

15 Single-well tests with constant or variable discharges and recovery tests

A single-well test is a test in which no piezometers are used. Water-level changes during pumping or recovery are measured only in the well itself. The drawdown in a pumped well, however, is influenced by well losses (Chapter 14) and well-bore storage. In the hydraulics of well flow, the well is generally regarded as a line source or line sink, i.e. the well is assumed to have an infinitesimal radius so that the well-bore storage can be neglected. In reality, any well has a finite radius and thus a certain storage capacity. Well-bore storage is large when compared with the storage in an equal volume of aquifer material. In a single-well test, well-bore storage must be considered when analyzing the drawdown data.

Papadopoulos and Cooper (1967) observed that the influence of well-bore storage on the drawdown in a well decreases with time and becomes negligible at $t > 25r_c^2/KD$, where r_c is the radius of the unscreened part of the well, where the water level is changing.

To determine whether the early-time drawdown data are dominated by well-bore storage, a log-log plot of drawdown s_w versus pumping time t should be made. If the early-time drawdowns plot as a unit-slope straight line, we can conclude that well-bore storage effects exist.

The methods presented in Sections 15.1 and 15.2 take the linear well losses (skin effects) into account by using the effective well radius r_{ew} in the equations instead of the actual well radius r_w . Most methods are based on the assumption that non-linear well losses can be neglected. If not, the drawdown data must be corrected with the methods presented in Chapter 14.

Section 15.1 presents four methods of analysis for single-well constant-discharge tests. The Papadopoulos-Cooper curve-fitting method (Section 15.1.1) and Rushton-Singh's modified version of it (Section 15.1.2) are applicable for confined aquifers. Jacob's straight-line method (Section 15.1.3), does not require any corrections for non-linear well losses and can be used for confined or leaky aquifers, and so also can Hurr-Worthington's approximation method (Section 15.1.4). All four methods are applicable if the early-time data are affected by well-bore storage, provided that sufficient late-time data ($t > 25 r_c^2/KD$) are also available.

Section 15.2 treats variable-discharge tests. Birsoy-Summers's method (Section 15.2.1) can be used for confined aquifers. A special type of variable discharge test, the free-flowing-well test, can be analyzed by Jacob-Lohman's method (Section 15.2.2) for confined aquifers and by Hantush's method (Section 15.2.3) for leaky aquifers.

A recovery test is invaluable if the pumping test is performed without the use of piezometers.

The methods for analyzing residual drawdown data (Chapter 13) are straight-line methods. The transmissivity of the aquifer is calculated from the slope of a semi-log straight-line, i.e. from differences in residual drawdown. Those influences on the residual drawdown that are or become constant with time, i.e. well losses, partial penetration, do not affect the calculation of the transmissivity. The methods presented in Chapter 13 are also applicable to single-well recovery test data (Section 15.3). In apply-

ing these methods, one must make allowance for those influences on the residual draw-down that do not become constant with time, e.g. well-bore storage.

15.1 Constant-discharge tests

15.1.1 Confined aquifers, Papadopulos-Cooper's method

For a constant-discharge test in a well that fully penetrates a confined aquifer, Papadopulos and Cooper (1967) devised a curve-fitting method that takes the storage capacity of the well into account. The method is based on the following drawdown equation

$$s_w = \frac{Q}{4\pi KD} F(u_w, \alpha) \quad (15.1)$$

where

$$u_w = \frac{r_{cw}^2 S}{4KDt} \quad (15.2)$$

$$\alpha = \frac{r_{cw}^2 S}{r_c^2} \quad (15.3)$$

r_{cw} = effective radius of the screened (or otherwise open) part of the well; $r_{cw} = r_w e^{-skin}$

r_c = radius of the unscreened part of the well where the water level is changing

Values of the function $F(u_w, \alpha)$ are given in Annex 15.1.

The assumptions and conditions underlying the Papadopulos-Cooper method are:

- The assumptions listed at the beginning of Chapter 3, with the exception of the eighth assumption, which is replaced by:
 - The well diameter cannot be considered infinitesimal; hence, storage in the well cannot be neglected.

The following conditions are added:

- The flow to the well is in an unsteady state;
- The non-linear well losses are negligible.

Procedure 15.1

- On log-log paper and using Annex 15.1, plot the family of type curves $F(u_w, \alpha)$ versus $1/u_w$ for different values of α (Figure 15.1);
- On another sheet of log-log paper of the same scale, plot the data curve s_w versus t ;
- Match the data curve with one of the type curves;
- Choose an arbitrary point A on the superimposed sheets and note for that point the values of $F(u_w, \alpha)$, $1/u_w$, s_w , and t ; note also the value of α of the matching type curve;
- Substitute the values of $F(u_w, \alpha)$ and s_w , together with the known value of Q , into Equation 15.1 and calculate KD .

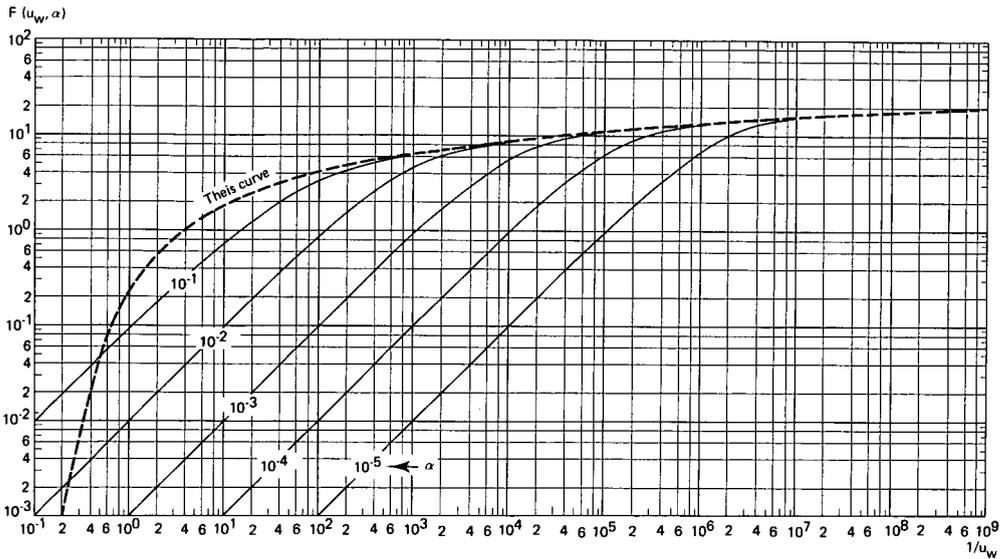


Figure 15.1 Family of Papadopulos-Cooper's type curves: $F(u_w, \alpha)$ versus $1/u_w$ for different values of α

Remarks

- The early-time, almost straight portion of the type curves corresponds to the period when most of the water is derived from storage within the well. Points on the data curve that coincide with these parts of the type curves do not adequately reflect the aquifer characteristics;
- If r_{ew} is known (i.e. if the skin factor or the linear well loss coefficient B_1 is known), in theory a value of S can be calculated by introducing the values of r_{ew} , $1/u_w$, t , and KD into Equation 15.2 or by introducing the values of r_c , r_{ew} , and α into Equation 15.3. The values of S calculated in these two ways should show a close agreement. However, since the form of the type curves differs only very slightly when α differs by an order of magnitude, the value of S determined by this method has questionable reliability.

15.1.2 Confined aquifers, Rushton-Singh's ratio method

Because of the similarities of the Papadopulos-Cooper type curves (Section 15.1.1), it may be difficult to match the data curve with the appropriate type curve. To overcome this difficulty, Rushton and Singh (1983) have proposed a more sensitive curve-fitting method in which the changes in the well drawdown with time are examined. Their well-drawdown ratio is

$$\frac{s_t}{s_{0.4t}}$$

where

- s_t = well drawdown at time t
- $s_{0.4t}$ = well drawdown at time $0.4t$
- t = time since the start of pumping

The values of this ratio are between 2.5 and 1.0. The upper value represents the situation at the beginning of the (constant discharge) test when all the pumped water is derived from well-bore storage. The lower value is approached at the end of the test when the changes in well drawdown with time have become very small.

The type curves used in the Rushton-Singh ratio method are based on values derived from a numerical model (see Annex 15.2).

Rushton-Singh's ratio method can be used if the same assumptions as those underlying the Papadopoulos-Cooper method (Section 15.1.1) are satisfied.

Procedure 15.2

- On semi-log paper and using Annex 15.2, plot the family of type curves $s_t/s_{0.4t}$ versus $4KDt/r_{ew}^2$ for different values of S (Figure 15.2);

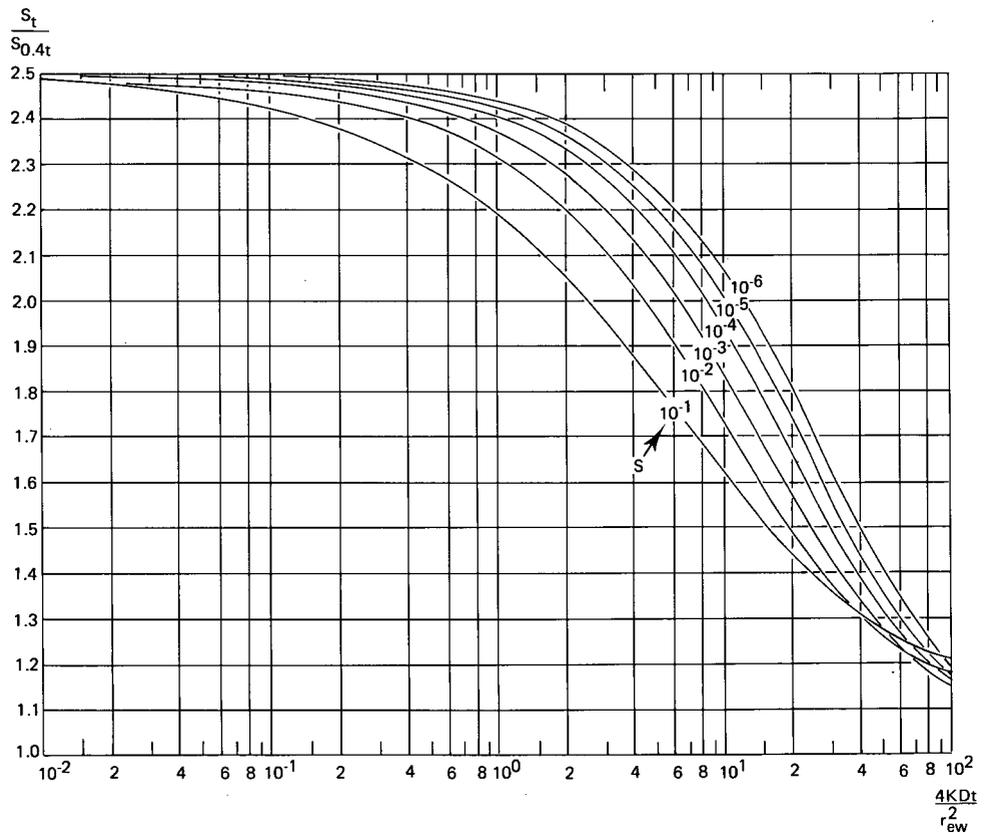


Figure 15.2 Family of Rushton-Singh's type curves for a constant discharge: $s_t/s_{0.4t}$ versus $4KDt/r_{ew}^2$ for different values of S

- Calculate the ratio $s_t/s_{0.4t}$ from the observed drawdowns for different values of t ;
- On another sheet of semi-log paper of the same scale, plot the data curve ($s_t/s_{0.4t}$) versus t ;
- Superimpose the data curve on the family of type curves and, with the horizontal coordinates $s_t/s_{0.4t} = 2.5$ and 1.0 of both plots coinciding, adjust until a position is found where most of the plotted points of the data curve fall on one of the type curves;
- For $4KDt/r_{cw}^2 = 1.0$, read the corresponding value of t from the time axis of the data curve;
- Substitute the value of t together with the known or estimated value of r_{cw} into $4KDt/r_{cw}^2 = 1.0$ and calculate KD ;
- Read the value of S belonging to the best-matching type curve.

15.1.3 Confined and leaky aquifers, Jacob's straight-line method

Jacob's straight-line method (Section 3.2.2) can also be applied to single-well constant-discharge tests to estimate the aquifer transmissivity. However, not all the assumptions underlying the Jacob method are met if data from single-well tests are used. Therefore, the following additional conditions should also be satisfied:

- For single-well tests in confined aquifers

$$t > 25 r_c^2 / KD$$

If this time condition is met, the effect of well-bore storage can be neglected;

- For single-well tests in leaky aquifers

$$\frac{25r_c^2}{KD} < t < \frac{cS}{20} \left(= \frac{L^2S}{20KD} \right)$$

As long as $t < cS/20$, the influence of leakage is negligible.

Procedure 15.3

- On semi-log paper, plot the observed values of s_w versus the corresponding time t (t on logarithmic scale) and draw a straight line through the plotted points;
- Determine the slope of the straight line, i.e. the drawdown difference Δs_w per log cycle of time;
- Substitute the values of Q and Δs_w into $KD = 2.30Q/4\pi\Delta s_w$, and calculate KD .

Remarks

- The drawdown in the well reacts strongly to even minor variations in the discharge rate. Therefore, a constant discharge is an essential condition for the use of the Jacob method;
- There is no need to correct the observed drawdowns for well losses before applying the Jacob method; the aquifer transmissivity is determined from drawdown differences Δs_w , which are not influenced by well losses as long as the discharge is constant;
- In theory, Jacob's method can also be applied if the well is partially penetrating, provided that late-time ($t > D^2S/2KD$) data are used. According to Hantush (1964), the additional drawdown due to partial penetration will be constant for $t > D^2S/$

- 2KD and hence will not influence the value of Δs_w as used in Jacob's method;
- Instead of using the time condition $t > 25r_c^2/KD$ to determine when the effect of well-bore storage can be neglected, we can use the 'one and one-half log cycle rule of thumb' (Ramey 1976). On a diagnostic log-log plot, the early-time data may plot as a unit-slope straight line ($\Delta s_w/\Delta t = 1$), indicating that the drawdown data are dominated by well-bore storage. According to Ramey, the end of this unit-slope straight line is about 1.5 log cycles prior to the start of the semi-log straight line as used in the Jacob method.

Example 15.1

To illustrate the Jacob method, we shall use data from a single-well constant-discharge test conducted in a leaky aquifer in Hoogezand, The Netherlands (after Mulder 1983). Mulder's observations were made with electronic equipment that allowed very precise measurements of s_w and Q to be made every five seconds. The recorded drawdown data are given in Table 15.1.

Table 15.1 Single-well constant-discharge test 'Hoogezand', The Netherlands (from Mulder 1983)

t (s)	s_w (m)	Q m ³ /hr	t	s_w	Q
1	0.108	25.893	178	1.947	29.229
5	1.064	19.991	220	1.950	29.161
10	1.484	30.431	251	1.955	29.286
15	1.721	29.551	286	1.955	28.942
20	1.791	29.248	328	1.960	29.142
25	1.820	28.891	388	1.970	28.963
30	1.843	29.003	508	1.970	28.581
45	1.895	28.547	568	1.972	29.012
60	1.909	28.446	628	1.976	28.893
75	1.916	28.186	688	1.973	28.787
90	1.919	28.135	748	1.976	28.977
148	1.939	27.765			

Figure 15.3 shows a semi-log plot of the drawdown s_w against the corresponding time, with a straight line fitted through the plotted points. The slope of this line, Δs_w , is 0.07 m per log cycle of time. The transmissivity is calculated from

$$KD = \frac{2.30Q}{4\pi\Delta s_w} = \frac{2.30 \times 28.7 \times 24}{4\pi \times 0.07} = 1800 \text{ m}^2/\text{d}$$

Jacob's straight-line method is applicable to data from single-well tests in leaky aquifers, provided that

$$\frac{25r_c^2}{KD} < t < \frac{cS}{20}$$

Substituting the value of the radius of the well ($r_c = 0.185 \text{ m}$) and the calculated transmissivity into $25r_c^2/KD$ yields

$$t > \frac{25 \times 0.185^2}{1800} = 0.00048 \text{ d} \quad \text{or } t > 41 \text{ s}$$

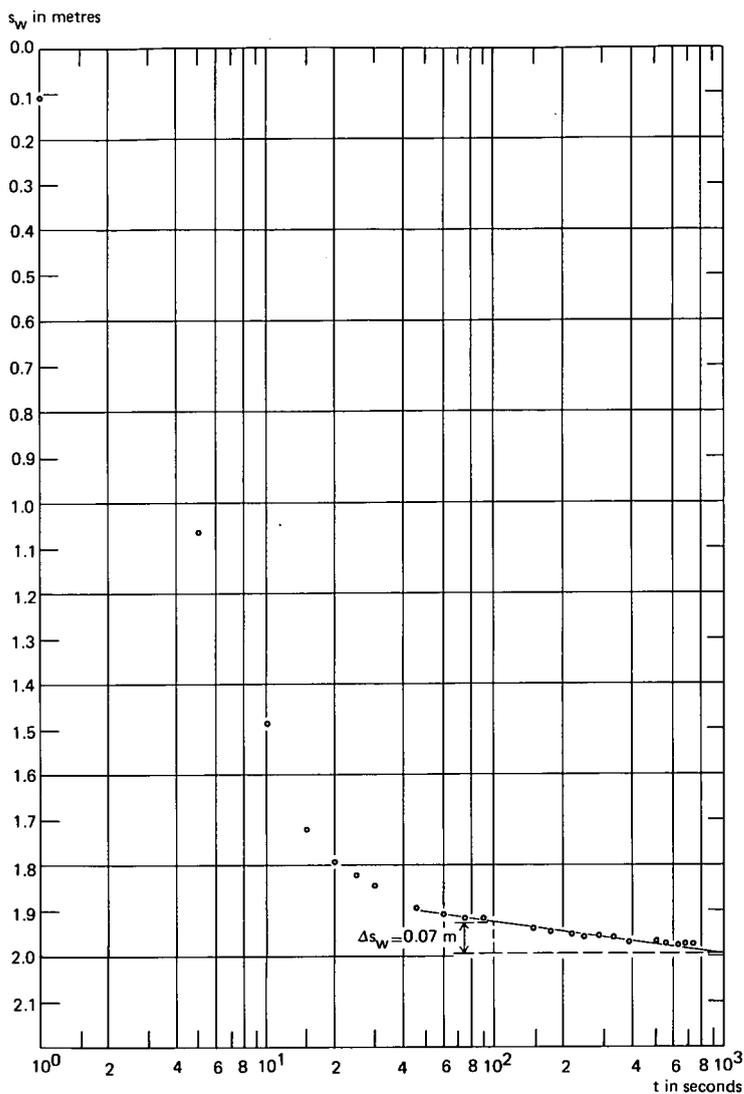


Figure 15.3 Analysis of data from the single-well constant discharge test 'Hoogezand' with the Jacob method

According to Mulder (1983), the values of c and S can be estimated at $c = 1000$ days and $S = 4 \times 10^{-4}$. The drawdown in the well is not influenced by leakage as long as

$$t < \frac{cS}{20} = \frac{1000 \times 4 \times 10^{-4}}{20} \text{ d} \quad \text{or } t < 1728 \text{ s}$$

Hence, for $t > 41$ s, Jacob's method can be applied to the drawdown data from the test 'Hoogezand'.

15.1.4 Confined and leaky aquifers, Hurr-Worthington's method

The unsteady-state flow to a small-diameter well pumping a confined aquifer can be described by a modified Theis equation, provided that the non-linear well losses are negligible. The equation is written as

$$s_w = \frac{Q}{4\pi KD} W(u_w), \quad (15.4)$$

where

$$u_w = \frac{r_{cw}^2 S}{4KDt} \quad (15.5)$$

Rearranging Equation 15.4 gives

$$W(u_w) = \frac{4\pi KDs_w}{Q} \quad (15.6)$$

Hurr (1966) demonstrated that multiplying both sides of Equation 15.6 by u_w eliminates KD from the right-hand side of the equation

$$u_w W(u_w) = \frac{4\pi KDs_w}{Q} \times \frac{r_{cw}^2 S}{4KDt} = \frac{\pi r_{cw}^2 S}{t} \times \frac{s_w}{Q} \quad (15.7)$$

A table of corresponding values of u_w and $u_w W(u_w)$ is given in Annex 15.3 and a graph in Figure 15.4.

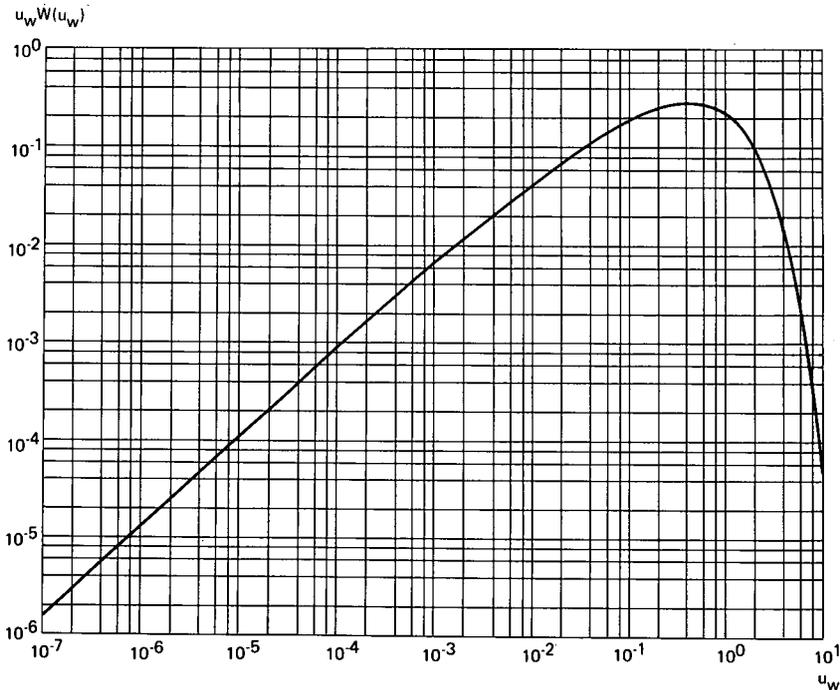


Figure 15.4 Graph of corresponding values of u_w and $u_w W(u_w)$

Hurr (1966) outlined a procedure for estimating the transmissivity of a confined aquifer from a single drawdown observation in the pumped well. In 1981, Worthington incorporated Hurr's procedure in a method for estimating the transmissivity of (thin) leaky aquifers from single-well drawdown data.

In leaky aquifers, the drawdown data can be affected by well losses, by well-bore storage phenomena during early pumping times, and by leakage during late pumping times.

According to Worthington (1981), after the drawdown data have been corrected for non-linear well losses, one can calculate 'pseudo-transmissivities' by applying Hurr's procedure to a sequence of the corrected data. Both well-bore storage effects and leakage effects reduce the drawdown in the well and will therefore lead to calculated pseudo-transmissivities that are greater than the aquifer transmissivity. A semi-log plot of pseudo-transmissivities versus time shows a minimum (Figure 15.5). A flat minimum indicates the time during which the well-bore storage effects have become negligible and leakage effects have not yet manifested themselves: the minimum value of the pseudo-transmissivity gives the value of the aquifer transmissivity. If well-bore storage and leakage effects overlap, the lowest pseudo-transmissivity is the best estimate of a leaky aquifer's transmissivity.

The unsteady-state drawdown data from confined aquifers can also be used to construct a semi-log plot of pseudo-transmissivities versus time to account for the early-time well-bore storage effects.

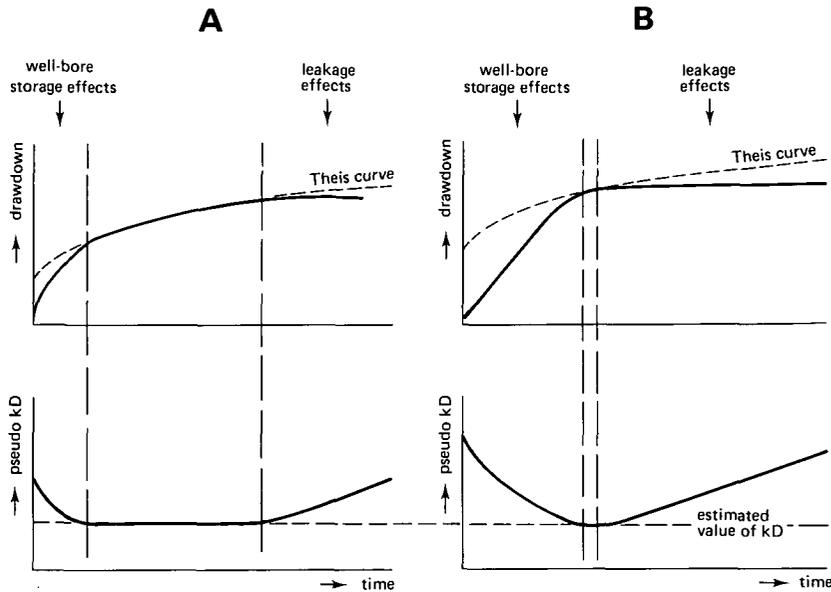


Figure 15.5 Drawdown data and calculated 'pseudo-transmissivities'

A: Moderately affected by well storage and leakage

B: Severely affected by well storage and leakage

(after Worthington 1981)

Hurr-Worthington's method is based on the following assumptions and conditions:

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and eighth assumptions, which are replaced by:
 - The aquifer is confined or leaky;
 - The storage in the well cannot be neglected.

The following conditions are added:

- The flow to the well is in an unsteady-state;
- The non-linear well losses are negligible;
- The storativity is known or can be estimated with reasonable accuracy.

Procedure 15.4

- Calculate pseudo-transmissivity values by applying the following procedure proposed by Hurr to a sequence of observed drawdown data:
 - For a single drawdown observation, calculate $u_w W(u_w)$ from Equation 15.7 for known or estimated values of S and r_{ew} , and the corresponding values of t , s_w , and Q ;
 - Knowing $u_w W(u_w)$, determine the corresponding value of u_w from Annex 15.3 or Figure 15.4;
 - Substitute the values of u_w , r_{ew} , t , and S into Equation 15.5 and calculate the pseudo-transmissivity;
- On semi-log paper, plot the pseudo-transmissivity values versus the corresponding t (t on the logarithmic scale). Determine the minimum value of the pseudo-transmissivity from the plot. This is the best estimate of the aquifer's transmissivity.

Remarks

- The Hurr procedure permits the calculation of the (pseudo) transmissivity from a single drawdown observation in the pumped well, provided that the storativity can be estimated with reasonable accuracy. The accuracy required declines with declining values of u_w . For $u_w/S < 0.001$, the influence of S on the calculated values of KD becomes negligible;
- If the non-linear well losses are not negligible, the observed unsteady-state drawdowns should be corrected before the Hurr-Worthington method is applied.

Example 15.2

To illustrate the Hurr-Worthington method, we shall use the drawdown data from the first step of the step-drawdown test 'Well 1' (see Example 14.1). During the first step, the well was pumped at a discharge rate of 1306 m³/d. Because the non-linear well losses were not negligible ($CQ^2 = 1.4 \times 10^{-7} \times 1306^2 = 0.239$ m), the drawdown data have to be corrected according to the calculations made in Example 14.2.

To calculate (pseudo-)transmissivities, we apply Hurr's procedure to the data from each corrected drawdown observation. First, we calculate the values of $u_w W(u_w)$ from Equation 15.7 for $Q = 1306$ m³/d and the assumed values of $S = 10^{-4}$ and $r_{ew} = 0.25$ m. Then, using the graph of corresponding values of u_w and $u_w W(u_w)$ (Figure 15.4) and the table in Annex 15.3, we find the corresponding values of u_w . From Equation 15.5, we calculate the pseudo-transmissivities (Table 15.2).

Table 15.2 Pseudo-transmissivity values calculated from data obtained during the first step of step-draw-down test 'Well 1'

Time (min)	s_w (m)	$(s_w)_{\text{corr}}^*$ $= s_w - 0.239$ (m)	$u_w W(u_w)$	u_w	(pseudo) KD (m ² /d)
5	1.303	1.064	4.6×10^{-6}	3.2×10^{-7}	1406
6	2.289	2.050	7.4×10^{-6}	5.4×10^{-7}	694
7	3.117	2.878	8.9×10^{-6}	6.5×10^{-7}	495
8	3.345	3.106	8.4×10^{-6}	6.1×10^{-7}	461
9	3.486	3.247	7.8×10^{-6}	5.6×10^{-7}	446
10	3.521	3.282	7.1×10^{-6}	5.1×10^{-7}	441
12	3.592	3.353	6.0×10^{-6}	4.2×10^{-7}	446
14	3.627	3.388	5.2×10^{-6}	3.6×10^{-7}	446
16	3.733	3.494	4.7×10^{-6}	3.3×10^{-7}	426
18	3.768	3.529	4.2×10^{-6}	2.9×10^{-7}	431
20	3.836	3.597	3.9×10^{-6}	2.7×10^{-7}	417
25	3.873	3.634	3.1×10^{-6}	2.1×10^{-7}	429
30	4.014	3.775	2.7×10^{-6}	1.8×10^{-7}	417
35	3.803	3.564	2.2×10^{-6}	1.45×10^{-7}	443
40	4.043	3.804	2.1×10^{-6}	1.4×10^{-7}	402
45	4.261	4.022	1.9×10^{-6}	1.25×10^{-7}	400
50	4.261	4.022	1.7×10^{-6}	1.1×10^{-7}	409
55	4.190	3.951	1.6×10^{-6}	1.05×10^{-7}	390
60	4.120	3.881	1.4×10^{-6}	9×10^{-8}	417
70	4.120	3.881	1.2×10^{-6}	7.6×10^{-8}	423
80	4.226	3.987	1.1×10^{-6}	7.0×10^{-8}	402
90	4.226	3.987	9.6×10^{-7}	6.0×10^{-8}	417
100	4.226	3.987	8.6×10^{-7}	5.4×10^{-8}	417
120	4.402	4.163	7.5×10^{-7}	4.6×10^{-8}	408
150	4.402	4.163	6.0×10^{-7}	3.6×10^{-8}	417
180	4.683	4.444	5.3×10^{-7}	3.2×10^{-8}	391

* Well loss = $CQ^2 = 1.4 \times 10^{-7} \times (1306)^2 = 0.239$ m

Subsequently, we plot the calculated pseudo-transmissivities versus time on semi-log paper (Figure 15.6), from which we can see that during the first eight minutes of pumping, the drawdown in the well was clearly affected by well-bore storage effects. Our estimate of the aquifer transmissivity is 410 m²/d.

15.2 Variable-discharge tests

15.2.1 Confined aquifers, Birsoy-Summers's method

Birsoy-Summers's method (Section 12.1.1) can also be used for analyzing single-well tests with variable discharges. The parameters s and r should be replaced by s_w and r_{cw} in all the equations.

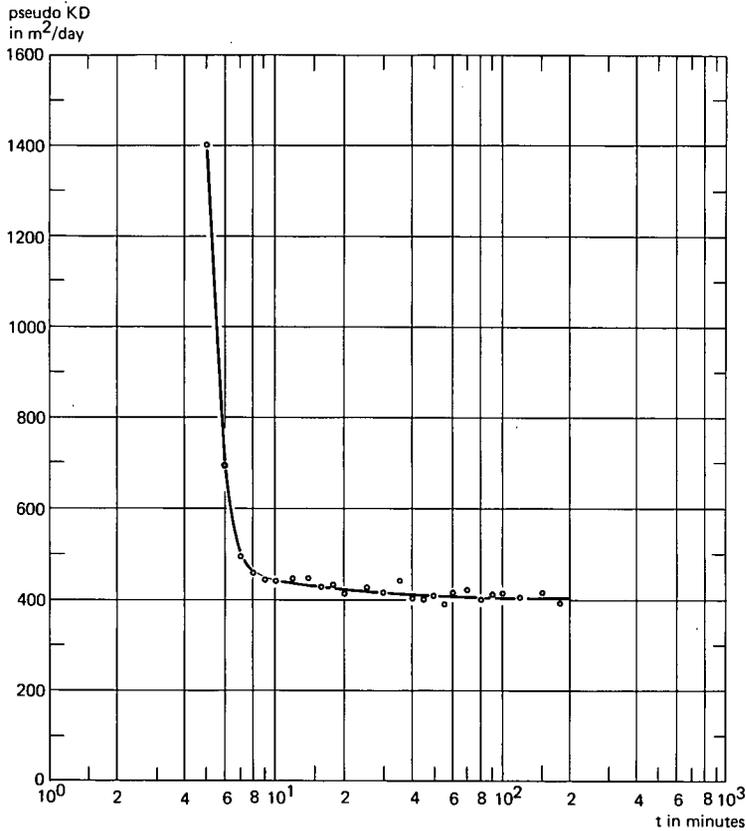


Figure 15.6 Analysis of data from the first step of the step-drawdown test 'Well 1' with the Hurr-Worthington method: determination of the aquifer's transmissivity

15.2.2 Confined aquifers, Jacob-Lohman's free-flowing-well method

Jacob and Lohman (1952) derived the following equation for the discharge of a free-flowing well

$$Q = 2\pi K D s_w G(u_w) \quad (15.8)$$

where

s_w = constant drawdown in the well (= difference between static head measured during shut-in of the well and the outflow opening of the well)

$G(u_w)$ = Jacob-Lohman's free-flowing-well discharge function for confined aquifers

$$u_w = \frac{r_{cw}^2 S}{4KDt}$$

r_{ew} = effective radius of the well

According to Jacob and Lohman, the function $G(u_w)$ can be approximated by $2/W(u_w)$ for all but extremely small values of t . If, in addition, $u_w < 0.01$, Equation 15.8 can be expressed as

$$Q = \frac{4\pi K D s_w}{2.30 \log(2.25 K D t / r_{ew}^2)} \text{ or } \frac{s_w}{Q} = \frac{2.30}{4\pi K D} \log \frac{2.25 K D t}{r_{ew}^2} \quad (15.9)$$

A semi-log plot of s_w/Q versus t (t on logarithmic scale) will thus yield a straight line. A method analogous to the Jacob straight-line method (Section 3.2.2) can therefore be used to analyze the data from a free-flowing well discharging from a confined aquifer.

The Jacob-Lohman method can be used if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the fifth assumption, which is replaced by:
 - At the beginning of the test ($t = 0$), the water level in the free-flowing well is lowered instantaneously. At $t > 0$, the drawdown in the well is constant, and its discharge is variable.

The following conditions are added:

- The flow to the well is in an unsteady state;
- $u_w < 0.01$.

Procedure 15.5

- On semi-log paper, plot the values of s_w/Q versus t (t on logarithmic scale);
- Fit a straight line through the plotted points;
- Extend the straight line until it intercepts the time-axis where $s_w/Q = 0$ at the point t_0 ;
- Introduce the value of the slope of the straight line $\Delta(s_w/Q)$ (i.e. the difference of s_w/Q per log cycle of time) into Equation 15.10 and solve for KD

$$KD = \frac{2.30}{4\pi \Delta(s_w/Q)} \quad (15.10)$$

- Calculate the storativity S from

$$S = \frac{2.25 K D t_0}{r_{ew}^2} \quad (15.11)$$

Remark

- If the value of r_{ew} is not known, S cannot be determined by this method.

15.2.3 Leaky aquifers, Hantush's free-flowing-well method

The variable discharge of a free-flowing well tapping a leaky aquifer is given by Hantush (1959a) as

$$Q = 2\pi KD s_w G(u_w, r_{cw}/L) \quad (15.12)$$

where

s_w = constant drawdown in well
 $G(u_w, r_{cw}/L)$ = Hantush's free-flowing-well discharge function for leaky aquifers

$$u_w = \frac{r_{cw}^2 S}{4KDt} \quad (15.13)$$

Annex 15.4 presents values of the function $G(u_w, r_{cw}/L)$ for different values of $1/u_w$ and r_{cw}/L , as given by Hantush (1959a, 1964; see also Reed 1980). A family of type curves can be plotted from that annex.

The Hantush method for determining a leaky aquifer's parameters KD , S , and c can be applied if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 4, with the exception of the fifth assumption, which is replaced by:
 - At the beginning of the test ($t = 0$), the water level in the free-flowing well is lowered instantaneously. At $t > 0$, the drawdown in the well is constant, and its discharge is variable;

The following conditions are added:

- The flow to the well is in an unsteady state;
- The aquitard is incompressible, i.e. changes in aquitard storage are negligible.

Procedure 15.6

- On log-log paper and using Annex 15.4, draw a family of type curves by plotting $G(u_w, r_{cw}/L)$ versus $1/u_w$ for a range of values of r_{cw}/L ;
- On another sheet of log-log paper of the same scale, prepare the data curve by plotting the values of Q against the corresponding time t ;
- Match the data plot with one of the type curves. Note the value of r_{cw}/L for that type curve;
- Select an arbitrary point A on the overlapping portion of the two sheets and note the values of $G(u_w, r_{cw}/L)$, $1/u_w$, Q , and t for that point;
- Substitute the values of Q and $G(u_w, r_{cw}/L)$ and the value of s_w into Equation 15.12 and calculate KD ;
- Substitute the values of KD , t , $1/u_w$, and r_{cw} into Equation 15.13 and calculate S ;
- Substitute the value of r_{cw}/L corresponding to the type curve and the values of r_{cw} and KD into $r_{cw}/L = r_{cw}/\sqrt{KDc}$, and calculate c .

Remark

- If the effective well radius r_{cw} is not known, the values of S and c cannot be obtained.

15.3 Recovery tests

15.3.1 Theis's recovery method

The Theis recovery method (Section 13.1.1) is also applicable to data from single-well

recovery tests conducted in confined, leaky or unconfined aquifers.

The method can be used if the following assumptions and conditions are met:

- The assumptions listed at the beginning of Chapter 3, adjusted for recovery tests, with the exception of the eighth assumption, which is replaced by:
 - $t_p > 25 r_c^2/KD$;
 - $t' > 25 r_c^2/KD$.

The following conditions are added:

- The aquifer is confined, leaky or unconfined.
For leaky aquifers, the sum of the pumping and recovery times should be $t_p + t' \leq L^2S/20KD$ or $t_p + t' \leq cS/20$ (Section 13.1.2).
For unconfined aquifers only late-time recovery data can be used (Section 13.1.3);
- The flow to the well is in an unsteady state;
- $u < 0.01$, i.e. $t_p > 25 r_w^2S/KD$;
- $u' < 0.01$, i.e. $t' > 25 r_w^2S/KD$ (see also Section 3.2.2).

Procedure 15.7

- For each observed value of s'_w , calculate the corresponding value of t/t' ;
- Plot s'_w versus t/t' on semi-log paper (t/t' on the logarithmic scale);
- Fit a straight line through the plotted points;
- Determine the slope of the straight line, i.e. the residual drawdown difference $\Delta s'_w$ per log cycle of t/t' ;
- Substitute the known values of Q and $\Delta s'_w$ into Equation 15.14 $\Delta s'_w = 2.30Q/4\pi KD$, and calculate KD .

Remarks

- Storage in the well may influence s'_w at the beginning of a recovery test. If the conditions $t_p > 25 r_c^2/KD$ and $t' > 25 r_c^2/KD$ are met, a semi-log plot of s'_w versus t/t' yields a straight-line and Theis's recovery method is applicable. Because the observed recovery data should plot as a straight-line for at least one log cycle of t/t' , Uffink (1982) recommends that both t_p and t' should be at least $500 r_c^2/KD$;
- If the pumped well is partially penetrating, the Theis recovery method can be used, provided that both t_p and t' are greater than $D^2S/2KD$ (Section 13.1.4);
- If the recovery test follows a constant-drawdown test instead of a constant-discharge test, the discharge at the moment before the pump is shut down should be used in Equation 15.14 (Rushton and Rathod 1980).

15.3.2 Birsoy-Summers's recovery method

Residual drawdown data from the recovery phase of single-well variable-discharge tests conducted in confined aquifers can be analyzed by the Birsoy-Summers recovery method (Section 13.3.1), provided that s' is replaced by s'_w in all equations.

15.3.3 Eden-Hazel's recovery method

The Eden-Hazel method for step-drawdown tests (Section 14.1.2) is also applicable to the data from the recovery phase of such a test.

The Eden-Hazel recovery method can be used if the following assumptions and conditions are met:

- The assumptions listed at the beginning of Chapter 3, as adjusted for recovery tests, with the exception of the fifth assumption, which is replaced by:
 - Prior to the recovery test, the aquifer is pumped step-wise.

The following conditions are added:

- The flow to the well is in unsteady state;
- $u < 0.01$ (see Section 3.2.2);
- $u' < 0.01$.

Procedure 15.8

- Calculate for the recovery phase (i.e. $t > t_n$) the values of H_n from Equation 14.15, using the measured discharges and times;
- On arithmetic paper, plot the observed residual drawdown $s'_{w(n)}$ versus the corresponding calculated values of H_n ;
- Draw a straight line through the plotted points;
- Determine the slope of the straight line, $\Delta s'_{w(n)}/\Delta H_n$;
- Calculate KD from

$$\frac{\Delta s'_{w(n)}}{\Delta H_n} = \frac{2.30}{4\pi KD}$$

Example 15.3

We shall illustrate the Eden-Hazel recovery method with the data of the step-drawdown test 'Well 1' (Table 14.1 and Table 15.3).

For the recovery phase of the step-drawdown test, Equation 14.5 becomes

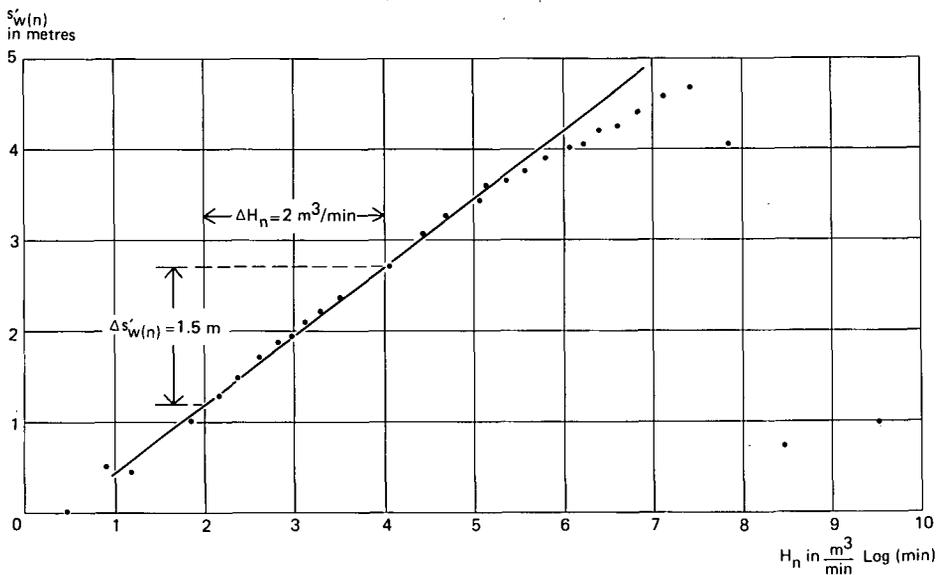


Figure 15.7 Analysis of data from the recovery phase of the step-drawdown test 'Well 1' with the Eden-Hazel recovery method

$$\begin{aligned}
 H_n = & \left(\frac{1360}{1440}\right)\log(t) + \left(\frac{387}{1440}\right)\log(t-180) + \left(\frac{730}{1440}\right)\log(t-360) \\
 & + \left(\frac{838}{1440}\right)\log(t-540) + \left(\frac{833}{1440}\right)\log(t-720) + \left(\frac{925}{1440}\right)\log(t-900) \\
 & - \left(\frac{5019}{1440}\right)\log(t-180) \text{ (m}^3\text{/min) log(min)}
 \end{aligned}$$

Table 15.3 shows the result of the calculations for $t > t_n$.
 Figure 15.7 gives the arithmetic plot of the $s'_{w(n)}$ versus H_n .
 The slope of the straight line is

$$\frac{\Delta s'_{w(n)}}{\Delta H_n} = \frac{1.5}{2} \times \frac{1}{1440} = 5.2 \times 10^{-4} \text{ d/m}^2$$

$$\text{The transmissivity KD} = \frac{2.30}{4\pi \times 5.2 \times 10^{-4}} = 352 \text{ m}^2/\text{d}$$

Table 15.3 Values of H_n calculated for the recovery phase of step-drawdown test 'Well 1'

t (min)	H_n (m ³ /min) log(min)	$s'_{w(n)}$ (m)
1081	9.515	0.599
1082	8.469	1.233
1083	7.859	4.050
1084	7.427	4.683
1085	7.092	4.578
1086	6.820	4.402
1087	6.590	4.261
1088	6.391	4.226
1089	6.216	4.050
1090	6.060	4.014
1092	5.791	3.909
1094	5.564	3.768
1096	5.369	3.662
1098	5.197	3.627
1100	5.045	3.416
1105	4.723	3.275
1110	4.463	3.064
1115	4.246	-
1120	4.059	2.711
1125	3.896	-
1130	3.752	-
1135	3.623	-
1140	3.506	2.359
1150	3.301	2.218
1160	3.127	2.078
1170	2.977	1.937
1180	2.844	1.866
1200	2.620	1.726
1230	2.356	1.479
1260	2.150	1.303
1320	1.843	1.021
1560	1.209	0.458
1800	0.914	0.528
2650	0.499	0.035

