18 Single vertical fractures

18.1 Introduction

If a well intersects a single vertical fracture, the aquifer's unsteady drawdown response to pumping differs significantly from that predicted by the Theis solution (Chapter 3). This well-flow problem has long been a subject of research in the petroleum industry, especially after it had been discovered that if an oil well is artificially fractured ('hydraulic fracturing') its yield can be raised substantially. Various solutions to this problem have been proposed, but most of them produced erroneous results. A major step forward was taken when the fracture was assumed to be a plane, vertical fracture of relatively short length and infinite hydraulic conductivity. (A plane fracture is one of zero width, which means that fracture storage can be neglected.) This made it possible to analyze the system as an 'equivalent', anisotropic, homogeneous, porous medium, with a single fracture of high permeability intersected by the pumped well.

The concept underlying the analytical solutions is as follows: The aquifer is homogeneous, isotropic, and of large lateral extent, and is bounded above and below by impermeable beds. A single plane, vertical fracture of relatively short length dissects the aquifer from top to bottom (Figure 18.1A). The pumped well intersects the fracture midway. The fracture is assumed to have an infinite (or very large) hydraulic conductivity. This means that the drawdown in the fracture is uniform over its entire length at any instant of time (i.e. there is no hydraulic gradient in the fracture). This uniform drawdown induces a flow from the aquifer into the fracture. At early pumping times, this flow is one-dimensional (i.e. it is horizontal, parallel, and perpendicular to the fracture) (Figure 18.1B). All along the fracture, a uniform flux condition is assumed to exist (i.e. water from the aquifer enters the fracture at the same rate per unit area).

Groundwater hydrology recognizes a similar situation: that of a constant groundwater discharge into an open channel that fully penetrates a homogeneous unconsolidated aquifer. Solutions to this flow problem have been presented by Theis (1935), Edelman (1947; 1972), Ferris (1950), and Ferris et al. (1962). It is hardly surprising that the solutions that have been developed for early-time drawdowns in a single vertical fracture are identical to those found by the above authors (Jenkins and Prentice 1982).

As pumping continues, the flow pattern changes from parallel flow to pseudo-radial flow (Figure 18.1C), regardless of the fracture's hydraulic conductivity. During this period, most of the well discharge originates from areas farther removed from the fracture. Often, uneconomic pumping times are required to attain pseudo-radial flow, but once it has been attained, the classical methods of analysis can be applied.

The methods presented in this chapter are all based on the following general assumptions and conditions:

- The general assumptions and conditions listed in Section 17.1.

And:

- The aquifer is confined, homogeneous, and isotropic, and is fully penetrated by a single vertical fracture;

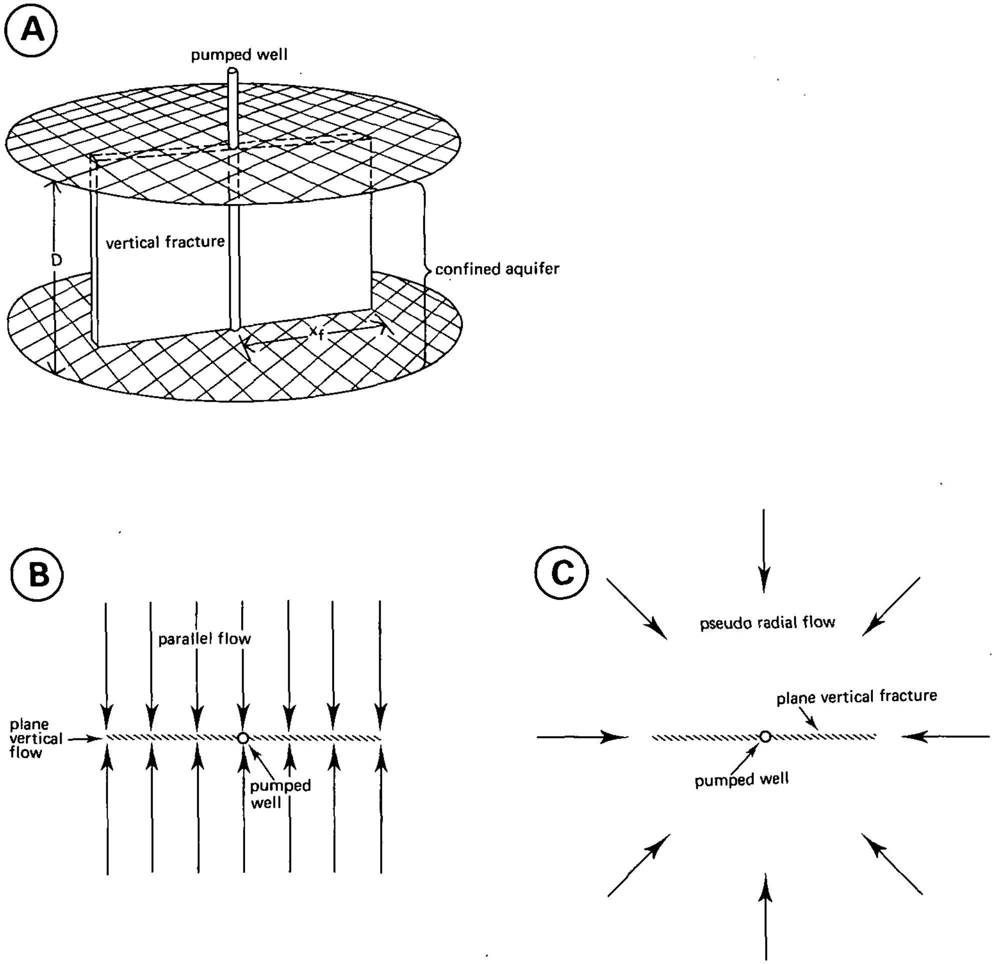

Figure 18.1 A well that intersects a single, vertical, plane fracture of finite length and infinite hydraulic conductivity
A: The well-fracture-aquifer system
B: The parallel flow system at early pumping times
C: The pseudo-radial flow system at late pumping times

- The fracture is plane (i.e. storage in the fracture can be neglected), and its horizontal extent is finite;
- The well is located on the axis of the fracture;
- With decline of head, water is instantaneously removed from storage in the aquifer;
- Water from the aquifer enters the fracture at the same rate per unit area (i.e. a uniform flux exists along the fracture, or the fracture conductivity is high although not infinite);

The first method in this chapter, in Section 18.2, is that of Gringarten and Witherspoon

(1972), which uses the drawdown data from observation wells placed at specific locations with respect to the pumped well. Next, in Section 18.3, is the method of Gringarten and Ramey (1974); it uses drawdown data from the pumped well only, neglecting well losses and well-bore storage effects. Finally, in Section 18.4, we present the Ramey and Gringarten method (1976), which allows for well-bore storage effects in the pumped well.

18.2 Gringarten-Witherspoon's curve-fitting method for observation wells

For a well pumping a single, plane, vertical fracture in an otherwise homogeneous, isotropic, confined aquifer (Figure 18.2), Gringarten and Witherspoon (1972) obtained the following general solution for the drawdown in an observation well

$$s = \frac{Q}{4\pi T} F(u_{vf}, r') \tag{18.1}$$

where

$$u_{vf} = \frac{T\,t}{Sx_f^2} \tag{18.2}$$

$$r' = \frac{\sqrt{x^2 + y^2}}{x_f} \tag{18.3}$$

S = storativity of the aquifer, dimensionless
T = transmissivity of the aquifer (m^2/d)
x_f = half length of the vertical fracture (m)
x,y = distance between observation well and pumped well, measured along the x and y axis, respectively (m)

From Equations 18.1 and 18.2, it can be seen that the drawdown in an observation well depends not only on the parameter u_{vf} (i.e. on the aquifer characteristics T and S, the vertical fracture half-length x_f, and the pumping time t), but also on the geometrical relationship between the location of the observation well and that of the fracture.

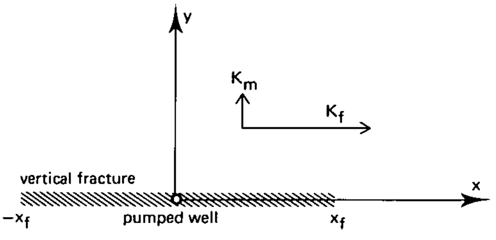

Figure 18.2 Plan view of a pumped well that intersects a plane, vertical fracture of finite length and infinite hydraulic conductivity

For observation wells in three different locations (Figure 18.3), Gringarten and Witherspoon developed simplified expressions for the drawdown derived from Equation 18.1.

For an observation well located along the x axis ($r' = x/x_f$), the drawdown function $F(u_{vf},r')$ in Equation 18.1 reads

$$F(u_{vf},r') = \frac{\sqrt{\pi}}{2} \int_0^{u_{vf}} \left[\operatorname{erf}\left(\frac{1-r'}{2\sqrt{\tau}}\right) + \operatorname{erf}\left(\frac{1+r'}{2\sqrt{\tau}}\right) \right] \frac{d\tau}{\sqrt{\tau}} \tag{18.4}$$

For an observation well located along the y axis ($r' = y/x_f$), the drawdown function $F(u_{vf},r')$ in Equation 18.1 reads

$$F(u_{vf},r') = \sqrt{\pi} \int_0^{u_{vf}} \operatorname{erf}\left(\frac{1}{2\sqrt{\tau}}\right) \exp\left[-\frac{(r')^2}{4\tau}\right] \frac{d\tau}{\sqrt{\tau}} \tag{18.5}$$

For an observation well located along a line through the pumped well and making an angle of 45° with the direction of the fracture ($r' = x\sqrt{2}/x_f = y\sqrt{2}/x_f$), the drawdown function $F(u_{vf},r')$ in Equation 18.1 reads

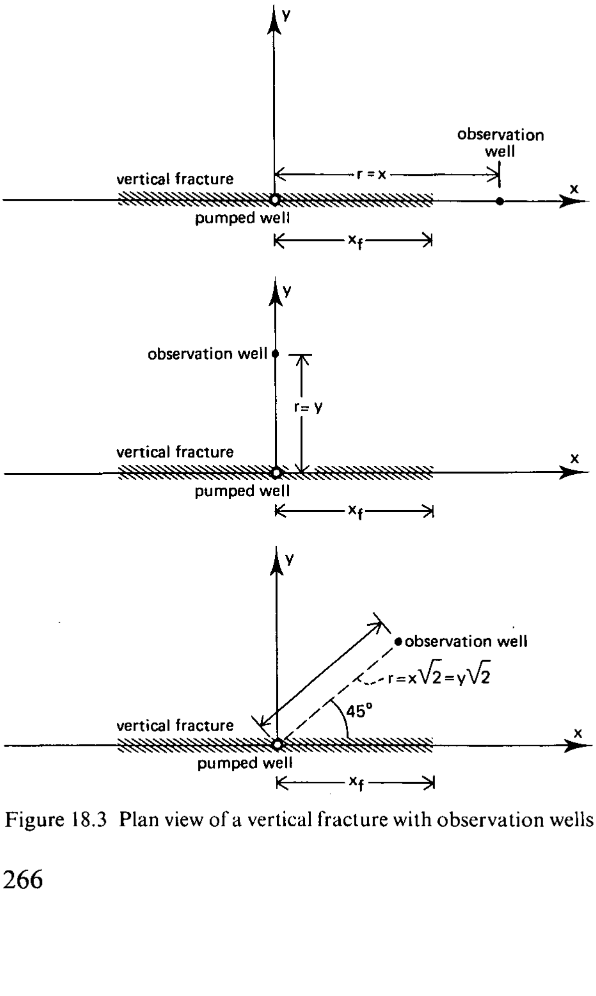

Figure 18.3 Plan view of a vertical fracture with observation wells at three different locations

$$F(u_{vf},r') = \frac{\sqrt{\pi}}{2}\int_0^{u_{vf}} \exp\left[-\frac{\left(\frac{r'}{\sqrt{2}}\right)^2}{4\tau}\right] \cdot \left[\mathrm{erf}\left(\frac{1-\left(\frac{r'}{\sqrt{2}}\right)}{2\sqrt{\tau}}\right) + \mathrm{erf}\left(\frac{1+\left(\frac{r'}{\sqrt{2}}\right)}{2\sqrt{\tau}}\right)\right] \frac{d\tau}{\sqrt{\tau}}$$

Figures 18.4, 18.5, and 18.6 show the three different families of type curves developed from Equations 18.4, 18.5, and 18.6, respectively (Gringarten and Witherspoon 1972; see also Thiery et al. 1983). For the three locations of observation well, Annex 18.1 gives values of the function $F(u_{vf},r')$ for different values of u_{vf} and r'.

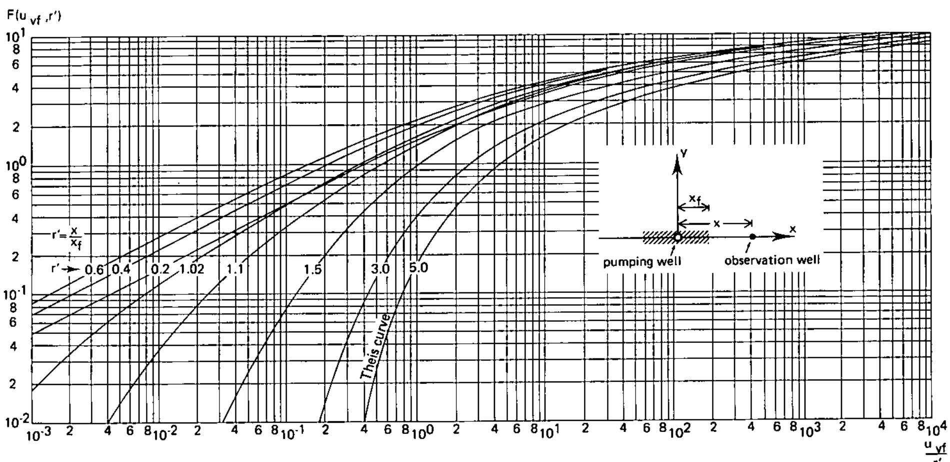

Figure 18.4 Gringarten-Witherspoon's type curves for a vertical fracture with an observation well located on the x axis (after Merton 1987)

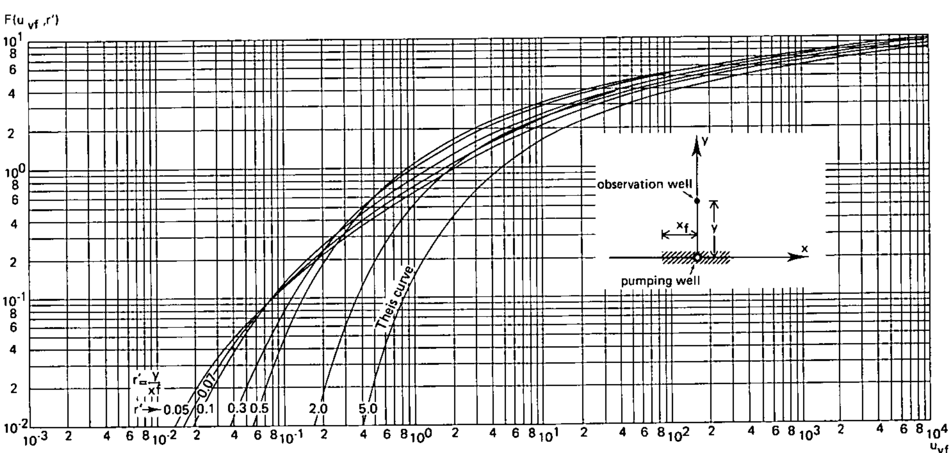

Figure 18.5 Gringarten-Witherspoon's type curves for a vertical fracture with an observation well located on the y axis (after Merton 1987)

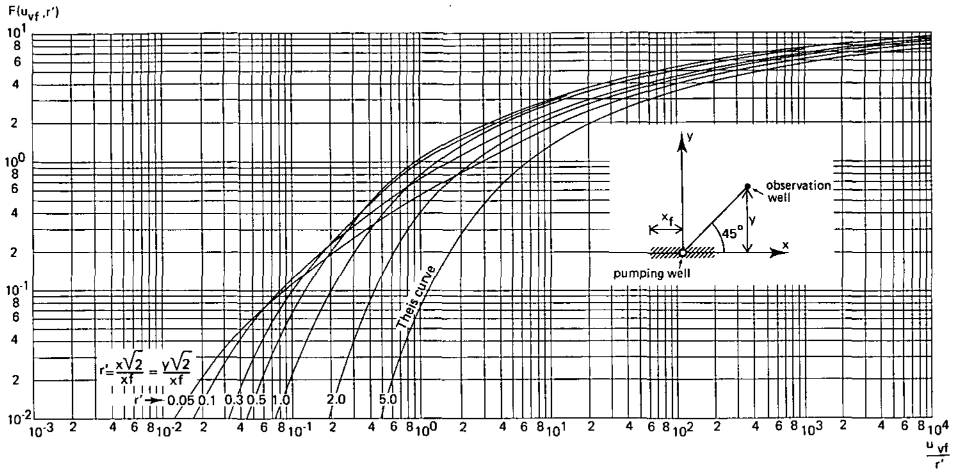

Figure 18.6 Gringarten-Witherspoon's type curves for a vertical fracture with an observation well located at 45° from the centre of the fracture (after Merton 1987)

The type curves in Figures 18.4, 18.5, and 18.6 clearly indicate that the drawdown response in an observation well differs from that in a pumped well. As long as an observation well does not intersect the same fracture as the pumped well, the log-log plot of the drawdown in the observation well does not yield an initial straight line of slope 0.5. Far enough from the pumped well (i.e. $r' > 5$), the drawdown response becomes identical to that for radial flow to a pumped well in the Theis equation (Equation 3.5). In other words, beyond a distance $r' = 5$, the influence of the fracture on the drawdown is negligible.

The Gringarten-Witherspoon curve-fitting method can be used if the assumptions and conditions listed in Section 18.1 are met.

Procedure 18.1

- If the location of the observation well is known with respect to the location of the fracture, choose the appropriate set of type curves (for $r' = x/x_f$; $r' = y/x_f$; or $r' = x\sqrt{2}/x_f = y\sqrt{2}/x_f$);
- Using Annex 18.1, prepare the selected family of type curves on log-log paper by plotting $F(u_{vf},r')$ versus u_{vf}/r' for different values of r';
- On another sheet of log-log paper of the same scale, plot s versus t for the observation well;
- Match the data plot with one of the type curves and note the value of r' for that curve;
- Knowing r and r', calculate the fracture half-length, x_f, from $r' = r/x_f$;
- Select a matchpoint A on the superimposed sheets and note for A the values of $F(u_{vf},r')$, u_{vf}/r', s, and t;
- Substitute the values of $F(u_{vf},r')$ and s and the known value of Q into Equation 18.1 and calculate T;
- Knowing u_{vf}/r' and r', calculate the value of u_{vf};

– Substitute the values of u_{vf}, t, x_f, and T into Equation 18.2 and solve for S.

If the geometrical relationship between the observation wells and the fracture is not known, a trial-and-error matching procedure will have to be applied to all three sets of type curves. Data from at least two observation wells are required for this purpose. The trial-and-error procedure should be continued until matching positions are found that yield approximations of the fracture location and its dimensions, and estimates of the aquifer parameters consistent with all available observation-well data.

Remarks

- For $r' \geq 5$, no real value of r' (and consequently of x_f) can be found with the Gringarten-Witherspoon method alone because no separate type curves for $r' \geq 5$ can be distinguished. It will only be possible to calculate a maximum value of x_f. If data from the pumped well are also available, however, the product Sx_f^2 can be obtained (Section 18.3). Then, knowing S from the observation-well data, and also knowing Sx_f^2, one can calculate x_f. It should be noted, however, that calculated values of x_f are not precise and are often underestimated (Gringarten et al. 1975);
- For $r' \geq 5$, the observation-well data can be analyzed with the Theis method (Section 3.2.1), from which the aquifer parameters T and S can be obtained.

18.3 Gringarten et al.'s curve-fitting method for the pumped well

For a well intersecting a single, plane, vertical fracture in an otherwise homogeneous, isotropic, confined aquifer (Figure 18.1A), Gringarten and Ramey. (1974) obtained the following general solution for the drawdown in the pumped well

$$s_w = \frac{Q}{4\pi T} F(u_{vf}) \tag{18.7}$$

where

$$F(u_{vf}) = 2\sqrt{\pi u_{vf}}\,\mathrm{erf}\left(\frac{1}{2\sqrt{u_{vf}}}\right) - \mathrm{Ei}\left(-\frac{1}{4u_{vf}}\right) \tag{18.8}$$

and

$$-\mathrm{Ei}(-x) = \int_0^x \frac{e^{-u}}{u}\,du = \text{the exponential integral of x}$$

Equation 18.8 is the reduced form of Equations 18.4 to 18.6 for $r' = 0$. Values of the function $F(u_{vf})$ for different values of u_{vf} are given in Annex 18.2. Figure 18.7 shows a log-log plot of $F(u_{vf})$ versus u_{vf}.

At early pumping times, when the drawdown in the well is governed by the horizontal parallel flow from the aquifer into the fracture, the drawdown can be written as

$$s_w = \frac{Q}{4\pi T} F(u_{vf}) \qquad (18.7)$$

where

$$F(u_{vf}) = 2\sqrt{\pi u_{vf}} \tag{18.9}$$

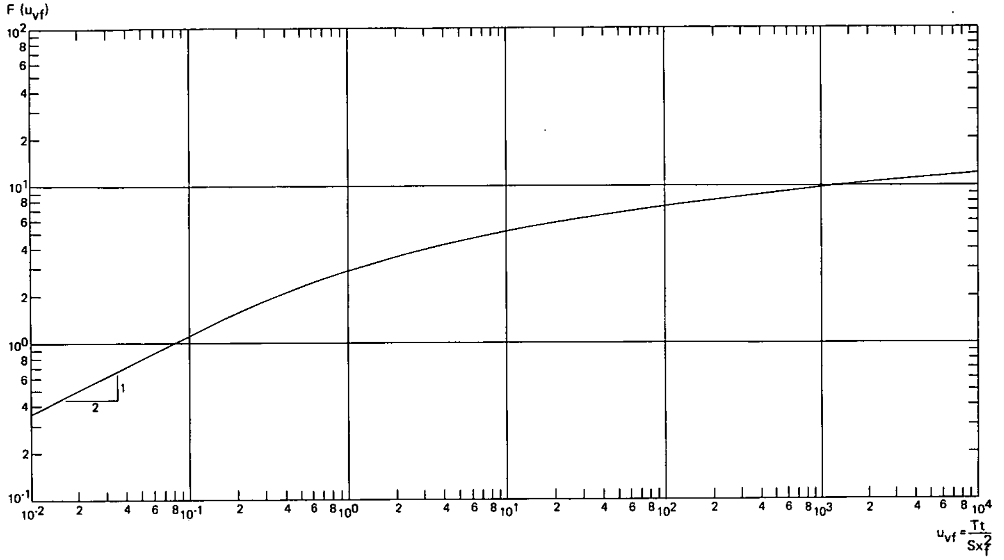

Figure 18.7 Gringarten et al.'s type curve $F(u_{vf})$ versus u_{vf} for a vertical fracture

or

$$\log F(u_{vf}) = 0.5 \log (u_{vf}) + \text{constant}$$

and consequently

$$s_w = \frac{Q}{2\sqrt{\pi T S x_f^2}} \sqrt{t} \qquad (18.10)$$

or

$$\log s_w = 0.5 \log(t) + \text{constant}$$

As Equations 18.9 and 18.10 show, on a log-log plot of $F(u_{vf})$ versus u_{vf} (Figure 18.7) (and on the corresponding data plot), the early-time parallel-flow period is characterized by a straight line with a slope of 0.5. The parallel-flow period ends at approximately $u_{vf} = 1.6 \times 10^{-1}$(Gringarten and Ramey. 1975). If the aquifer has a low transmissivity and the fracture is elongated, the parallel-flow period may last relatively long.

The pseudo-radial-flow period starts at $u_{vf} = 2$ (Gringarten et al. 1975). During this period, the drawdown in the well varies according to the Theis equation for radial flow in a pumped, homogeneous, isotropic, confined aquifer (Equation 3.5), plus a constant, and can be approximated by the following expression (Gringarten and Ramey. 1974)

$$s_w = \frac{2.30Q}{4\pi T} \log \frac{16.59Tt}{Sx_f^2} \qquad (18.11)$$

The log-log plot of $F(u_{vf})$ versus u_{vf} (Figure 18.7) is used as a type curve to determine T and the product Sx_f^2.

Gringarten et al.'s method is based on the following assumptions and conditions:
- The general assumptions and conditions listed in Section 18.1.

And:
- The diameter of the well is very small (i.e. well-bore storage can be neglected);
- The well losses are negligible.

Procedure 18.2
- Using Annex 18.2, prepare a type curve on log-log paper by plotting $F(u_{vf})$ versus u_{vf};
- On another sheet of log-log paper of the same scale, prepare the data curve by plotting s_w versus t;
- Match the data curve with the type curve and select a matchpoint A on the superimposed sheets; note for A the values of $F(u_{vf})$, u_{vf}, s_w, and t;
- Substitute the values of $F(u_{vf})$ and s_w and the known value of Q into Equation 18.7 and calculate T;
- Substitute the values of u_{vf} and t and the calculated value of T into Equation 18.2 and solve for the product Sx_f^2.

For large values of pumping time (i.e. for $t \geq 2Sx_f^2/T$), the data can be analyzed with Procedure 18.3, which is similar to Procedure 3.4 of the Jacob method (Section 3.2.2).

Procedure 18.3
- If the semi-log plot of s_w versus t yields a straight line, determine the slope of this line, Δs_w;
- Calculate the aquifer transmissivity from $T = 2.30Q/4\pi\Delta s_w$;
- As T is known and the value of t_o can be read from the graph, find Sx_f^2 from $Sx_f^2 = 16.59\, Tt_o$.

Remarks
- No separate values of x_f and S can be found with Gringarten et al.'s method. To obtain such values, one must have drawdown data from at least two observation wells. (See method in Section 18.2);
- Procedures 18.2 and 18.3 can only be applied to data from perfect wells (i.e. wells that have no well losses). Such wells seldom exist, but Procedure 18.3, being applied to late-time drawdown data, allows the aquifer transmissivity to be found;
- If the early-time drawdown data are influenced by well-bore storage, the initial straight line in the data plot may not have a slope of 0.5, but instead a slope of 1, which indicates a large storage volume connected with the well. This corresponds to a fracture of large dimensions rather than the assumed plane fracture. Gringarten et al.'s method will then not be applicable and the data should be analyzed by the method in Section 18.4.

18.4 Ramey-Gringarten's curve-fitting method

For a well intersecting a non-plane vertical fracture in a homogeneous, isotropic, confined aquifer, Ramey and Gringarten (1976) developed a method that takes the storage effects of the fracture into account. Their equation reads

$$s_w = \frac{Q}{4\pi T} F(u_{vf}, C'_{vf}) \tag{18.12}$$

where

$$C'_{vf} = \frac{C_{vf}}{Sx_f^2} \tag{18.13}$$

C_{vf} = a storage constant = $\Delta V / s_w$ = ratio of change in volume of water in the well plus vertical fracture, and the corresponding drawdown (m^2)

Ramey and Gringarten developed their equation by assuming a large-diameter well and a plane vertical fracture of infinite conductivity. In practice, however, the apparent storage effect, C_{vf}, is due not only to the total volume of the well, but also to the pore volume of the fracture.

The family of type curves drawn on the basis of Equation 18.12 is shown in Figure 18.8. Annex 18.3 gives a table of the values of $F(u_{vf}, C'_{vf})$ for different values of u_{vf}

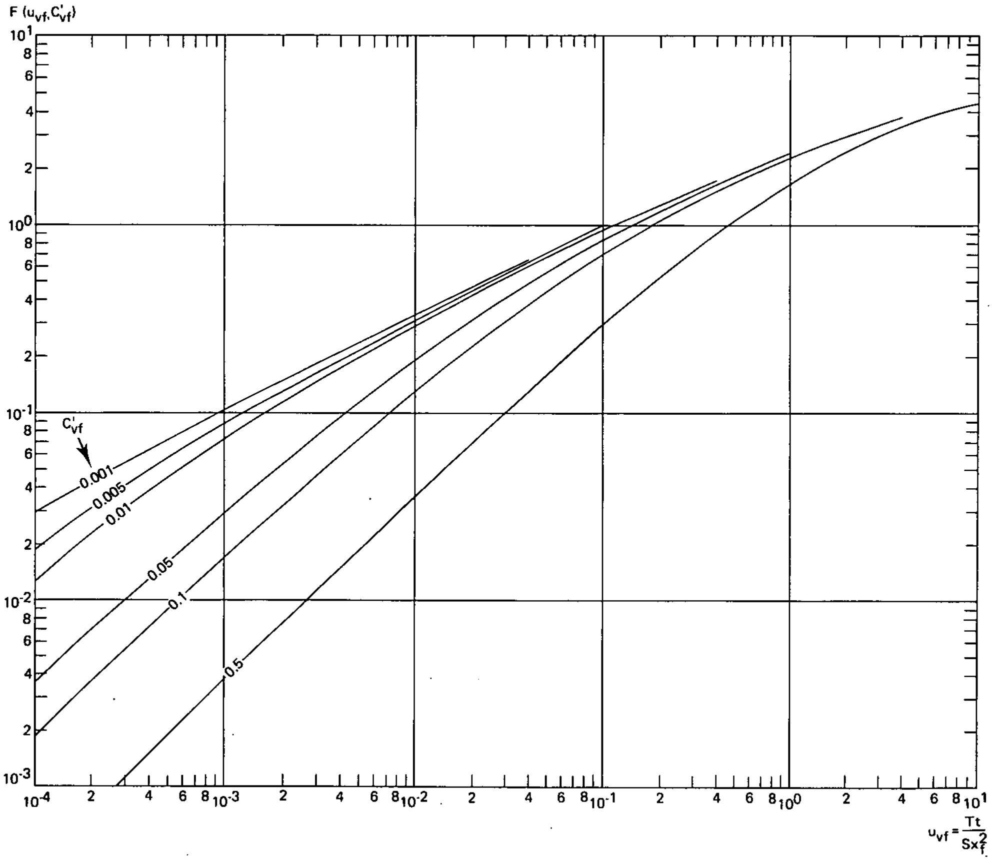

Figure 18.8 Ramey-Gringarten's family of type curves $F(u_{vf}, C'_{vf})$ versus u_{vf} for different values of C'_{vf} for a vertical fracture, taking well-bore storage effects into account

and C'_{vf}. For $C'_{vf} = 0$, the type curve is similar to the Gringarten et al. type curve (Figure 18.7) for a vertical fracture with negligible storage capacity. For values of $C'_{vf} > 0$, the type curves (and in theory also the log-log data plot) will exhibit three different segments (Figure 18.8). Initially, the curves follow a straight line of unit slope, indicating the period during which the storage effects prevail. This straight line gradually passes into another straight line with a slope of 0.5, representing the horizontal parallel-flow period. Finally, when one is using semi-log paper, a straight-line segment also appears, which corresponds to the period of pseudo-radial flow. The slope of this line is 1.15.

Ramey and Gringarten's curve-fitting method is applicable if the following assumptions and conditions are satisfied:
- The general assumptions and conditions listed in Section 18.1.

And:
- The well losses are negligible.

Procedure 18.4
- Using Annex 18.3, prepare a family of type curves on log-log paper by plotting $F(u_{vf},C'_{vf})$ versus u_{vf} for different values of C'_{vf};
- On another sheet of log-log paper of the same scale, plot s_w versus t;
- Match the data curve with one of the type curves and note the value of C'_{vf} for that type curve;
- Select a matchpoint A on the superimposed sheets and note for A the values of $F(u_{vf},C'_{vf})$, u_{vf}, s_w, and t;
- Substitute the values of $F(u_{vf},C'_{vf})$, s_w, and Q into Equation 18.12 and calculate T;
- Substitute the values of u_{vf}, t, and T into Equation 18.2, $Sx_f^2 = Tt/u_{vf}$, and calculate the product Sx_f^2;
- Knowing C'_{vf} and Sx_f^2, calculate the storage constant C_{vf} from Equation 18.13, $C_{vf} = C'_{vf} \times Sx_f^2$.

Discussion

It should not be forgotten that the above (and many other) methods have been developed primarily for a better understanding of the behaviour of hydraulically fractured geological formations in deep oil reservoirs. Although field examples are scanty in the literature, Gringarten et al. (1975) state that the type-curve approach has been successfully applied to many wells that intersect natural or hydraulic vertical fractures. Nevertheless, there are still certain problems associated with wells in fractures. Fracture storativity and fracture hydraulic conductivity cannot be determined, because, in the theoretical concept, the former is assumed to be infinitely small and the latter is assumed to be infinitely great. The assumption of an infinite hydraulic conductivity in the fracture is not very realistic, certainly not if the assumption of a plane fracture (no width) is made or if the fracture is mineral-filled, as is often so in nature. In reality, a certain hydraulic gradient will exist in the pumped fracture. The so-called uniform-flux solution must therefore be interpreted as giving the appearance of a fracture with high, but not infinite, conductivity. This solution seems, indeed, to match drawdown behaviour of wells intersecting natural fractures better than the infinite-conductivity solution does.

It has also been experienced that computed fracture lengths were far too short, which indicates that still other solutions will be necessary before fracture behaviour can be analyzed completely. Finally, naturally fractured formations that were generally broken, but not in a way as to exhibit separated planar fractures, usually do not show the characteristic early-time drawdown response that follows from the theoretical concept described above.