sqrt{KDc}$ = leakage factor in m (6.29)

x = projection of distance r in m between pumped well and piezometer, along the direction of the channel

w = width of the channel in m

Presented in Annex 6.5 are values of the function F(u,x/L) for different values of u and x/L, as given by Vandenberg (1976). These values can be plotted as a family of type curves (Figure 6.10).

The Vandenberg curve-fitting method can be used if the following assumptions and conditions are satisfied:

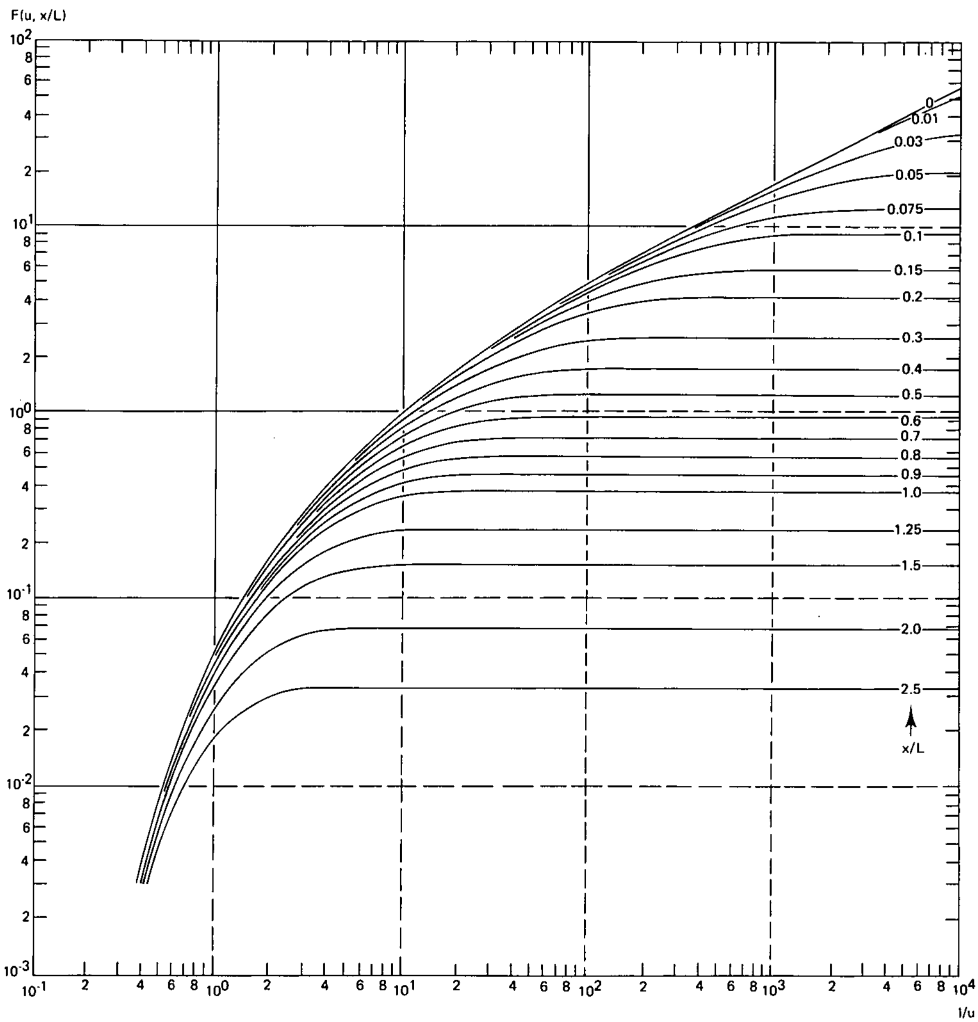

Figure 6.10 Family of Vandenberg's type curves F(u,x/L) versus 1/u for different values of x/L

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and second assumptions, which are replaced by:
  - The aquifer is leaky;
  - Within the zone influenced by the pumping test, the aquifer is bounded by two straight parallel fully penetrating barrier boundaries.

The following conditions are added:
- The flow to the well is in unsteady state;
- The width and direction of the aquifer are both known with sufficient accuracy;
- $x/w > 1$.

Procedure 6.4
- Using Annex 6.5, construct on log-log paper a family of Vandenberg type curves by plotting $F(u,x/L)$ versus $1/u$ for a range of values of x/L;
- On another sheet of log-log paper of the same scale, plot s versus t for a single piezometer at a projected distance x from the pumped well;
- Match the observed data curve with one of the type curves;
- Select a match point on the superimposed sheets, and note for this point the values of $F(u,x/L)$, $1/u$, s, and t. Note also the value of x/L of the selected type curve;
- Substitute the values of $F(u,x/L)$ and s, together with the known values of Q, x, and w into Equation 6.26 and calculate KD;
- Substitute the values of u and t, together with the known values of KD and x, into Equation 6.28 and calculate S;
- Knowing x/L and x, calculate L;
- Calculate c from Equation 6.29;
- Repeat the procedure for all available piezometers ($x/w > 1$). The calculated values of KD, S, and c should show reasonable agreement.

Remarks
- If the direction of the channel is known, but not its width w, the same procedure as above can be followed, except that instead of calculating KD and S, the products KDw and Sw are calculated;
- If the direction of the channel is not known and the data from only one piezometer are available, the distance r may be used instead of x. For those cases where $r \gg w$, only a small error will be introduced;
- When $x/L = 0$, i.e. when $L \rightarrow \infty$, the drawdown function (Equation 6.26) becomes the drawdown function for parallel flow in a confined channel aquifer

$$s = \frac{Qx}{2KDw} F(u) \tag{6.30}$$

  where

$$F(u) = \exp(-u/\sqrt{\pi u}) - \mathrm{erfc}(\sqrt{u}) \tag{6.31}$$

  With the type curve $F(u,x/L)$ versus $1/u$ for $x/L = 0$ (Annex 6.5), the values of KD and S of confined parallel channel aquifers can be determined;
- If $x/w < 1$, Equation 6.26 is not sufficiently accurate and the following drawdown equation for a system of real and image wells should be used (Vandenberg 1976; see also Bukhari et al. 1969)

$$s = \frac{Q}{4\pi KD} [W(u,r/L) + \sum_{i=1}^{\infty} W(u_i,r_i/L)] \tag{6.32}$$

where W(u,r/L) is the function for radial flow towards a well in a leaky aquifer of infinite extent.

Type curves can be constructed from the exact solution of Equation 6.32. For each particular configuration of pumped well and piezometer, however, a different set of curves is required. Vandenberg (1976) provides 16 sets of type curves and gives a listing and user's guide for a Fortran program that will plot a set of type curves for any well/piezometer configuration.