4 Leaky aquifers

In nature, leaky aquifers occur far more frequently than the perfectly confined aquifers discussed in the previous chapter. Confining layers overlying or underlying an aquifer are seldom completely impermeable; instead, most of them leak to some extent. When a well in a leaky aquifer is pumped, water is withdrawn not only from the aquifer, but also from the overlying and underlying layers. In deep sedimentary basins, it is common for a leaky aquifer to be just one part of a multi-layered aquifer system as was shown in Figure 1.1E.

For the purpose of this chapter, we shall consider the three-layered system shown in Figure 4.1. The system consists of two aquifers, separated by an aquitard. The lower aquifer rests on an aquiclude. A well fully penetrates the lower aquifer and is screened over the total thickness of the aquifer. The well is not screened in the upper unconfined aquifer. Before the start of pumping, the system is at rest, i.e. the piezometric surface of the lower aquifer coincides with the watertable in the upper aquifer.

When the well is pumped, the hydraulic head in the lower aquifer will drop, thereby creating a hydraulic gradient not only in the aquifer itself, but also in the aquitard. The flow induced by the pumping is assumed to be vertical in the aquitard and horizontal in the aquifer. The error introduced by this assumption is usually less than 5 per cent if the hydraulic conductivity of the aquifer is two or more orders of magnitude greater than that of the aquitard (Neuman and Witherspoon 1969a).

The water that the pumped aquifer contributes to the well discharge comes from storage within that aquifer. The water contributed by the aquitard comes from storage within the aquitard and leakage through it from the overlying unpumped aquifer.

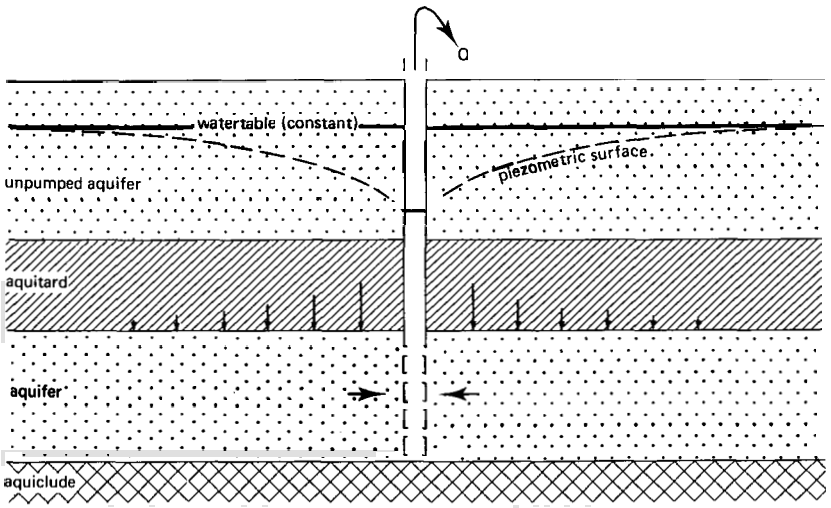

Figure 4.1 Cross-section of a pumped leaky aquifer

As pumping continues, more of the water comes from leakage from the unpumped aquifer and relatively less from aquitard storage. After a certain time, the well discharge comes into equilibrium with the leakage through the aquitard and a steady-state flow is attained. Under such conditions, the aquitard serves merely as a water-transmitting medium, and the water contributed from its storage can be neglected.

Solutions to the steady-state flow problem (Section 4.1) have been found on the basis of two very restrictive assumptions. The first is that, during pumping, the watertable in the upper aquifer remains constant; the second is that the rate of leakage into the leaky aquifer is proportional to the hydraulic gradient across the aquitard. But, as pumping continues, the watertable in the upper aquifer will drop because more and more of its water will be leaking through the aquitard into the pumped aquifer. The assumption of a constant watertable will only be satisfied if the upper aquifer is replenished by an outside source, say from surface water distributed over the aquifer via a system of narrowly spaced ditches. If the watertable can thus be kept constant as pumping continues, the well discharge will eventually be supplied entirely from the upper aquifer and steady-state flow will be attained. If the watertable cannot be controlled and does not remain constant and if pumping times are long, neglecting the drawdown in the upper aquifer can lead to considerable errors, unless its transmissivity is significantly greater than that of the pumped aquifer (Neuman and Witherspoon 1969b).

The second assumption completely ignores the storage capacity of the aquitard. This is justified when the flow to the well has become steady and the amount of water supplied from storage in the aquitard has become negligibly small (Section 4.1).

As long as the flow is unsteady, the effects of aquitard storage cannot be neglected. Yet, two of the solutions for unsteady flow (Sections 4.2.1 and 4.2.2) do neglect these effects, although, as pointed out by Neuman and Witherspoon (1972), this can result in:

- An overestimation of the hydraulic conductivity of the leaky aquifer;
- An underestimation of the hydraulic conductivity of the aquitard;
- A false impression of inhomogeneity in the leaky aquifer.

The other two methods do take the storage capacity of the aquitard into account. They are the Hantush curve-fitting method, which determines aquifer and aquitard characteristics (Section 4.2.3), and the Neuman-Witherspoon ratio method, which determines only the aquitard characteristics (Section 4.2.4). All four solutions for unsteady flow assume a constant watertable.

For a proper analysis of a pumping test in a leaky aquifer, piezometers are required in the leaky aquifer, in the aquitard, and in the upper aquifer.

The assumptions and conditions underlying the methods in this chapter are:

- The aquifer is leaky;
- The aquifer and the aquitard have a seemingly infinite areal extent;
- The aquifer and the aquitard are homogeneous, isotropic, and of uniform thickness over the area influenced by the test;
- Prior to pumping, the piezometric surface and the watertable are horizontal over the area that will be influenced by the test;
- The aquifer is pumped at a constant discharge rate;

- The well penetrates the entire thickness of the aquifer and thus receives water by horizontal flow;
- The flow in the aquitard is vertical;
- The drawdown in the unpumped aquifer (or in the aquitard, if there is no unpumped aquifer) is negligible.

And for unsteady-state conditions:

- The water removed from storage in the aquifer and the water supplied by leakage from the aquitard is discharged instantaneously with decline of head;
- The diameter of the well is very small, i.e. the storage in the well can be neglected.

The methods will be illustrated with data from the pumping test 'Dalem', The Netherlands (De Ridder 1961). Figure 4.2 shows a lithostratigraphical section of the test site as derived from the drilling data. The Kedichem Formation is regarded as the aquiclude. The Holocene layers form the aquitard overlying the leaky aquifer. The reader will note that there is no aquifer overlying the aquitard as in Figure 4.1. Instead, the aquitard extends to the surface where a system of narrowly spaced drainage ditches ensured a relatively constant watertable in the aquitard during the test.

The site lies about 1500 m north of the River Waal. The level of this river is affected by the tide and so too is the piezometric surface of the aquifer because it is in hydraulic connection with the river. The well was fitted with two screens. During the test, the lower screen was sealed and the entry of water was restricted to the upper screen, placed from 11 to 19 m below the surface. For 24 hours prior to pumping, the water levels in the piezometers were observed to determine the effect of the tide on the hydraulic head in the aquifer. By extrapolation of these data, time-tide curves for the

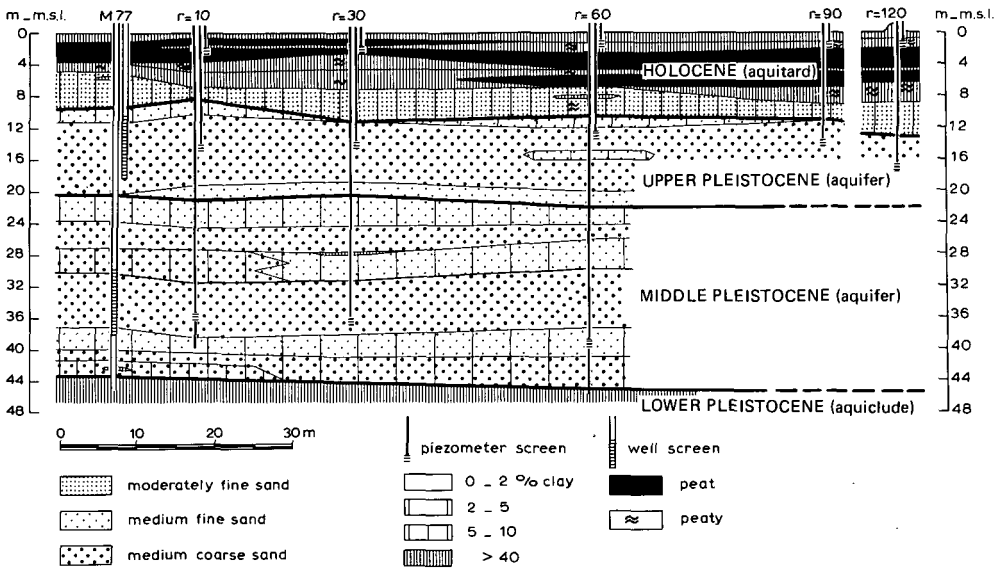

Figure 4.2 Lithostratigraphical cross-section of the pumping-test site 'Dalem', The Netherlands (after De Ridder 1961)

pumping period were established to allow a correction of the measured drawdowns (see Example 2.2). The data from the piezometers near the well were influenced by the effects of the well's partial penetration, for which allowance also had to be made (Example 10.1). The aquifer was pumped for 8 hours at a constant discharge of Q = 31.70 m^3/hr (or 761 m^3/d). The steady-state drawdown, which had not yet been reached, could be extrapolated from the time-drawdown curves.

4.1 Steady-state flow

The two methods presented below, both of which use steady-state drawdown data, allow the characteristics of the aquifer and the aquitard to be determined.

4.1.1 De Glee's method

For the steady-state drawdown in an aquifer with leakage from an aquitard proportional to the hydraulic gradient across the aquitard, De Glee (1930, 1951; see also Anonymous 1964, pp 35-41) derived the following formula

$$s_m = \frac{Q}{2\pi KD} K_0(\frac{r}{L}) \tag{4.1}$$

where

s_m = steady-state (stabilized) drawdown in m in a piezometer at distance r in m from the well

Q = discharge of the well in m^3/d

L = $\sqrt{KDc}$: leakage factor in m (4.2)

c = D′/K′: hydraulic resistance of the aquitard in d

D′ = saturated thickness of the aquitard in m

K′ = hydraulic conductivity of the aquitard for vertical flow in m/d

$K_0(x)$ = modified Bessel function of the second kind and of zero order (Hankel function)

The values of $K_0(x)$ for different values of x can be found in Annex 4.1.

De Glee's method can be applied if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of this chapter;
- The flow to the well is in steady state;
- L > 3D.

Procedure 4.1
- Using Annex 4.1, prepare a type curve by plotting values of $K_0(x)$ versus values of x on log-log paper;
- On another sheet of log-log paper of the same scale, plot the steady-state (stabilized) drawdown in each piezometer s_m versus its corresponding value of r;

- Match the data plot with the type curve;
- Select an arbitrary point A on the overlapping portion of the sheets and note for A the values of s, r, $K_0(r/L)$, and $r/L(=x)$. It is convenient to select as point A the point where $K_0(r/L) = 1$ and $r/L = 1$;
- Calculate KD by substituting the known value of Q and the values of s_m and $K_0(r/L)$ into Equation 4.1;
- Calculate c by substituting the calculated value of KD and the values of r and r/L into Equation 4.2, written as

$$c = \frac{L^2}{KD} = \frac{1}{(r/L)^2} \times \frac{r^2}{KD}$$

Example 4.1
When the pump at 'Dalem' was shut down, steady-state drawdown had not yet been fully reached, but could be extrapolated from the time-drawdown curves. Table 4.1 gives the extrapolated steady-state drawdowns in the piezometers that had screens at a depth of 14 m (unless otherwise stated), corrected for the effects of the tide in the river and for partial penetration.

Table 4.1 Corrected extrapolated steady-state drawdowns of pumping test 'Dalem' (after De Ridder 1961)

Piezometer	P_{10}	P_{10}*	P_{30}	P_{30}*	P_{60}	P_{90}	P_{120}	P_{400}*
Drawdown in m	0.310	0.252	0.235	0.213	0.170	0.147	0.132	0.059

* screen depth 36 m

For this example, we first plot the drawdowns listed in Table 4.1 versus the corresponding distances, which we then fit with De Glee's type curve $K_0(x)$ versus x (Figure 4.3). As match point A, we choose the point where $K_0(r/L) = 1$ and $r/L = 1$. On the observed data sheet, point A has the coordinates $s_m = 0.057$ m and $r = 1100$ m. Substituting these values and the known value of $Q = 761$ m³/d into Equation 4.1, we obtain

$$KD = \frac{Q}{2\pi s_m} K_0\left(\frac{r}{L}\right) = \frac{761}{2 \times 3.14 \times 0.057} \times 1 = 2126 \text{ m}^2/\text{d}$$

Further, $r/L = 1$, $L = r = 1100$ m. Hence

$$c = \frac{L^2}{KD} = \frac{(1100)^2}{2126} = 569 \text{ d}$$

4.1.2 Hantush-Jacob's method

Unaware of the work done many years earlier by De Glee, Hantush and Jacob (1955) also derived Equation 4.1. Hantush (1956, 1964) noted that if r/L is small ($r/L \leq$

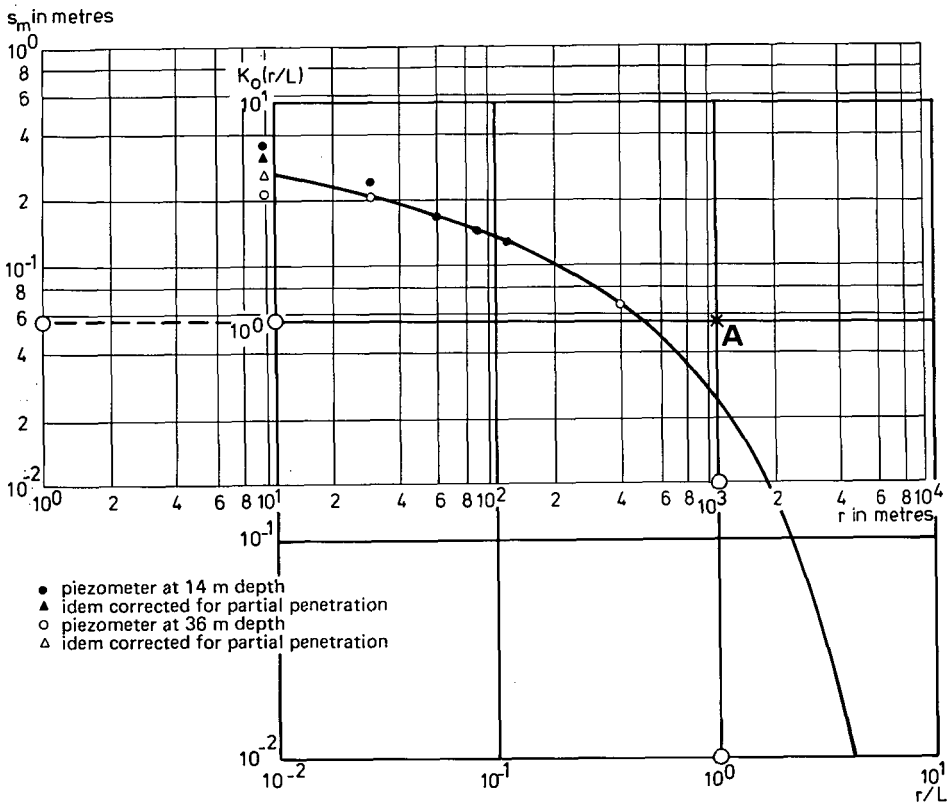

Figure 4.3 Analysis of data from pumping test 'Dalem' with the De Glee method

0.05), Equation 4.1 can, for practical purposes, be approximated by

$$s_m \approx \frac{2.30Q}{2\pi KD}\left(\log 1.12\frac{L}{r}\right) \tag{4.3}$$

For $r/L < 0.16$, 0.22, 0.33, and 0.45, the errors in using this equation instead of Equation 4.1 are less than 1, 2, 5, and 10 per cent, respectively (Huisman 1972). A plot of s_m against r on semi-log paper, with r on the logarithmic scale, will show a straight-line relationship in the range where r/L is small (Figure 4.4). In the range where r/L is large, the points fall on a curve that approaches the zero-drawdown axis asymptotically.

The slope of the straight portion of the curve, i.e. the drawdown difference Δs_m per log cycle of r, is expressed by

$$\Delta s_m = \frac{2.30Q}{2\pi KD} \tag{4.4}$$

The extended straight-line portion of the curve intercepts the r axis where the drawdown is zero. At the interception point, $s_m = 0$ and $r = r_0$ and thus Equation 4.3 reduces to

$$0 = \frac{2.30Q}{2\pi KD}\left(\log 1.12\frac{L}{r_0}\right)$$

from which it follows that

$$1.12\frac{L}{r_0} = \frac{1.12}{r_0}\sqrt{KDc} = 1$$

and hence

$$c = \frac{(r_0/1.12)^2}{KD} \tag{4.5}$$

The Hantush-Jacob method can be used if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of this chapter;
- The flow to the well is in steady state;
- $L > 3D$;
- $r/L \leq 0.05$.

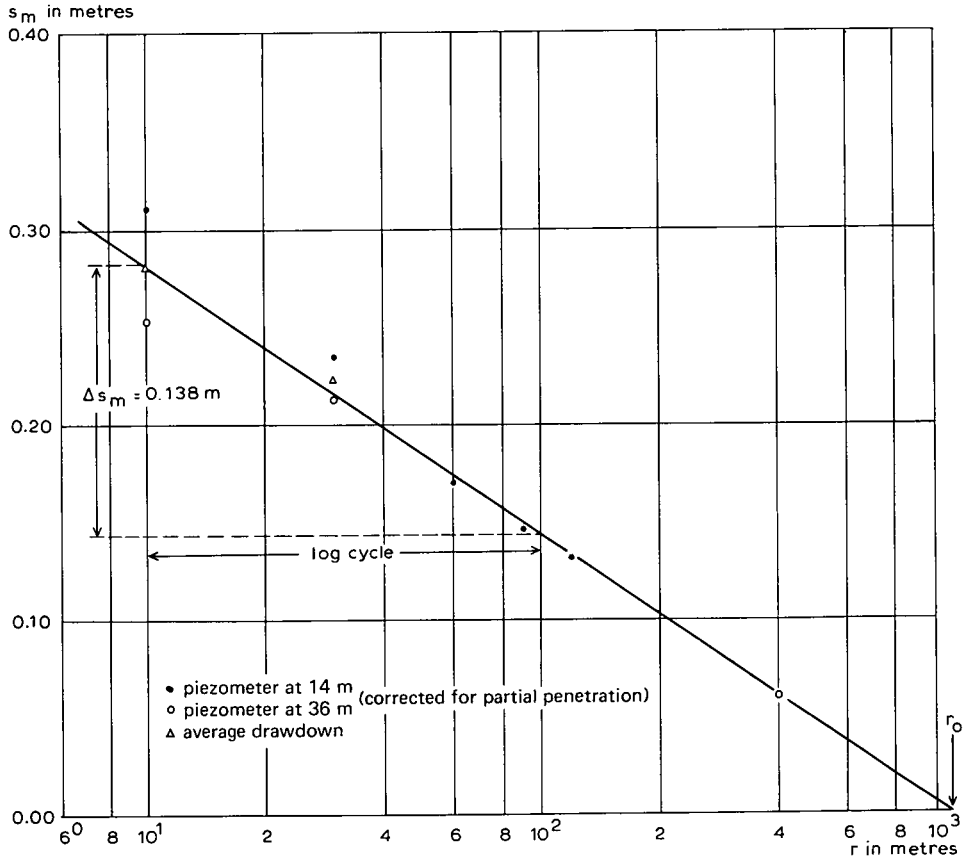

Figure 4.4 Analysis of data from pumping test 'Dalem' with the Hantush-Jacob method

Procedure 4.2

- On semi-log paper, plot s_m versus r (r on logarithmic scale);
- Draw the best-fit straight line through the points;
- Determine the slope of the straight line (Figure 4.4);
- Substitute the value of Δs_m and the known value of Q into Equation 4.4 and solve for KD;
- Extend the straight line until it intercepts the r axis and read the value of r_0;
- Calculate the hydraulic resistance of the aquitard c by substituting the values of r_0 and KD into Equation 4.5.

Another way to calculate c is:

- Select any point on the straight line and note its coordinates s_m and r;
- Substitute these values, together with the known values of Q and KD into Equation 4.3 and solve for L;
- Since $L = \sqrt{KDc}$, calculate c.

Example 4.2

For this example, using data from the pumping test 'Dalem', we first plot the steady-state drawdown data listed in Table 4.1 on semi-log paper versus the corresponding distances. For the piezometer at 10 m from the well, we use the average of the drawdowns measured at depths of 14 and 36 m, and do the same for the piezometer at 30 m from the well. After fitting a straight line through the plotted points, we read from the graph (Figure 4.4) the drawdown difference per log cycle of r

$$\Delta s_m = 0.281 - 0.143 = 0.138 \text{ m}$$

Further, $Q = 761 \text{ m}^3/\text{d}$. Substituting these data into Equation 4.4, we obtain

$$KD = \frac{2.30Q}{2\pi\Delta s_m} = \frac{2.30 \times 761}{2 \times 3.14 \times 0.138} = 2020 \text{ m}^2/\text{d}$$

The fitted straight line intercepts the zero-drawdown axis at the point $r_0 = 1100$ m. Substitution into Equation 4.5 gives

$$c = \frac{(r_o/1.12)^2}{KD} = \frac{(1100/1.12)^2}{2020} = 478 \text{ d}$$

and L is calculated from $1.12\frac{L}{r_0} = 1$ or $L = \frac{1100}{1.12} = 982$ m.

This result is an approximation because this method can only be used for values of $r/L \leq 0.05$, a rather restrictive limiting condition, as we said earlier. If errors in the calculated hydraulic parameters are to be less than 1 per cent, the value of r/L should be less than 0.16. This means that the data from the five piezometers at $r \leq 0.16 \times 982 = 157$ m can be used.

4.2 Unsteady-state flow

Until steady-state flow is reached, the water discharged by the well is derived not only from leakage through the aquitard, but also from a reduction in storage within both the aquitard and the pumped aquifer.

The methods available for analyzing data of unsteady-state flow are the Walton curve-fitting method, the Hantush inflection-point method (both of which, however, neglect the aquitard storage), the Hantush curve-fitting method, and the Neuman and Witherspoon ratio method (both of which do take aquitard storage into account).

4.2.1 Walton's method

With the effects of aquitard storage considered negligible, the drawdown due to pumping in a leaky aquifer is described by the following formula (Hantush and Jacob 1955)

$$s = \frac{Q}{4\pi KD} \int_u^\infty \frac{1}{y} \exp\left(-y - \frac{r^2}{4L^2 y}\right) dy$$

or

$$s = \frac{Q}{4\pi KD} W(u,r/L) \tag{4.6}$$

where

$$u = \frac{r^2 S}{4KDt} \tag{4.7}$$

Equation 4.6 has the same form as the Theis well function (Equation 3.5), but there are two parameters in the integral: u and r/L. Equation 4.6 approaches the Theis well function for large values of L, when the exponential term $r^2/4L^2y$ approaches zero.

On the basis of Equation 4.6, Walton (1962) developed a modification of the Theis curve-fitting method, but instead of using one type curve, Walton uses a type curve for each value of r/L. This family of type curves (Figure 4.5) can be drawn from the tables of values for the function W(u,r/L) as published by Hantush (1956) and presented in Annex 4.2.

Walton's method can be applied if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of this chapter;
- The aquitard is incompressible, i.e. the changes in aquitard storage are negligible;
- The flow to the well is in unsteady state.

Procedure 4.3

- Using Annex 4.2, plot on log-log paper W(u,r/L) versus 1/u for different values of r/L; this gives a family of type curves (Figure 4.5);
- Plot for one of the piezometers the drawdown s versus the corresponding time t on another sheet of log-log paper of the same scale; this gives the observed time-drawdown data curve;
- Match the observed data curve with one of the type curves (Figure 4.6);
- Select a match point A and note for A the values of W(u,r/L), 1/u, s, and t;
- Substitute the values of W(u,r/L) and s and the known value of Q into Equation 4.6 and calculate KD;
- Substitute the value of KD, the reciprocal value of 1/u, and the values of t and r into Equation 4.7 and solve for S;

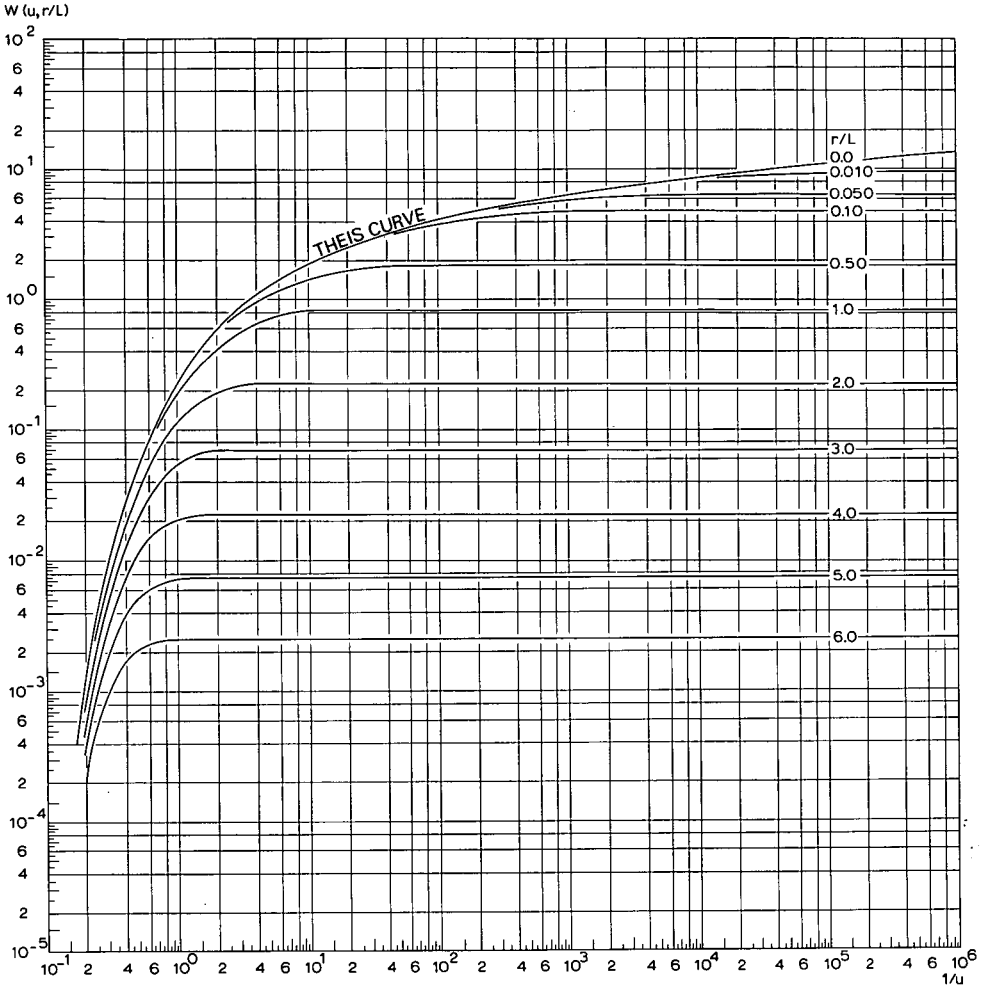

Figure 4.5 Family of Walton's type curves W(u,r/L) versus 1/u for different values of r/L

- From the type curve that best fits the observed data curve, take the numerical value of r/L and calculate L. Then, because $L = \sqrt{KDc}$, calculate c;
- Repeat the procedure for all piezometers. The calculated values of KD, S, and c should show reasonable agreement.

Remark

- To obtain the unique fitting position of the data plot with one of the type curves, enough of the observed data should fall within the period when leakage effects are negligible, or r/L should be rather large.

Example 4.3

Compiled from the pumping test 'Dalem', Table 4.2 presents the corrected drawdown data of the piezometers at 30, 60, 90, and 120 m from the well. Using the data from the piezometer at 90 m, we plot the drawdown data against the corresponding values of t on log-log paper. A comparison with the Walton family of type curves shows that the plotted points fall along the curve for r/L = 0.1 (Figure 4.6). The point where W(u,r/L) = 1 and 1/u = 10^2 is chosen as match point A_{90}. On the observed data sheet, this point has the coordinates s = 0.035 m and t = 0.22 d. Introducing the appropriate numerical values into Equations 4.6 and 4.7 yields

$$KD = \frac{Q}{4\pi s} W(u,r/L) = \frac{761}{4 \times 3.14 \times 0.035} \times 1 = 1731 \text{ m}^2/\text{d}$$

and

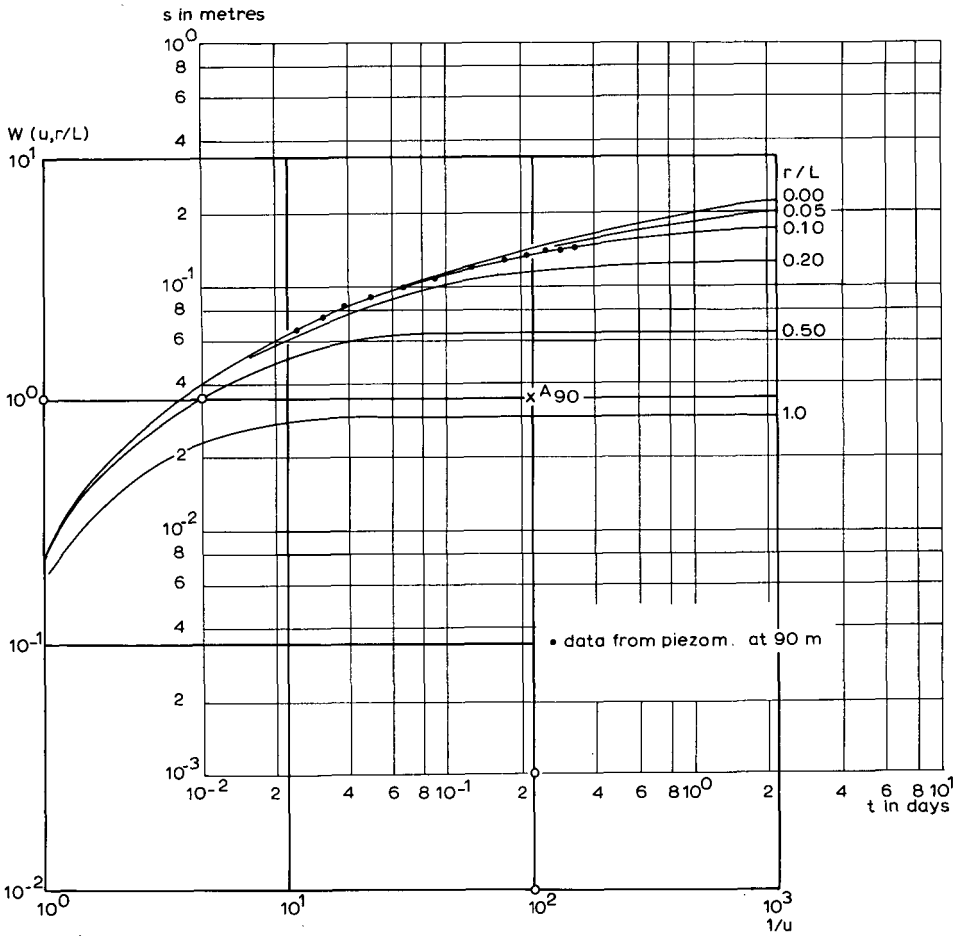

Figure 4.6 Analysis of data from pumping test 'Dalem' (r = 90 m) with the Walton method

$$S = \frac{4KDt}{r^2} u = \frac{4 \times 1731 \times 0.22}{90^2} \times \frac{1}{10^2} = 1.9 \times 10^{-3}$$

Further, because r = 90 m and r/L = 0.1, it follows that L = 900 m and hence $c = L^2/KD = (900)^2/1731 = 468$ d.

Table 4.2 Drawdown data from pumping test 'Dalem', The Netherlands (after De Ridder 1961)

Time (d)	Drawdown (m)	Time (d)	Drawdown (m)
Piezometer at 30 m distance and 14 m depth			
0	0		
1.53×10^{-2}	0.138	8.68×10^{-2}	0.190
1.81	0.141	1.25×10^{-1}	0.201
2.29	0.150	1.67	0.210
2.92	0.156	2.08	0.217
3.61	0.163	2.50	0.220
4.58	0.171	2.92	0.224
6.60×10^{-2}	0.180	3.33×10^{-1}	0.228
extrapolated steady-state drawdown			0.235 m
Piezometer at 60 m distance and 14 m depth			
0	0	8.82×10^{-2}	0.127
1.88×10^{-2}	0.081	1.25×10^{-1}	0.137
2.36	0.089	1.67	0.148
2.99	0.094	2.08	0.155
3.68	0.101	2.50	0.158
4.72	0.109	2.92	0.160
6.67×10^{-2}	0.120	3.33×10^{-1}	0.164
extrapolated steady-state drawdown			0.170 m
Piezometer at 90 m distance and 14 m depth			
0	0		
2.43×10^{-2}	0.069	1.25×10^{-1}	0.120
3.06	0.077	1.67	0.129
3.75	0.083	2.08	0.136
4.68	0.091	2.50	0.141
6.74	0.100	2.92	0.142
8.96×10^{-2}	0.109	3.33×10^{-1}	0.143
extrapolated steady-state drawdown			0.147 m
Piezometer at 120 m distance and 14 m depth			
0	0		
2.50×10^{-2}	0.057	1.25×10^{-1}	0.105
3.13	0.063	1.67	0.113
3.82	0.068	2.08	0.122
5.00	0.075	2.50	0.125
6.81	0.086	2.92	0.127
9.03×10^{-2}	0.092	3.33×10^{-1}	0.129
Extrapolated steady-state drawdown			0.132 m

4.2.2 Hantush's inflection-point method

Hantush (1956) developed several procedures for the analysis of pumping test data in leaky aquifers, all of them based on Equation 4.6

$$s = \frac{Q}{4\pi KD} W(u,r/L)$$

One of these procedures (Procedure 4.4) uses the drawdown data from a single piezometer; the other (Procedure 4.5) uses the data from at least two piezometers. To determine the inflection point P (which will be discussed further below), the steady-state drawdown s_m should be known, either from direct observations or from extrapolation. The curve of s versus t on semi-log paper has an inflection point P where the following relations hold

$$s_p = 0.5\, s_m = \frac{Q}{4\pi KD} K_0\left(\frac{r}{L}\right) \tag{4.8}$$

where K_0 is the modified Bessel function of the second kind and zero order

$$u_p = \frac{r^2 S}{4KDt_p} = \frac{r}{2L} \tag{4.9}$$

The slope of the curve at the inflection point Δs_p is given by

$$\Delta s_p = \frac{2.30Q}{4\pi KD} e^{-r/L} \tag{4.10}$$

or

$$r = 2.30L\left(\log\frac{2.30Q}{4\pi KD} - \log \Delta s_p\right) \tag{4.11}$$

At the inflection point, the relation between the drawdown and the slope of the curve is given by

$$2.30\frac{s_p}{\Delta s_p} = e^{r/L} K_0(r/L) \tag{4.12}$$

In Equations 4.8 to 4.12, the index p means 'at the inflection point'. Further, Δs stands for the slope of a straight line.

Either of Hantush's procedures of the inflection-point method can be used if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of this chapter;
- The aquitard is incompressible, i.e. changes in aquitard storage are negligible;
- The flow to the well is in unsteady state;
- It must be possible to extrapolate the steady-state drawdown for each piezometer.

Procedure 4.4
- For one of the piezometers, plot s versus t on semi-log paper (t on logarithmic scale) and draw the curve that best fits through the plotted points (Figure 4.7);

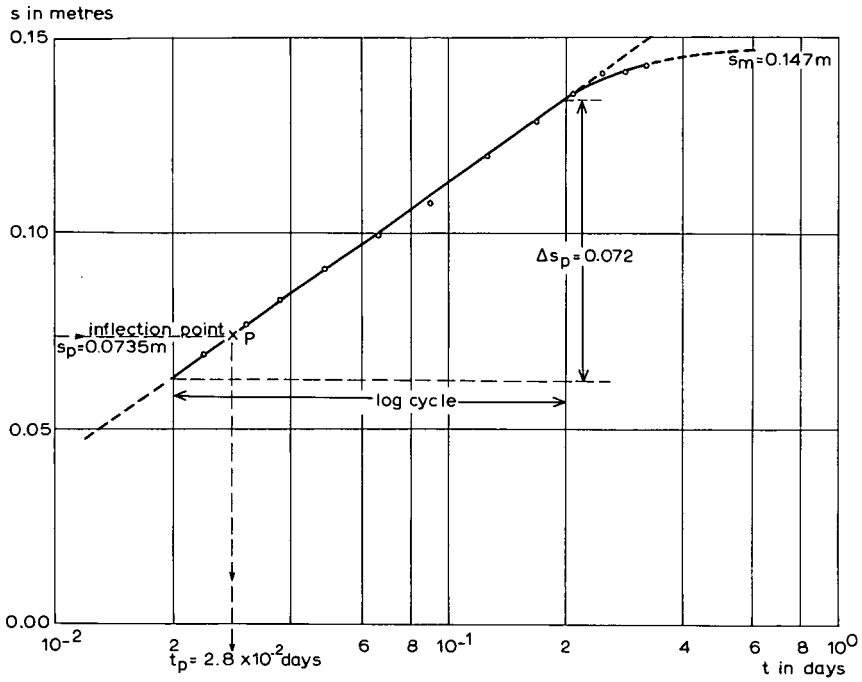

Figure 4.7 Analysis of data from pumping test 'Dalem' (r = 90 m) with Procedure 4.4 of the Hantush inflection-point method

- Determine the value of the maximum drawdown s_m by extrapolation. This is only possible if the period of the test was long enough;
- Calculate s_p with Equation 4.8: $s_p = (0.5)s_m$. The value of s_p on the curve locates the inflection point P;
- Read the value of t_p at the inflection point from the time-axis;
- Determine the slope Δs_p of the curve at the inflection point. This can be closely approximated by reading the drawdown difference per log cycle of time over the straight portion of the curve on which the inflection point lies, or over the tangent to the curve at the inflection point;
- Substitute the values of s_p and Δs_p into Equation 4.12 and find r/L by interpolation from the table of the function $e^x K_0(x)$ in Annex 4.1;
- Knowing r/L and r, calculate L;
- Knowing Q, s_p, Δs_p, and r/L, calculate KD from Equation 4.10, using the table of the function e^{-x} in Annex 4.1, or from Equation 4.8, using the table of the function $K_0(x)$ in Annex 4.1;
- Knowing KD, t_p, r, and r/L, calculate S from Equation 4.9;
- Knowing KD and L, calculate c from the relation $c = L^2/KD$.

Remarks

- The accuracy of the calculated hydraulic characteristics depends on the accuracy

of the extrapolation of s_m. The calculations should therefore be checked by substituting the values of S, L, and KD into Equations 4.6 and 4.7.

Calculations of s should be made for different values of t. If the values of t are not too small, the values of s should fall on the observed data curve. If the calculated data deviate from the observed data, the extrapolation of s_m should be adjusted. Sometimes, the observed data curve can be drawn somewhat steeper or flatter through the plotted points, and so Δs_p can be adjusted too. With the new values of s_m and/or Δs_p, the calculation is repeated.

Example 4.4

From the pumping test 'Dalem', we use the data from the piezometer at 90 m (Table 4.2). We first plot the drawdown data of this piezometer versus t on semi-log paper (Figure 4.7) and then find the maximum (or steady-state) drawdown by extrapolation ($s_m = 0.147$ m). According to Equation 4.8, the drawdown at the inflection point $s_p = 0.5\ s_m = 0.0735$ m. Plotting this point on the time-drawdown curve, we obtain $t_p = 2.8 \times 10^{-2}$ d.

Through the inflection point of the curve, we draw a tangent line to the curve, which matches here with the straight portion of the curve itself. The slope of this tangent line $\Delta s_p = 0.072$ m.

Introducing these values into Equation 4.12 gives

$$2.30\frac{s_p}{\Delta s_p} = 2.30 \times \frac{0.0735}{0.072} = 2.34 = e^{r/L}K_0(r/L)$$

Annex 4.1 gives $r/L = 0.15$, and because $r = 90$ m, it follows that $L = 90/0.15 = 600$ m.

Further, $Q = 761\ m^3/d$ is given, and the value of $e^{-r/L} = e^{-0.15} = 0.86$ is found from Annex 4.1. Substituting these values into Equation 4.10 yields

$$KD = \frac{2.30Q}{4\pi\Delta s_p}e^{-r/L} = \frac{2.30 \times 761}{4 \times 3.14 \times 0.072} \times 0.86 = 1665\ m^2/d$$

and consequently

$$c = \frac{L^2}{KD} = \frac{(600)^2}{1665} = 216\ d$$

Introducing the appropriate values into Equation 4.9 gives

$$S = \frac{r4KDt_p}{2Lr^2} = \frac{90}{2 \times 600} \times \frac{4 \times 1665 \times 2.8 \times 10^{-2}}{90^2} = 1.7 \times 10^{-3}$$

To verify the extrapolated steady-state drawdown, we calculate the drawdown at a chosen moment, using Equations 4.6 and 4.7. If we choose $t = 0.1$ d, then

$$u = \frac{r^2S}{4KDt} = \frac{90^2 \times 1.7 \times 10^{-3}}{4 \times 1665 \times 10^{-1}} = 0.02$$

According to Annex 4.2, $W(u,r/L) = 3.11$ (for $u = 0.02$ and $r/L = 0.15$). Thus

$$s_{(t=0.1)} = \frac{Q}{4\pi KD}W(u,r/L) = \frac{761}{4 \times 3.14 \times 1665} \times 3.11 = 0.113\ m$$

The point $t = 0.1$, $s = 0.113$ falls on the time-drawdown curve and justifies the extrapolated value of s_m. In practice, several points should be tried.

Procedure 4.5

- On semi-log paper, plot s versus t for each piezometer (t on logarithmic scale) and draw curves through the plotted points (Figure 4.8);
- Determine the slope of the straight portion of each curve Δs;
- On semi-log paper, plot r versus Δs (Δs on logarithmic scale) and draw the best-fit straight line through the plotted points. (This line is the graphic representation of Equation 4.11);
- Determine the slope of this line Δr, i.e. the difference of r per log cycle of Δs (Figure 4.9);
- Extend the straight line until it intercepts the absciss where $r = 0$ and $\Delta s = (\Delta s)_o$. Read the value of $(\Delta s)_o$;
- Knowing the values of Δr and $(\Delta s)_o$, calculate L from

$$L = \frac{1}{2.30}\Delta r \tag{4.13}$$

and KD from

$$KD = 2.30\frac{Q}{4\pi(\Delta s)_o} \tag{4.14}$$

- Knowing KD and L, calculate c from the relation $c = L^2/KD$;

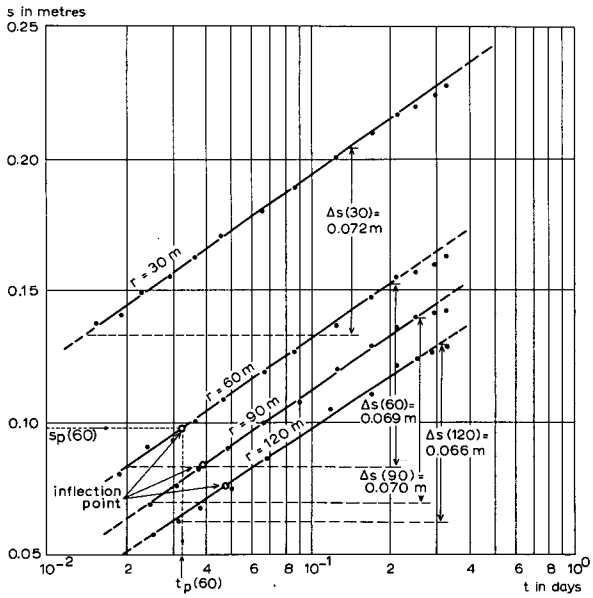

Figure 4.8 Analysis of data from pumping test 'Dalem' with Procedure 4.5 of the Hantush inflection-point method: determination of values of Δs for different values of r

- With the known values of Q, r, KD, and L, calculate s_p for each piezometer, using Equation 4.8: $s_p = (Q/4\pi KD)K_0(r/L)$ and the table for the function $K_0(x)$ in Annex 4.1;
- Plot each s_p value on its corresponding time-drawdown curve and read t_p on the absciss;
- Knowing the values of KD, r, r/L, and t_p, calculate S from Equation 4.9: $(r^2S)/(4KDt_p) = 0.5(r/L)$.

Example 4.5

From the pumping test 'Dalem', we use data from the piezometers at 30, 60, 90, and 120 m (Table 4.2). Figure 4.8 shows a time-drawdown plot for each of the piezometers on semi-log paper. Determining the slope of the straight portion of each curve, we obtain:

$\Delta s\,(30\text{ m}) = 0.072\text{ m}$ $\qquad$ $\Delta s\,(90\text{ m}) = 0.070\text{ m}$

$\Delta s\,(60\text{ m}) = 0.069\text{ m}$ $\qquad$ $\Delta s\,(120\text{ m}) = 0.066\text{ m}$

In Figure 4.9, the values of Δs are plotted versus r on semi-log paper and a straight line is fitted through the plotted points. Because of its steepness, the slope is measured as the difference of r over 1/20 log cycle of Δs. (If 1 log cycle measures 10 cm, 1/20 log cycle is 0.5 cm). The difference of r per 1/20 log cycle of Δs equals 120 m, or the difference of r per log cycle of Δs, i.e. Δr equals 2400 m. The straight line intersects the Δs axis where $r = 0$ in the point $(\Delta s)_o = 0.074$ m. Substitution of these values into Equations 4.13 and 4.14 gives

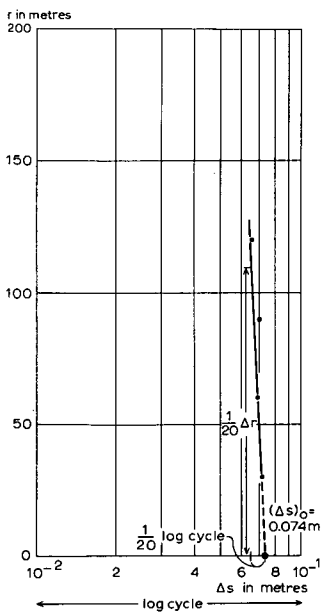

Figure 4.9 Idem: determination of the value of Δr

$$L = \frac{1}{2.30} r = \frac{1}{2.30} \times 2400 = 1043 \text{ m}$$

and because $Q = 761 \text{ m}^3/\text{d}$

$$KD = \frac{2.30Q}{4\pi(\Delta s)_o} = \frac{2.30 \times 761}{4 \times 3.14 \times 0.074} = 1883 \text{ m}^2/\text{d}$$

finally

$$c = \frac{L^2}{KD} = \frac{(1043)^2}{1883} = 578 \text{ d}$$

The value of r/L is calculated for each piezometer, and the corresponding values of $K_0(r/L)$ are found in Annex 4.1. The results are listed in Table 4.3.

Table 4.3 Data to be substituted into Equations 4.8 and 4.9

r (m)	r/L	$K_0(r/L)$	s_p (m)	t_p (d)	s_m (m)
30	0.0288	3.668	0.1180	outside figure	0.236
60	0.0575	2.984	0.0960	3.25×10^{-2}	0.192
90	0.0863	2.576	0.0829	3.85×10^{-2}	0.166
120	0.1150	2.290	0.0737	4.70×10^{-2}	0.147

The drawdown s_p at the inflection point of the curve through the observed data, as plotted in Figure 4.8 for the piezometer at 60 m, is calculated from Equation 4.8

$$s_p(60) = \frac{Q}{4\pi KD} K_0(r/L) = \frac{761}{4 \times 3.14 \times 1883} \times 2.984 = 0.0960 \text{ m}$$

The point on this curve for which $s = 0.0960$ m is determined; this is the inflection point. On the absciss, the value of t_p at the inflection point is $t_p(60) = 3.25 \times 10^{-2}$ d. From Equation 4.8, it follows that $s_m(60) = 2s_p(60) = 0.192$ m. This calculation was also made for the other piezometers. These results are also listed in Table 4.3. Substitution of the values of t_p into Equation 4.9 yields values of S. For example, for $r = 60$ m,

$$S = \frac{r}{2L} \frac{4KDt_p}{r^2} = \frac{60}{2 \times 1043} \times \frac{4 \times 1883 \times 3.25 \times 10^{-2}}{60^2} = 2.0 \times 10^{-3}$$

In the same way, for $r = 90$ m and for $r = 120$ m, the values of S are 1.5×10^{-3} and 1.4×10^{-3}, respectively. The average value of S is 1.6×10^{-3}.
It will be noted that the calculated values for the steady-state drawdown are somewhat higher than the extrapolated values from Table 4.1.

4.2.3 Hantush's curve-fitting method

Hantush (1960) presented a method of analysis that takes into account the storage

changes in the aquitard. For small values of pumping time, he gives the following drawdown equation for unsteady flow

$$s = \frac{Q}{4\pi KD} W(u,\beta) \tag{4.15}$$

where

$$u = \frac{r^2 S}{4KDt} \tag{4.16}$$

$$\beta = \frac{r}{4}\sqrt{\frac{K'/D'}{KD} \times \frac{S'}{S}} \tag{4.17}$$

S' = aquitard storativity

$$W(u,\beta) = \int_u^\infty \frac{e^{-y}}{y} \operatorname{erfc} \frac{\beta\sqrt{u}}{\sqrt{y(y-u)}} dy$$

Values of the function $W(u,\beta)$ are presented in Annex 4.3.

Hantush's curve-fitting method can be used if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of this chapter;
- The flow to the well is in an unsteady state;
- The aquitard is compressible, i.e. the changes in aquitard storage are appreciable;
- $t < S'D'/10K'$.

Only the early-time drawdown data should be used so as to satisfy the assumption that the drawdown in the aquitard (or overlying unpumped aquifer) is negligible.

Procedure 4.6
- Using Annex 4.3, construct on log-log paper the family of type curves $W(u,\beta)$ versus $1/u$ for different values of β (Figure 4.10);
- On another sheet of log-log paper of the same scale, plot s versus t for one of the piezometers;
- Match the observed data plot with one of the type curves (Figure 4.11);
- Select an arbitrary point A on the overlapping portion of the two sheets and note the values of $W(u,\beta)$, $1/u$, s, and t for this point. Note the value of β on the selected type curve;
- Substitute the values of $W(u,\beta)$ and s and the known value of Q into Equation 4.15 and calculate KD;
- Substitute the values of KD, t, r, and the reciprocal value of $1/u$ into Equation 4.16 and solve for S;
- Substitute the values of β, KD, S, r, and D' into Equation 4.17 and solve for $K'S'$.

Remarks
- It is difficult to obtain a unique match of the two curves because the shapes of the type curves change gradually with β (β values are practically indeterminate in the range $\beta = 0 \rightarrow \beta = 0.5$, because the curves are very similar);

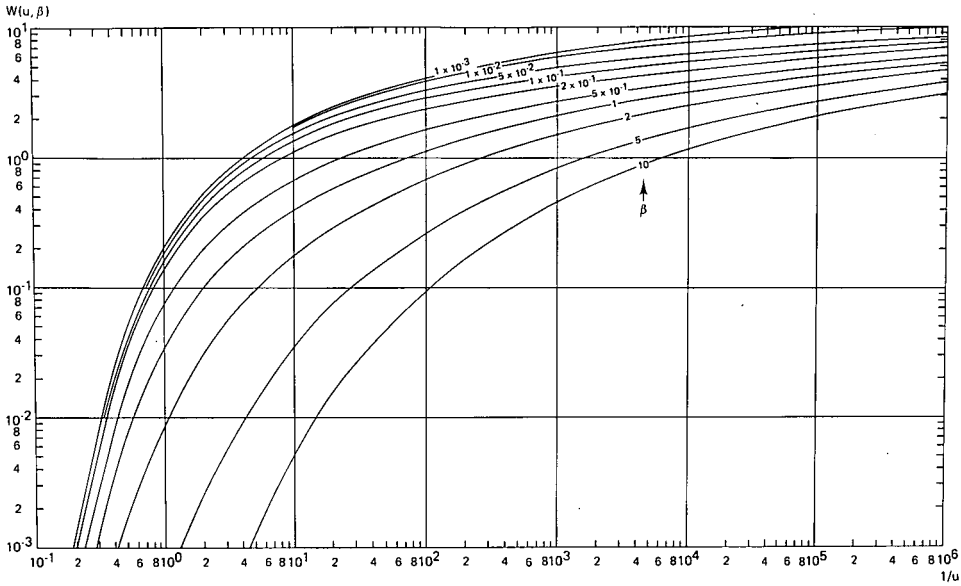

Figure 4.10 Family of Hantush's type curves W(u,β) versus 1/u for different values of β

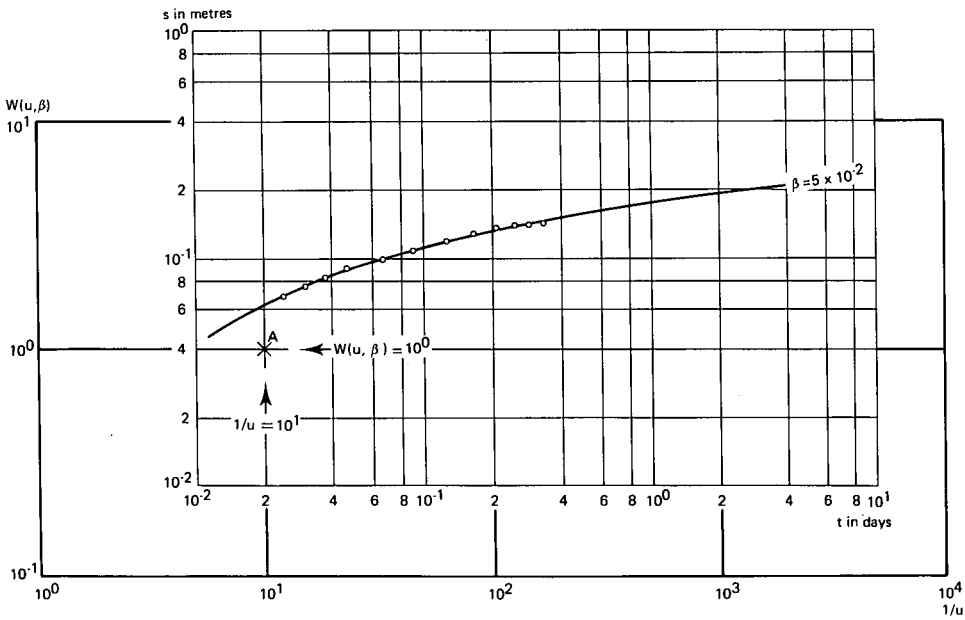

Figure 4.11 Analysis of data from pumping test 'Dalem' (r = 90 m) with the Hantush curve-fitting method

- As K′ approaches zero, the limit of Equation 4.15 is equal to the Theis equation $s = (Q/4\pi KD)W(u)$. If the ratio of the storativity of the aquitard and the storativity of the leaky aquifer is small ($S'/S < 0.01$), the effect of any storage changes in the aquitard on the drawdown in the aquifer is very small. In that case, and for small values of pumping time, the Theis formula (Equation 3.5) can be used (see also Section 4.2.4).

Example 4.6
From the pumping test 'Dalem' we use the drawdown data from the piezometer at 90 m (Table 4.2), plotting on log-log paper the drawdown data against the corresponding values of t (Figure 4.11). A comparison of the data plot with the Hantush family of type curves shows that the best fit of the plotted points is obtained with the curve $\beta = 5 \times 10^{-2}$. We choose a match point A, whose coordinates are $W(u,\beta) = 10^0$, $1/u = 10$, $s = 4 \times 10^{-2}$ m, and $t = 2 \times 10^{-2}$ d. Substituting these values, together with the values of $Q = 761$ m³/d and $r = 90$ m, into Equations 4.15, 4.16, and 4.17, we obtain

$$KD = \frac{Q}{4\pi s} W(u,\beta) = \frac{761}{4 \times 3.14 \times 4 \times 10^{-2}} 10^0 = 1515 \text{ m}^2/\text{d}$$

$$S = \frac{4KDtu}{r^2} = \frac{4 \times 1515 \times 2 \times 10^{-2} \times 10^{-1}}{90^2} = 1.5 \times 10^{-3}$$

$$\frac{K'S'}{D'} = \beta^2(4/r)^2KDS = (5 \times 10^{-2})^2 \times (4/90)^2 \times 1515 \times 1.5 \times 10^{-3}$$
$$= 1.1 \times 10^{-5} \text{ d}^{-1}$$

The thickness of the aquitard $D' = 8$ m (Figure 4.2). Hence, $K'S' = 9 \times 10^{-5}$ m/d.

To check whether the condition $t < S'D'/10K'$ is fulfilled, we need more calculated parameters. Using the value of $c = D'/K' = 450$ d (see Section 4.3), we can calculate an approximate value of S′

$$\frac{K'S'}{D'} = 1.1 \times 10^{-5} \text{ d}^{-1}$$

$$S' = 450 \times 1.1 \times 10^{-5} = 5 \times 10^{-3}$$

Hence

$$t < 5 \times 10^{-3} \times 450 \times 0.1 \text{ or } t < 0.225 \text{ d}$$

If this time condition is to be satisfied, the drawdown data measured at $t = 2.50 \times 10^{-1}$, 2.92×10^{-1}, and 3.33×10^{-1} d should not be used in the analysis (Figure 4.11).
Note: Because the data curve matches with a type curve in the range $\beta = 0 \rightarrow \beta = 0.5$, not too much value should be attached to the exact value of β, nor to the calculated value of K′S′.

4.2.4 Neuman-Witherspoon's method

Neuman and Witherspoon (1972) developed a method for determining the hydraulic

characteristics of aquitards at small values of pumping time when the drawdown in the overlying unconfined aquifer is still negligible. The method is based on a theory developed for a so-called slightly leaky aquifer (Neuman and Witherspoon 1968), where the drawdown function in the pumped aquifer is given by the Theis equation (Equation 3.5), and the drawdown in the aquitard of very low permeability is described by

$$s_c = \frac{Q}{4\pi KD} W(u,u_c) \tag{4.18}$$

where

$$W(u,u_c) = \frac{2}{\sqrt{\pi}} \int_{\sqrt{u_c}}^{\infty} -Ei\left(-\frac{uy^2}{y^2-u_c}\right) e^{-y^2}\, dy$$

$$u_c = \frac{z^2 S'}{4K'D't} \tag{4.19}$$

$\frac{K'D'}{S'}$ = hydraulic diffusivity of the aquitard in m^2/d

z = vertical distance from aquifer-aquitard boundary to piezometer in the aquitard in m

At the same elapsed time and the same radial distance from the well, the ratio of the drawdown in the aquitard and the drawdown in the pumped aquifer is

$$\frac{s_c}{s} = \frac{W(u,u_c)}{W(u)}$$

Figure 4.12 shows curves of $W(u,u_c)/W(u)$ versus $1/u_c$ for different values of u. These curves have been prepared from values given by Witherspoon et al. (1967) and are presented in Annex 4.4. Knowing the ratio s_c/s from the observed drawdown data and a previously determined value of u for the aquifer, we can read a value of $1/u_c$ from Figure 4.12. By substituting the value of $1/u_c$ into Equation 4.19, we can determine the hydraulic diffusivity of the aquitard of very low permeability.

Neuman and Witherspoon (1972) showed that their ratio method, although developed for a slightly leaky aquifer, can also be used for a very leaky aquifer. The only requirement is that, in Equation 4.17, $\beta \leq 1.0$ because, as long as $\beta \leq 1.0$, the ratio s_c/s is found to be independent of β for all practical values of u_c. As β is directly proportional to the radial distance r from the well to the piezometer, r should be small ($r < 100$ m).

The Neuman-Witherspoon ratio method can be applied if the following assumptions and conditions are fulfilled:

- The assumptions listed at the beginning of this chapter;
- The flow to the well is in an unsteady state;
- The aquitard is compressible, i.e. the changes in aquitard storage are appreciable;
- $\beta < 1.0$, i.e. the radial distance from the well to the piezometers should be small ($r < 100$ m);
- $t < S'D'/10K'$.

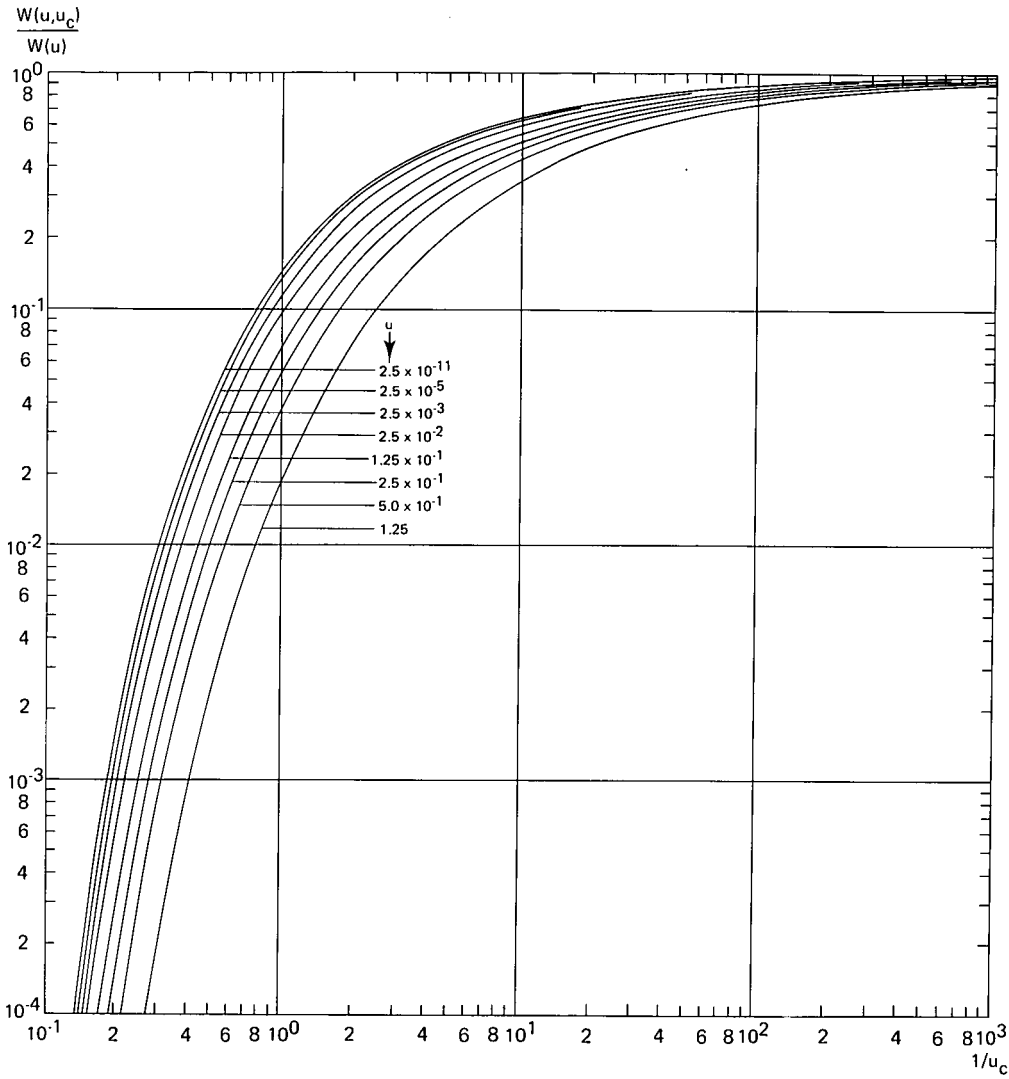

Figure 4.12 Neuman-Witherspoon's nomogram showing the relation of $W(u,u_c)/W(u)$ versus $1/u_c$ for different values of u

Procedure 4.7

- Calculate the transmissivity KD and the storativity S of the aquifer with one of the methods described in Section 4.2, using the early-time drawdown data of the aquifer;
- For a selected value of r ($r < 100$ m), prepare a table of values of the drawdown in the aquifer s, in the overlying aquitard s_c, and, if possible, in the overlying unconfined aquifer s_u for different values of t (see Remarks below);
- Select a time t and calculate for this value of t the value of the ratio s_c/s and the value of $u = r^2S/4KDt$;

- Knowing $s_c/s = W(u,u_c)/W(u)$ and u, determine the corresponding value of $1/u_c$, using Figure 4.12;
- Substitute the value of $1/u_c$ and the values of z and t into Equation 4.19, written as

$$\frac{K'D'}{S'} = \frac{1}{u_c} \times \frac{z^2}{4t}$$

  and calculate the hydraulic diffusivity of the aquitard $K'D'/S'$;
- Repeat the calculation of $K'D'/S'$ for different values of t, i.e. for different values of s_c/s and u. Take the arithmetic mean of the results;
- Repeat the procedure if data from more than one set of piezometers are available. Take the arithmetic mean of the results.

Remarks

- To check whether the selected value of t falls in the period in which the method is valid, the calculated values of S′, D′, and K′ have to be substituted into $t < S'D'/10K'$. Neuman and Witherspoon (1969a) showed that this time criterion is rather conservative. It is also possible to use drawdown data from piezometers in the unpumped unconfined aquifer and to read the time limit from the data plot of s_u versus t on log-log paper. However, if KD of the unpumped aquifer is relatively large, the drawdown s_u will be too small to determine the time limit reliably;
- According to Neuman and Witherspoon (1972), the KD and S values of a leaky aquifer can be determined with the methods of analysis based on the Theis solution (Section 3.2). They state that the errors introduced by these methods will be small if the earliest available drawdown data, collected close to the pumped well, are used;
- Neuman and Witherspoon (1972) also observed that when $u < 2.5 \times 10^{-3}$ the curves in Figure 4.12 are so close to each other that they can be assumed to be practically independent of u. Then, even a crude estimate of u will be sufficient for the ratio method to yield satisfactory results;
- The ratio method is also applicable to multiple leaky aquifer systems, provided that the sum of the β values related to the overlying and/or underlying aquitards is less than 1.

Example 4.7

The data are taken from the pumping test 'Dalem'. At 30 m from the well, piezometers were placed at depths of 2 and 14 m below ground surface. The drawdowns in them at $t = 4.58 \times 10^{-2}$ d are $s_c = 0.009$ m and $s = 0.171$ m, respectively. The values of the aquifer characteristics are taken from Table 4.4: KD = 1800 m^2/d and $S = 1.7 \times 10^{-3}$. Consequently

$$u = \frac{r^2S}{4KDt} = \frac{30^2 \times 1.7 \times 10^{-3}}{4 \times 1800 \times 4.58 \times 10^{-2}} = 4.6 \times 10^{-3}$$

and

$$\frac{s_c}{s} = \frac{0.009}{0.171} = 5.3 \times 10^{-2}$$

Plotting the value of $s_c/s = 5.3 \times 10^{-2}$ on the $W(u,u_c)/W(u)$ axis of the plot in Figure

4.12 and knowing the value of $u = 4.6 \times 10^{-3}$, we can read the value of $1/u_c$ from the horizontal axis of this plot: $1/u_c = 6.4 \times 10^{-1}$.

As the depth of the piezometer in the aquitard is 2 m below ground surface and $D' = 8$ m, it follows that $z = 6$ m. Consequently, the hydraulic diffusivity of the aquitard is

$$\frac{K'D'}{S'} = \frac{1}{u_c} \times \frac{z^2}{4t} = 6.4 \times 10^{-1} \times \frac{6^2}{4 \times 4.58 \times 10^{-2}} = 126\ \mathrm{m^2/d}$$

The Neuman-Witherspoon method is only applicable if $t < S'D'/10K'$. From $K'D'/S' = 126\ \mathrm{m^2/d}$ and $D' = 8$ m, it follows that

$$t < 0.1\left(\frac{K'D'}{S'} \times \frac{1}{(D')^2}\right)^{-1}, \text{or } t < 0.1\,(126 \times 1/8^2)^{-1} = 0.05\ \mathrm{d}$$

Hence, the time condition is fulfilled (the pumping time t used in the calculation was 4.58×10^{-2} d). As the radial distance of the piezometer to the well is 30 m, the condition $r < 100$ m is also satisfied.

4.3 Summary

Using data from the pumping test 'Dalem', we have illustrated the methods of analyzing steady and unsteady flow to a well in a leaky aquifer. Table 4.4 summarizes the values we obtained for the hydraulic characteristics of both the aquifer and the aquitard.

Table 4.4 Hydraulic characteristics of the leaky aquifer system at 'Dalem', calculated with the different methods

Method	Data from piezometer	KD (m^2/d)	S	L (m)	c (d)	K'S' (m/d)	K'D'/S' (m^2/d)
De Glee	All	2126	–	1100	569	–	–
Hantush-Jacob	All	2020	–	982	478	–	–
Walton	90	1731	1.9×10^{-3}	900	468	–	–
Hantush inflection-point 1	90	1665	1.7×10^{-3}	600	216	–	–
Hantush inflection-point 2	All	1883	1.6×10^{-3}	1043	578	–	–
Hantush curve-fitting	90	1515	1.5×10^{-3}	–	–	9×10^{-5}	–
Neuman-Witherspoon	30	–	–	–	–	–	126

We could thus conclude that the leaky aquifer system at 'Dalem' has the following (average) hydraulic characteristics:

Aquifer: KD	$= 1800\ m^2/d$	Aquitard: c	$= 450\ d$
S	$= 1.7 \times 10^{-3}$	K′D′/S′	$= 126\ m^2/d$
L	$= 900\ m$		

From the aquitard characteristics, we could calculate values of K′ and S′:

$$K' = D'/c = 8/450 = 1.8 \times 10^{-2}\ m/d$$
$$S' = K'D'/126 = 1.1 \times 10^{-3}$$

It will be noted that the different methods produce somewhat different results. This is due to inevitable inaccuracies in the observed and corrected or extrapolated data used in the calculations, but also, and especially, to the use of graphical methods. The steady-state drawdowns used in our examples, for instance, were extrapolated values and not measured values. These extrapolated values can be checked with Procedure 4.5 of the Hantush inflection-point method, but this requires a lot of straight lines having to be fitted through observed and calculated data that do not fall exactly on a straight line. Consequently, there are slightly different positions possible for these lines, which are still acceptable as fitted straight lines, but give different values of the hydraulic parameters.

The same difficulties are encountered when observed data plots have to be matched with a type curve or a family of type curves. In these cases too, slightly different matching positions are possible, with different match-point coordinates as a result, and thus different values for the hydraulic parameters. Because of such matching problems, the value of K′S′ in Table 4.4 is not considered to be very reliable.

Most of the methods described in this chapter only require data from the pumped aquifer. But, as already stated by Neuman and Witherspoon (1969b), such data are not sufficient to characterize a leaky system: the calculations should also be based on drawdown data from the aquitard and, if present, from the overlying unconfined unpumped aquifer, whose watertable will not remain constant, except for ideal situations, which are rare in nature.

Moreover, it should be kept in mind that, in practice, the assumptions underlying the methods are not always entirely satisfied. One of the assumptions, for instance, is that the aquifer is homogeneous, isotropic, and of uniform thickness, but it will be obvious that for an aquifer made up of alluvial sand and gravel, this assumption is not usually correct and that its hydraulic characteristics will vary from one place to another.

Summarizing, we can state that the average results of the calculations presented above are the most accurate values possible, and that, given the lithological character of the aquifer, aiming for any higher degree of accuracy would be to pursue an illusion.