14 Well-performance tests

The drawdown in a pumped well consists of two components: the aquifer losses and the well losses. A well-performance test is conducted to determine these losses.

Aquifer losses are the head losses that occur in the aquifer where the flow is laminar. They are time-dependent and vary linearly with the well discharge. In practice, the extra head loss induced, for instance, by partial penetration of a well is also included in the aquifer losses.

Well losses are divided into linear and non-linear head losses (Figure 14.1). Linear well losses are caused by damage to the aquifer during drilling and completion of the well. They comprise, for example, head losses due to compaction of the aquifer material during drilling, head losses due to plugging of the aquifer with drilling mud, which reduce the permeability near the bore hole; head losses in the gravel pack; and head losses in the screen. Amongst the non-linear well losses are the friction losses that occur inside the well screen and in the suction pipe where the flow is turbulent, and the head losses that occur in the zone adjacent to the well where the flow is usually also turbulent. All these well losses are responsible for the drawdown inside the well being much greater than one would expect on theoretical grounds.

Petroleum engineering recognizes the concept of 'skin effect' to account for the head losses in the vicinity of a well. The theory behind this concept is that the aquifer is assumed to be homogeneous up to the wall of the bore hole, while all head losses are assumed to be concentrated in a thin, resistant 'skin' against the wall of the bore hole.

In this chapter, we present two types of well-performance tests: the classical step-drawdown test (Section 14.1) and the recovery test (Section 14.2).

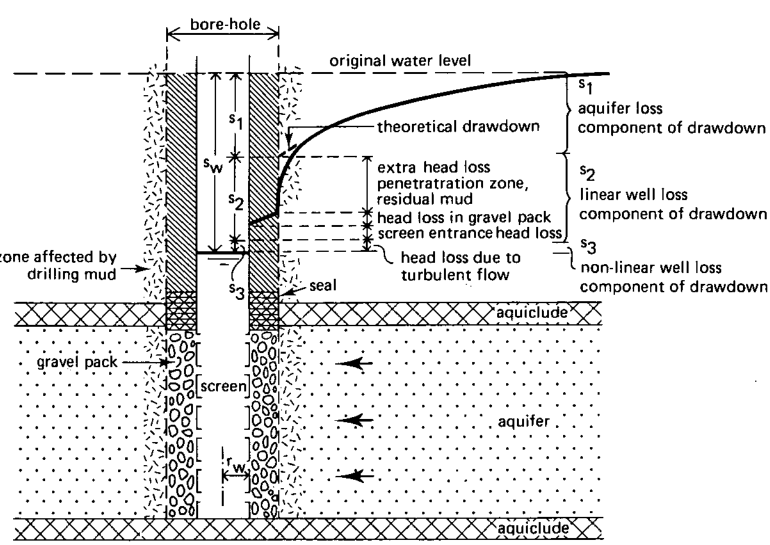

Figure 14.1 Various head losses in a pumped well

14.1 Step-drawdown test

A step-drawdown test is a single-well test in which the well is pumped at a low constant-discharge rate until the drawdown within the well stabilizes. The pumping rate is then increased to a higher constant-discharge rate and the well is pumped until the drawdown stabilizes once more. This process is repeated through at least three steps, which should all be of equal duration, say from 30 minutes to 2 hours each.

The step-drawdown test was first performed by Jacob (1947), who was primarily interested in finding out what the drawdown in a well would be if it were pumped at a rate that differs from the rate during the pumping test. For the drawdown in a pumped well, he gave the following equation

$$s_w = B(r_{ew},t)Q + CQ^2 \tag{14.1}$$

where

$B(r_{ew},t) = B_{1(r_w,t)} + B_2$

$B_{1(r_w,t)}$ = linear aquifer-loss coefficient

B_2 = linear well-loss coefficient
C = non-linear well-loss coefficient
r_{ew} = effective radius of the well
r_w = actual radius of the well
t = pumping time

Jacob combined the various linear head losses at the well into a single term, r_{ew}, the effective radius of the well. He defined this as the distance (measured radially from the axis of the well) at which the theoretical drawdown (based on the logarithmic head distribution) equals the drawdown just outside the well screen. From the data of a step-drawdown test, however, it is not possible to determine r_{ew} because one must also know the storativity of the aquifer, and this can only be obtained from observations in nearby piezometers.

Different researchers have found considerable variations in the flows in and outside of wells. Rorabaugh (1953) therefore suggested that Jacob's equation should read

$$s_w = BQ + CQ^P \tag{14.2}$$

where P can assume values of 1.5 to 3.5, depending on the value of Q (see also Lennox 1966). The value of $P = 2$, as proposed by Jacob is still widely accepted (Ramey 1982; Skinner 1988).

A step-drawdown test makes it possible to evaluate the parameters B and C, and eventually P.

Knowing B and C, we can predict the drawdown inside the well for any realistic discharge Q at a certain time t (B is time-dependent). We can then use the relationship between drawdown and discharge to choose, empirically, an optimum yield for the well, or to obtain information on the condition or efficiency of the well.

We can, for instance, express the relationship between drawdown and discharge as the specific capacity of a well, Q/s_w, which describes the productivity of both the aquifer and the well. The specific capacity is not a constant but decreases as pumping continues (Q is constant), and also decreases with increasing Q. The well efficiency, E_w, can be expressed as

$$E_w = \left\{\frac{B_1Q}{(B_1 + B_2)Q + CQ^P}\right\} \times 100\% \tag{14.3}$$

If a well exhibits no well losses, it is a perfect well. In practice, only the influence of the non-linear well losses on the efficiency can be established, because it is seldom possible to take B_1 and B_2 into account separately. As not all imperfections in well construction show up as non-linear flow resistance, the real degree of a well's imperfection cannot be determined from the well efficiency.

As used in well hydraulics, the concepts of linear and non-linear head loss components ($B_2Q + CQ^2$) relate to the concepts of skin effect and non-Darcyan flow (Ramey 1982). In well hydraulics parlance, the total drawdown inside a well due to well losses (also indicated as the apparent total skin effects) can be expressed as

$$B_2Q + CQ^2 = \frac{1}{2\pi KD}(\text{skin} + C'Q)Q \tag{14.4}$$

where

$C' = C \times 2\pi KD$ = non-linear well loss coefficient or high velocity coefficient

skin $= B_2 \times 2\pi KD$ = skin factor

Matthews and Russel (1967) relate the effective well radius, r_{ew}, to the skin factor by the equation

$$r_{ew} = r_w e^{-\text{skin}} \tag{14.5}$$

Various methods are available to analyze step-drawdown tests. The methods based on Jacob's equation (Equation 14.1) are the Hantush-Bierschenk method (Section 14.1.1) and the Eden-Hazel method (Section 14.1.2). The Hantush-Bierschenk method can determine values of B and C, and can be applied in confined, leaky, or unconfined aquifers. The Eden-Hazel method can be applied in confined aquifers and gives values of well-loss parameters as well as estimates of the transmissivity.

The methods based on Rorabaugh's equation (Equation 14.2) are the Rorabaugh trial-and-error straight line method (Section 14.1.3) and Sheahan's curve-fitting method (Section 14.1.4). They can be used in confined, leaky, or unconfined aquifers, and give values for B, C, and P. Analyzing data from a step-drawdown test does not yield separate values of B_1 and B_2. A recovery test, however, makes it possible to evaluate the skin factor (Section 14.2).

14.1.1 Hantush-Bierschenk's method

By applying the principle of superposition to Jacob's equation (Equation 14.1), Hantush (1964) expresses the drawdown $s_{w(n)}$ in a well during the n-th step of a step-drawdown test as

$$s_{w(n)} = \sum_{i=1}^{n} \Delta Q_i \, B(r_{ew}, t-t_i) + CQ_n^2 \tag{14.6}$$

where

$s_{w(n)}$ = total drawdown in the well during the n-th step at time t
r_{ew} = effective radius of the well
t_i = time at which the i-th step begins ($t_1 = 0$)
Q_n = constant discharge during the n-th step
Q_i = constant discharge during the i-th step of that preceding the n-th step
$\Delta Q_i = Q_i - Q_{i-1}$ = discharge increment beginning at time t_i

The sum of increments of drawdown taken at a fixed interval of time from the beginning of each step ($t - t_i = \Delta t$) can be obtained from Equation 14.6

$$\sum_{i=1}^{n} \Delta s_{w(i)} = s_{w(n)} = B(r_{ew},\Delta t)Q_n + CQ_n^2 \tag{14.7}$$

where

$\Delta s_{w(i)}$ = drawdown increment between the i-th step and that preceding it, taken at time $t_i + \Delta t$ from the beginning of the i-th step

Equation 14.7 can also be written as

$$\frac{s_{w(n)}}{Q_n} = B(r_{ew},\Delta t) + CQ_n \tag{14.8}$$

A plot of $s_{w(n)}/Q_n$ versus Q_n on arithmetic paper will yield a straight line whose slope is equal to C. From Equation 14.8 and the coordinates of any point on this line, B can be calculated.

The procedure suggested by Hantush (1964) and Bierschenk (1963) is applicable if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and fifth assumptions, which are replaced by:
  - The aquifer is confined, leaky or unconfined;
  - The aquifer is pumped step-wise at increased discharge rates;

The following conditions are added:

- The flow to the well is in an unsteady state;
- The non-linear well losses are appreciable and vary according to the expression CQ^2.

Procedure 14.1

- On semi-log paper, plot the observed drawdown in the well s_w against the corresponding time t (t on the logarithmic scale) (Figure 14.2);
- Extrapolate the curve through the plotted data of each step to the end of the next step;
- Determine the increments of drawdown $\Delta s_{w(i)}$ for each step by taking the difference between the observed drawdown at a fixed time interval Δt, taken from the beginning of each step, and the corresponding drawdown on the extrapolated curve of the preceding step;
- Determine the values of $s_{w(n)}$ corresponding to the discharge Q_n from $s_{w(n)} = \Delta s_{w(1)} + \Delta s_{w(2)} + \ldots + \Delta s_{w(n)}$. Subsequently, calculate the ratio $s_{w(n)}/Q_n$ for each step;

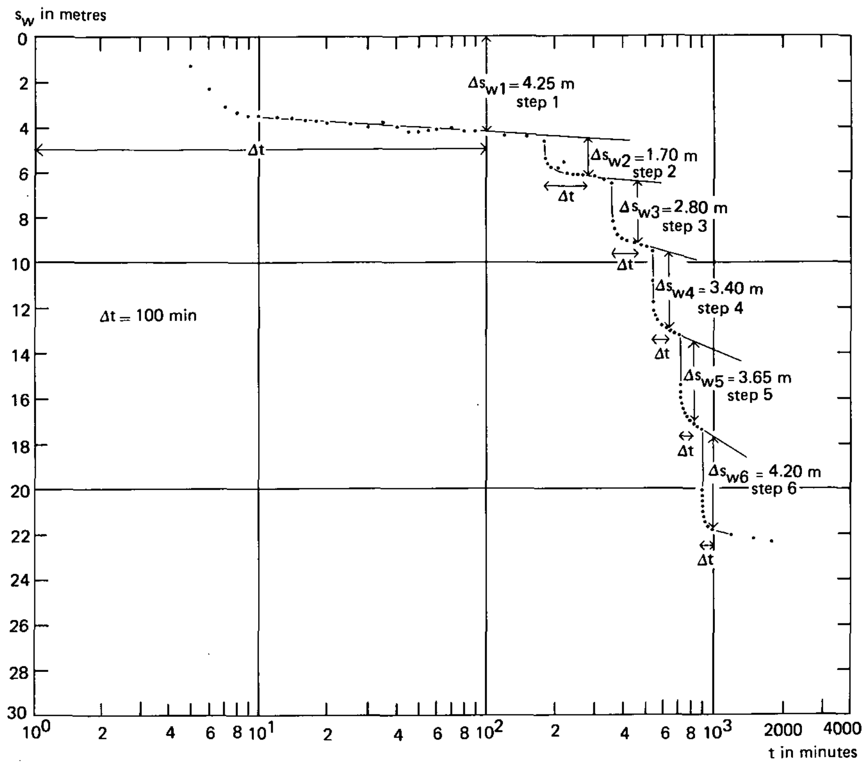

Figure 14.2 The Hantush-Bierschenk method: determination of the drawdown difference for each step

- On arithmetic paper, plot the values of $s_{w(n)}/Q_n$ versus the corresponding values of Q_n (Figure 14.3). Fit a straight line through the plotted points. (If the data do not fall on a straight line, a method based on the well loss component CQ^P should be used; see Sections 14.1.2, 14.1.3 or 14.1.4;
- Determine the slope of the straight line $\Delta(s_{w(n)}/Q_n)/\Delta Q_n$, which is the value of C;
- Extend the straight line until it intercepts the $Q = 0$ axis. The interception point on the $s_{w(n)}/Q_n$ axis gives the value of B.

Remarks

- The values of $\Delta s_{w(i)}$ depend on extrapolated data and are therefore subject to error;
- When a steady state is reached in each step, the drawdown in the well is no longer time-dependent. Hence, the observed steady-state drawdown and the corresponding discharge for each step can be used directly in the arithmetic plot of $s_{w(n)}/Q_n$ versus Q_n.

Example 14.1

To illustrate the Hantush-Bierschenk method, we shall use the data in Table 14.1. These data have been given by Clark (1977) for a step-drawdown test in 'Well 1', which taps a confined sandstone aquifer.

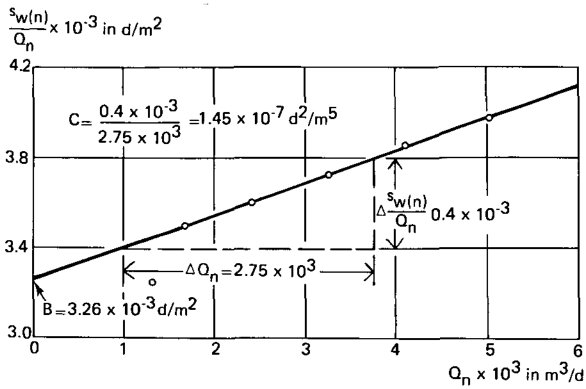

Figure 14.3 The Hantush-Bierschenk method: determination of the parameters B and C

Table 14.1 Step drawdown test data 'Well 1'. Reproduced by permission of the Geological Society from 'The analysis and planning of step-drawdown tests'. L. Clark, in Q.Jl. Engng. Geol. Vol. 10 (1977)

Time in minutes from beginning of step	Step 1 Q: 1306 (m^3/d)	2 1693 Drawdown in metres	3 2423	4 3261	5 4094	6 5019
1	–	5.458	8.170	10.881	15.318	20.036
2	–	5.529	8.240	11.797	15.494	20.248
3	–	5.564	8.346	11.902	15.598	20.389
4	–	5.599	8.451	12.008	15.740	20.529
5	1.303	5.634	8.486	12.078	15.846	20.600
6	2.289	5.669	8.557	12.149	15.881	20.660
7	3.117	5.669	8.557	12.149	15.952	20.741
8	3.345	5.705	8.592	12.184	16.022	20.811
9	3.486	5.740	8.672	12.219	16.022	20.882
10	3.521	5.740	8.672	12.325	16.093	20.917
12	3.592	5.810	8.663	12.360	16.198	20.952
14	3.627	5.810	8.698	12.395	16.268	21.022
16	3.733	5.824	8.733	12.430	16.304	21.128
18	3.768	5.845	8.839	12.430	16.374	21.163
20	3.836	5.810	8.874	12.501	16.409	21.198
25	3.873	5.824	8.874	12.508	16.586	21.304
30	4.014	5.824	8.979	12.606	16.621	21.375
35	3.803	5.881	8.979	12.712	16.691	21.480
40	4.043	5.591	8.994	12.747	16.726	21.551
45	4.261	5.591	9.050	12.783	16.776	21.619
50	4.261	6.092	9.050	12.818	16.797	21.656
55	4.190	6.092	9.120	12.853	16.902	–
60	4.120	6.176	9.120	12.853	16.938	21.663
70	4.120	6.162	9.155	12.888	16.973	21.691
80	4.226	6.176	9.191	12.923	17.079	21.762
90	4.226	6.169	9.191	12.994	17.079	21.832
100	4.226	6.169	9.226	12.994	17.114	21.903
120	4.402	6.176	9.261	13.099	17.219	22.008
150	4.402	6.374	9.367	13.205	17.325	22.184
180	4.683	6.514	9.578	13.240	17.395	22.325

Figure 14.2 shows the semi-log plot of the drawdown data versus time. From this plot, we determine the drawdown differences for each step and for a time-interval $\Delta t = 100$ min. We then calculate the specific drawdown values $s_{w(n)}/Q_n$ (Table 14.2). Plotting the $s_{w(n)}/Q_n$ values against the corresponding values of Q_n on arithmetic paper gives a straight line with a slope of 1.45×10^{-7} d^2/m^5 (= C) (Figure 14.3). The interception point of the straight line with the $Q_n = 0$ axis has a value of $s_{w(n)}/Q_n = 3.26 \times 10^{-3}$ d/m^2 (= B). Hence, we can write the drawdown equation for 'Well 1' as

$$s_w = (3.26 \times 10^{-3})\,Q + (1.45 \times 10^{-7})\,Q^2 \text{ (for t = 100 min).}$$

Table 14.2 Specific drawdown determined with the Hantush-Bierschenk method: step-drawdown test 'Well 1'

	$\Delta s_{w(n)}$	$s_{w(n)}$	Q_n	$s_{w(n)}/Q_n$
	m	m	m^3/d	d/m^2
Step 1	4.25	4.25	1306	3.25×10^{-3}
Step 2	1.70	5.95	1693	3.51×10^{-3}
Step 3	2.80	8.75	2423	3.61×10^{-3}
Step 4	3.40	12.15	3261	3.73×10^{-3}
Step 5	3.65	15.80	4094	3.86×10^{-3}
Step 6	4.20	20.00	5019	3.98×10^{-3}

($\Delta s_{w(n)}$ determined for $\Delta t = 100$ min)

14.1.2 Eden-Hazel's method (confined aquifers)

From step-drawdown tests in a fully penetrating well that taps a confined aquifer, the Eden-Hazel method (1973) can determine the well losses, and also the transmissivity of the aquifer. The method is based on Jacob's approximation of the Theis equation (Equation 3.7).

The drawdown in the well is given by the Jacob equation, now written as

$$s_w = \frac{2.30Q}{4\pi KD} \log \frac{2.25KDt}{r_{ew}^2 S}$$

This equation can also be written as

$$s_w = (a + b \log t)Q \tag{14.9}$$

where

$$a = \frac{2.30}{4\pi KD} \log \frac{2.25KD}{r_{ew}^2 S} \tag{14.10}$$

$$b = \frac{2.30}{4\pi KD} \tag{14.11}$$

Using the principle of superposition and Equation 14.9, we derive the drawdown at time t during the n-th step from

$$s_{w(n)} = \sum_{i=1}^{n} (\Delta Q_i) \{a + b \log(t-t_i)\} \tag{14.12}$$

or

$$s_{w(n)} = aQ_n + b \sum_{i=1}^{n} \Delta Q_i \log(t-t_i) \tag{14.13}$$

where

Q_n = constant discharge during the n-th step
Q_i = constant discharge during the i-th step of that preceding the n-th step
$\Delta Q_i = Q_i - Q_{i-1}$ = discharge increment beginning at time t_i
t_i = time at which the i-th step begins
t = time since the step-drawdown test started

The above equations do not account for the influence of non-linear well losses. Introducing these losses (CQ^2) into Equation 14.13 gives

$$s_{w(n)} = aQ_n + bH_n + CQ_n^2 \tag{14.14}$$

where

$$H_n = \sum_{i=1}^{n} \Delta Q_i \log(t-t_i) \tag{14.15}$$

The Eden-Hazel Procedure 14.2 can be used if the following assumptions and conditions are satisfied:

- The conditions listed at the beginning of Chapter 3, with the exception of the fifth assumption, which is replaced by:
  - The aquifer is pumped step-wise at increased discharge rates;

The following conditions are added:

- The flow to the well is in an unsteady state;
- $u < 0.01$;
- The non-linear well losses are appreciable and vary according to the expression CQ^2.

The Eden-Hazel Procedure 14.3 can be used if the last condition is replaced by:

- The non-linear well losses are appreciable and vary according to the expression CQ^P.

Procedure 14.2

- Calculate the values of H_n from Equation 14.15, using the measured discharges and times;
- On arithmetic paper, plot the observed drawdowns $s_{w(n)}$ versus the corresponding calculated values of H_n (Figure 14.4);
- Draw parallel straight lines of best fit through the plotted points, one straight line through each set of points (Figure 14.4);
- Determine the slope of the lines $\Delta s_{w(n)}/\Delta H_n$, which gives the value of b;
- Extend the lines until they intercept the $H_n = 0$ axis. The interception point (A_n) of each line is given by

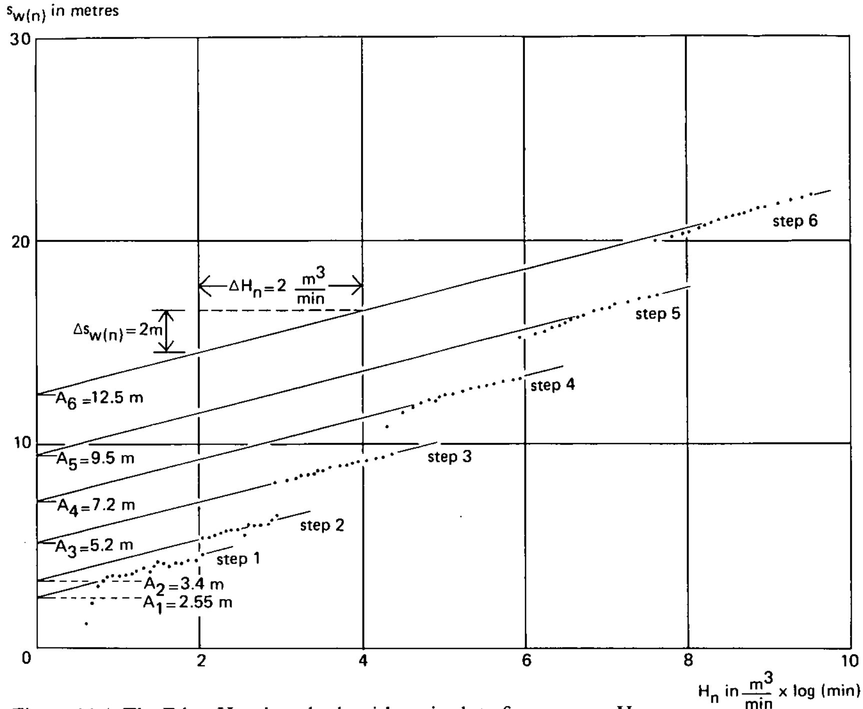

Figure 14.4 The Eden-Hazel method: arithmetic plot of $s_{w(n)}$ versus H_n

$$A_n = aQ_n + CQ_n^2, \qquad \text{or} \ \frac{A_n}{Q_n} = a + CQ_n \tag{14.16}$$

- Read the values of A_n;
- Calculate the ratio A_n/Q_n for each step (i.e. for each value of Q_n);
- On arithmetic paper, plot the values of A_n/Q_n versus the corresponding values of Q_n. Fit a straight line through the plotted points (Figure 14.5);
- Determine the slope of the straight line $\Delta(A_n/Q_n)/\Delta Q_n$, which is the value of C;
- Extend the straight line until it intersects the A_n/Q_n axis where $Q_n = 0$; the value of the intersection point is equal to a;
- Knowing b, calculate KD from Equation 14.11.

Procedure 14.3

- The Eden-Hazel method can also be used if the well losses vary with CQ^P, as may happen when well discharges are high (e.g. in a test to determine the maximum yield of a well). In Equations 14.14 and 14.16, CQ^2 should then be replaced by CQ^P. The adjusted Equation 14.16, after being rearranged in logarithmic form thus becomes

$$\log\left(\frac{A_n}{Q_n} - a\right) \doteq \log C + (P-1)\log Q_n$$

The last three steps of Procedure 14.2 are now replaced by:
- A plot of $[(A_n/Q_n) - a]$ values versus the corresponding values of Q_n on log-log paper should give a straight line whose slope $[\Delta\{(A_n/Q_n) - a\}/\Delta Q_n]$ can be determined. Because the slope equals P − 1, we can calculate P. The interception point of the extended straight line with the ordinate where $Q_n = 0$, gives the value of C. Knowing b from Procedure 14.2, we can calculate the transmissivity from Equation 14.11.

Remark
- The analysis of the data from the recovery phase of a step-drawdown test is incorporated in the Eden-Hazel method (Section 15.3.3).

Example 14.2
We shall illustrate the Eden-Hazel Procedure 14.2 with the data in Table 14.1. Using Equation 14.15, we calculate values of H_n. For example:
- For Step 1, Equation 14.15 becomes

$$H_1 = \frac{1306}{1440}\log t$$

$$\left[t = 50\,\text{min} \rightarrow H_1 = 1.541\,\frac{m^3}{min}\log(\text{min})\right]$$

- For Step 2

$$H_2 = \frac{1306}{1440}\log t + \frac{387}{1440}\log(t-180)$$

$$\left[t = 230\,\text{min} \rightarrow H_2 = 2.599\,\frac{m^3}{min}\log(\text{min})\right]$$

- For Step 6

$$H_6 = \frac{1306}{1440}\log t + \frac{387}{1440}\log(t-180) + \frac{730}{1440}\log(t-360) + \frac{838}{1440}\log(t-540) + \frac{833}{1440}\log(t-720) + \frac{925}{1440}\log(t-900)$$

$$\left[t = 950\,\text{min} \rightarrow H_6 = 8.859\,\frac{m^3}{min}\log(\text{min})\right]$$

Figure 14.4 gives the arithmetic plot of $s_{w(n)}$ versus H_n. The slope of the parallel straight lines is

$$b = \frac{\Delta s_{w(n)}}{\Delta H_n} = \frac{2}{2} \times \frac{1}{1440} = 6.9 \times 10^{-4}\,\text{d/m}^2$$

Introducing b into Equation 14.11 gives $KD = 2.30/4\pi \times 6.9 \times 10^{-4} = 265\,\text{m}^2/\text{d}$. The values of the intersection points A_n (Figure 14.4) are: $A_1 = 2.55$ m; $A_2 = 3.4$ m; $A_3 = 5.2$ m; $A_4 = 7.2$ m; $A_5 = 9.5$ m; and $A_6 = 12.5$ m. A plot of the calculated values of A_n/Q_n versus Q_n (Figure 14.5) gives a straight line with a slope $\Delta(A_n/Q_n)/\Delta Q_n = 0.28 \times 10^{-3}/2000 = 1.4 \times 10^{-7}$. Hence, $C = 1.4 \times 10^{-7}\,\text{d}^2/\text{m}^5$. At the intersection of the straight line and the ordinate where $Q_n = 0$, $a = 1.78 \times 10^{-3}\,\text{d/m}^2$.

After being pumped at a constant discharge Q for t days, the well has a drawdown

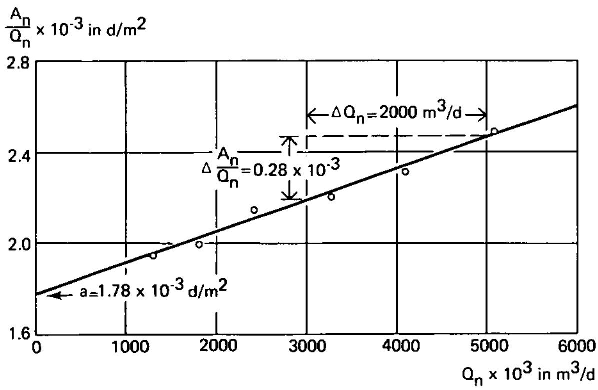

Figure 14.5 The Eden-Hazel method: arithmetic plot of A_n/Q_n versus Q_n

$s_w = \{(1.78 \times 10^{-3}) + (6.9 \times 10^{-4}) \log t\} Q + (1.4 \times 10^{-7})Q^2$. The estimated transmissivity of the aquifer KD = 265 m²/d.
Note: The separate analysis of the data from the recovery phase of the step-drawdown test on Well 1 gives KD = 352 m²/d (Section 15.3.3). In practice, the Eden-Hazel method should be applied to both the drawdown and recovery data.

14.1.3 Rorabaugh's method

If the principle of superposition is applied to Rorabaugh's equation (Equation 14.2), the expression for the drawdown corresponding to Equation 14.7 reads

$$\sum_{i=1}^{n} \Delta s_{w(i)} = s_{w(n)} = BQ_n + CQ^P_n \tag{14.17}$$

which can also be written as

$$\frac{s_{w(n)}}{Q_n} = B + CQ_n^{P-1} \tag{14.18}$$

or

$$\log\left[\frac{s_{w(n)}}{Q_n} - B\right] = \log C + (P-1) \log Q_n \tag{14.19}$$

A plot of $[(s_{w(n)}/Q_n) - B]$ versus Q_n on log-log paper will yield a straight line relationship (Figure 14.6).

The assumptions and conditions underlying Rorabaugh's method are:
- The assumptions listed at the beginning of Chapter 3, with the exception of the first and fifth assumptions, which are replaced by:
  - The aquifer is confined, leaky or unconfined;
  - The aquifer is pumped step-wise at increased discharge rates.

The following conditions are added:
- The flow to the well is in an unsteady state;
- The non-linear well losses are appreciable and vary according to the expression CQ^P.

Procedure 14.4
- On semi-log paper, plot the drawdowns s_w against the corresponding times t (t on the logarithmic scale);
- Extrapolate the curve through the plotted points of each step to the end of the next step;
- For each step, determine the increments of drawdown $\Delta s_{w(i)}$ by taking the difference between the observed drawdown at a fixed time interval Δt, taken from the beginning of that step, and the corresponding drawdown on the extrapolated drawdown curve of the preceding step;
- Determine the values of $s_{w(n)}$ corresponding to the discharge Q_n from $s_{w(n)} = \Delta s_{w(1)} + \Delta s_{w(2)} + ... + \Delta s_{w(n)}$;
- Assume a value of B_i and calculate $[(s_{w(n)}/Q_n) - B_i]$ for each step;
- On log-log paper, plot the values of $[(s_{w(n)}/Q_n) - B_i]$ versus the corresponding values of Q_n. Repeat this part of the procedure for different values of B_i. The value of B_i that gives the straightest line on the plot will be the correct value of B;
- Calculate the slope of the straight line $\Delta[(s_{w(n)}/Q_n) - B]/\Delta Q_n$. This equals (P-1), from which P can be obtained;
- Determine the value of the interception of the straight line with the $Q_n = 1$ axis. This value of $[(s_{w(n)}/Q_n) - B]$ is equal to C.

Remark
- When steady state is reached in each step, the observed steady-state drawdown and the corresponding discharge for each step can be used directly in a log-log plot of $[(s_{w(n)}/Q_n) - B_i]$ versus Q_n.

Example 14.3
To demonstrate the Rorabaugh method, we shall use the specific drawdown data and the corresponding discharge rates presented in Table 14.3 (after Sheahan 1971).

Values of $[(s_{w(n)}/Q_n) - B_i]$ have been calculated for $B_i = 0$; 0.8×10^{-3}; 1×10^{-3}; and 1.1×10^{-3} d/m^2 (Table 14.4). Figure 14.6 shows a log-log plot of $[(s_{w(n)}/Q)_n - B_i]$ versus Q_n. For $B_3 = 1 \times 10^{-3}$ d/m^2, the plotted points fall on a straight line. The slope of this line is

$$\frac{\Delta[(s_{w(n)}/Q_n) - B_3]}{\Delta Q_n} = \frac{\log 10^{-2} - \log 10^{-3}}{\log (17.500/5100)} = 1.85$$

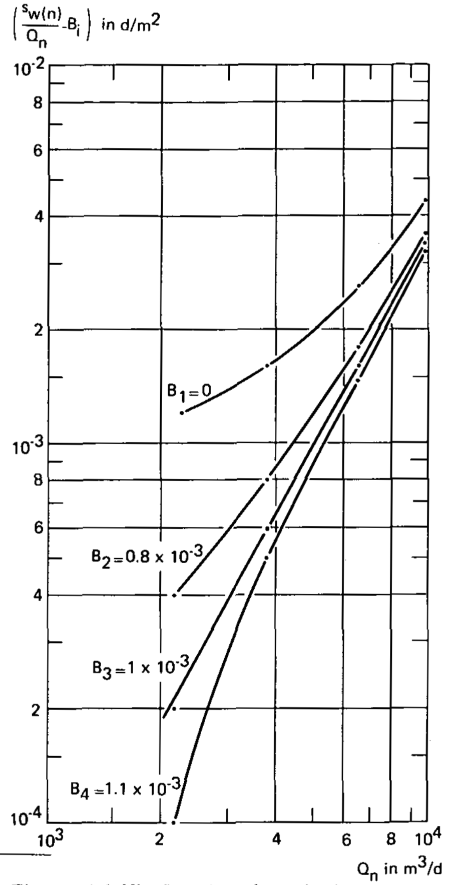

Figure 14.6 The Rorabaugh method

Table 14.3 Step-drawdown test data (from Sheahan 1971)

Total drawdown $s_{w(n)}$	Discharge Q_n	Specific drawdown $s_{w(n)}/Q_n$
(m)	(m^3/d)	(d/m^2)
2.62	2180	1.2×10^{-3}
6.10	3815	1.6×10^{-3}
17.22	6540	2.6×10^{-3}
42.98	9811	4.4×10^{-3}

Table 14.4 Values of $[(s_{w(n)}/Q_n) - B_i]$ and B_i as used in the analysis of Sheahan's step-drawdown test data with Rorabaugh's method

	$\frac{s_{w(1)}}{Q_1} - B_i$ (d/m²)	$\frac{s_{w(2)}}{Q_2} - B_i$ (d/m²)	$\frac{s_{w(3)}}{Q_3} - B_i$ (d/m²)	$\frac{s_{w(4)}}{Q_4} - B_i$ (d/m²)
$B_1 = 0$	1.2×10^{-3}	1.6×10^{-3}	2.6×10^{-3}	4.4×10^{-3}
$B_2 = 0.8 \times 10^{-3}$ d/m²	0.4×10^{-3}	0.8×10^{-3}	1.8×10^{-3}	3.6×10^{-3}
$B_3 = 1 \times 10^{-3}$ d/m²	0.2×10^{-3}	0.6×10^{-3}	1.6×10^{-3}	3.4×10^{-3}
$B_4 = 1.1 \times 10^{-3}$ d/m²	0.1×10^{-3}	0.5×10^{-3}	1.5×10^{-3}	3.3×10^{-3}

Because the slope of the line equals (P – 1), it follows that P = 2.85. The value of $[(s_{w(n)}/Q_n) - B]$ for $Q_n = 10^4$ m³/d is 3.55×10^{-3} d/m². Hence, the intersection of the line with the $Q_n = 1$ m³/d axis is four log cycles to the left. This corresponds with $4 \times 1.85 = 7.4$ log cycles below the point $[(s_{w(n)}/Q_n) - B] = 3.55 \times 10^{-3}$.

The interception point $[(s_{w(n)}/Q_n) - B]_j$ is calculated as follows: $\log [(s_{w(n)}/Q_n) - B]_j = \log 3.55 \times 10^{-3}) - \log (10^{7.4}) = -3 + 0.55 - 7 - 0.4 = -10 + 0.15$. Hence, $[(s_{w(n)}/Q_n) - B]_j = 1.4 \times 10^{-10}$, and $C = 1.4 \times 10^{-10}$ d²/m⁵.

The well drawdown equation is $s_w = (10 \times 10^{-4})Q + (1.4 \times 10^{-10})Q^{2.85}$.

14.1.4 Sheahan's method

Sheahan (1971) presented a curve-fitting method for determining B, C, and P of Rorabaugh's equation (Equation 14.18).

Assuming that B = 1, C = 1, P > 1, and that Q_i is defined for any value of P by $Q_i^{P-1} = 100$, we can calculate the ratio $s_{w(n)}/Q_n$ for selected values of Q_n ($Q_n < Q_i$) and P, using Equation 14.18 (see Annex 14.1). The values given in Annex 14.1 can be plotted on log-log paper as a family of type curves (Figure 14.7).

For those values of Q_n that equal Q_x, Equation 14.18 can be written as

$$\frac{s_{w(x)}}{Q_x} = B + CQ_x^{P-1} = 2B \tag{14.20}$$

and consequently

$$B = CQ_x^{P-1} = \frac{[s_{w(x)}/Q_x]}{2} \tag{14.21}$$

and

$$C = \frac{B}{Q_x^{P-1}} = \frac{(s_{w(x)}/Q_x)}{2Q_x^{P-1}} \tag{14.22}$$

For B = 1 and C = 1, Equation 14.21 gives $s_{w(x)}/Q_x = 2$, and from Equation 14.22 it follows that $Q_x^{P-1} = 1$, or $Q_x = 1$. Hence, for all values of P and assuming that B = 1 and C = 1, the ratio $s_{w(x)}/Q_x = 2$, and $Q_x = 1$ (see also Annex 14.1). All type curves based on the values in Annex 14.1 and plotted on log-log paper pass through the point $s_{w(n)}/Q_n = 2$; $Q_n = 1$. As this is inconvenient for the curve-matching procedure, the type curves are redrawn on plain paper in such a way that the common point expands into an 'index line', located at $s_{w(n)}/Q_n = 2$ (Figure 14.7).

Sheahan's curve-fitting method is applicable if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the first and fifth assumptions, which are replaced by:
  - The aquifer is confined, leaky or unconfined;
  - The aquifer is pumped step-wise at increased discharge rates.

The following conditions are added:

- The flow to the well is in an unsteady state;
- The non-linear well losses are appreciable and vary according to the expression CQ^P.

Procedure 14.5

- On a sheet of log-log paper, prepare the family of Sheahan type curves by plotting $s_{w(n)}/Q_n$ versus Q_n for different values of P, using Annex 14.1. Redraw the family

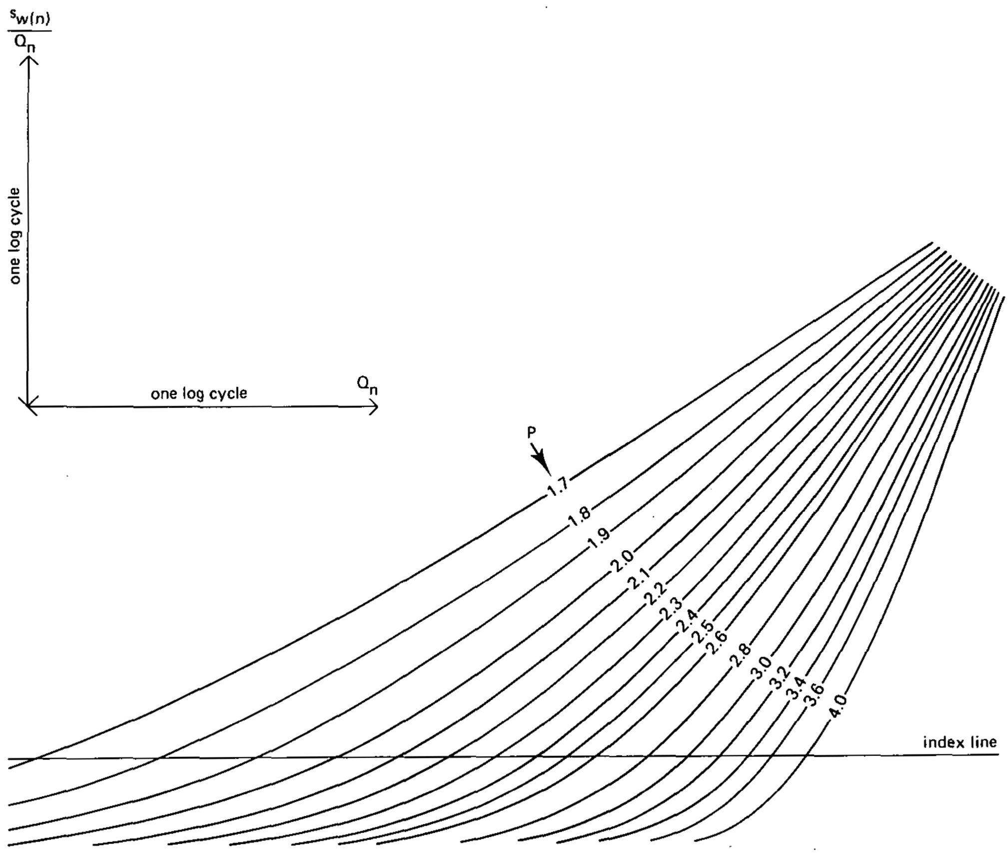

Figure 14.7 Family of Sheahan's type curves $s_{w(n)}/Q_n$ for different values of P (B = 1; C = 1; P > 1; $Q_n < Q_i$; $Q_i^{P-1} = 100$) (after Sheahan 1971)

of type curves on plain paper in such a way that the point $s_{w(n)}/Q_n = 2$; $Q_n = 1$ expands into an index line located at $s_{w(n)}/Q_n = 2$ (see Figure 14.7);

- On semi-log paper, plot the observed drawdowns in the well s_w against the corresponding times t (t on the logarithmic scale);
- Extrapolate the curve through the plotted points of each step to the end of the next step;
- Determine the increments of drawdown $\Delta s_{w(i)}$ for each step by taking the difference between the observed drawdown at a fixed time interval Δt, taken from the beginning of the step, and the corresponding drawdown on the extrapolated drawdown curve of the preceding step;
- Determine the values of $s_{w(n)}$ corresponding to the discharge Q_n from $s_{w(n)} = \Delta s_{w(1)} + \Delta s_{w(2)} + ... + \Delta s_{w(n)}$. Subsequently, calculate the ratio $s_{w(n)}/Q_n$ for each step;
- On log-log paper of the same scale as that used for the log-log plot of Sheahan's type curves, plot the calculated values of the ratio $s_{w(n)}/Q_n$ versus the corresponding values of Q_n;
- Match the data plot with one of the family of type curves and note the value of P for that type curve;
- For the intersection point of type curve and index line, read the corresponding coordinates from the data plot. This gives the values of $s_{w(x)}/Q_x$ and Q_x;
- Substitute the value of $s_{w(x)}/Q_x$ into Equation 14.21 and calculate B;
- Substitute the values of B, Q_x, and P into Equation 14.22 and calculate C.

Remarks

- The most accurate analysis of step-drawdown data is obtained if the plotted data fall on the type curve's portion of greatest curvature;
- For decreasing values of Q_n, the Sheahan type curves all approach the line $s_{w(n)}/Q_n = B$ asymptotically, indicating that for small values of Q_n, the well loss component CQ^P becomes negligibly small.

Example 14.4

When we plot the $s_{w(n)}/Q_n$ and Q_n data from Table 14.3 on log-log paper, we find that the best match with Sheahan's type curves is with the curve for P = 2.8 (Figure 14.8). The interception point (x) of Sheahan's index line and the curve (P = 2.8) through the observed data has the coordinates $s_{w(x)}/Q_x = 1.95 \times 10^{-3}$ d/m^2 and $Q_x = 4.9 \times 10^3$ m^3/d.

According to Equation 14.21

$$B = 0.5 \times \frac{s_{w(x)}}{Q_x} = 0.5 \times 1.95 \times 10^{-3} = 9.8 \times 10^{-4}\ \mathrm{d/m^2}$$

and according to Equation 14.22

$$C = \frac{(s_{w(x)}/Q_x)}{2Q_x^{P-1}} = \frac{1.95 \times 10^{-3}}{2(4.9 \times 10^3)^{(2.8-1)}} = 2.2 \times 10^{-10}\ \mathrm{d^2/m^5}$$

The drawdown equation can be written as

$$s_w = (9.8 \times 10^{-4})Q + (2.2 \times 10^{-10})Q^{2.8}$$

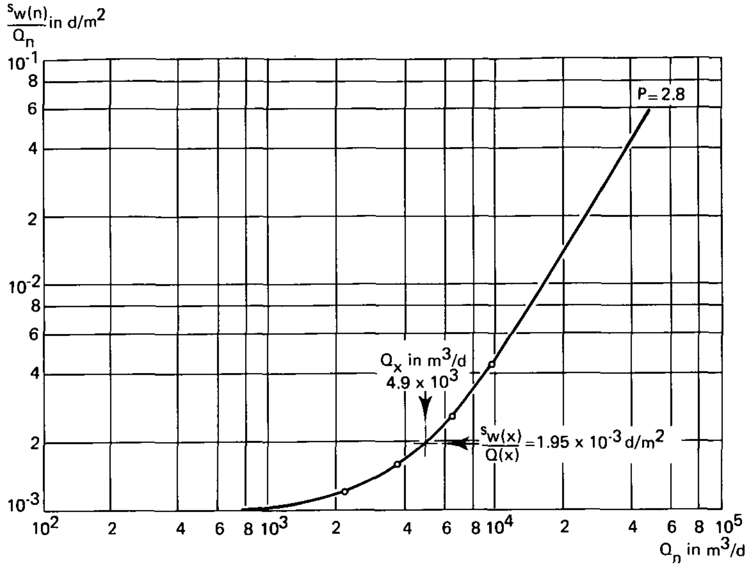

Figure 14.8 Sheahan's method

14.2 Recovery tests

14.2.1 Determination of the skin factor

If the effective radius of the well r_{ew} is larger than the real radius of the bore hole r_w, we speak of a positive skin effect. If it is smaller, the well is usually poorly developed or its screen is clogged, and we speak of a negative skin effect (De Marsily 1986).

In groundwater hydraulics, the skin effect is defined as the difference between the total drawdown observed in a well and the aquifer loss component, assuming that the non-linear well losses are negligible. Adding the skin effect to Jacob's equation (3.7) and assuming that the non-linear well losses are so small that they can be neglected, we obtain the following equation for the drawdown in a well that fully penetrates a confined aquifer and is pumped at a constant rate

$$s_w = \frac{Q}{4\pi KD} \ln \frac{2.25KDt}{r_w^2 S} + (\text{skin}) \frac{Q}{2\pi KD}$$

$$= \frac{Q}{4\pi KD}\left[\ln \frac{2.25KDt}{r_w^2 S} + 2(\text{skin})\right] \qquad (14.23)$$

where

skin $(Q/2\pi KD)$	= skin effect in m
skin	= skin factor (dimensionless)
r_w	= actual radius of the well in m

After the pump has been shut down, the residual drawdown s'_w in the well for $t' > 25r_w^2S/KD$ is

$$s'_w = \frac{Q}{4\pi KD}\left[\ln\frac{2.25KDt}{r_w^2 S} + 2\text{ skin}\right] - \frac{Q}{4\pi KD}\left[\ln\frac{2.25KDt'}{r_w^2 S} + 2\text{ skin}\right]$$
$$= \frac{2.30Q}{4\pi KD}\log\frac{t}{t'} \qquad (14.24)$$

where

t = time since pumping started
t' = time since pumping stopped

For $t' > 25r_w^2S/KD$, a semi-log plot of s'_w versus t/t' will yield a straight line. The transmissivity of the aquifer can be calculated from the slope of this line.
For time $t = t_p$ = total pumping time, Equation 14.23 becomes

$$s_w(t_p) = \frac{Q}{4\pi KD}\ln\frac{2.25KDt_p}{r_w^2 S} + \text{skin}\left(\frac{Q}{2\pi KD}\right) \qquad (14.25)$$

The difference between $s_w(t_p)$ and the residual drawdown s'_w at any time t', is

$$s_w(t_p) - s'_w = \frac{Q}{4\pi KD}\ln\frac{2.25KDt_p}{r_w^2 S} + \text{skin}\left(\frac{Q}{2\pi KD}\right) - \frac{Q}{4\pi KD}\ln\frac{t_p + t'}{t'} \qquad (14.26)$$

For $\frac{t_p + t'_i}{t'_i} = \frac{2.25KDt_p}{r_w^2 S}$ (14.27)

Equation 14.26 reduces to

$$s_w(t_p) - s'_{wi} = \text{skin}\left(\frac{Q}{2\pi KD}\right) \qquad (14.28)$$

The procedure for determining the skin factor has been described by various authors (e.g. Matthews and Russell 1967). It is applicable if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of Chapter 3, adjusted for recovery tests.

The following conditions are added:
- The aquifer is confined, leaky or unconfined;
- The flow to the well is in an unsteady state;
- $u < 0.01$;
- $u' < 0.01$;
- The linear well losses (i.e. the skin effect) are appreciable, and the non-linear well losses are negligible.

Procedure 14.6
- Follow Procedure 13.1 or Procedure 15.8 (the Theis recovery method) to determine KD;
  - On semi-log paper, plot the residual drawdown s'_w versus corresponding values of t/t' (t/t' on logarithmic scale);
  - Fit a straight line through the plotted points;

- Determine the slope of the straight line, i.e. the residual drawdown difference $\Delta s'_w$ per log cycle of t/t';
- Substitute the known values of Q and $\Delta s'_w$ into $\Delta s'_w = 2.30Q/4\pi KD$, and calculate KD;

– Determine the ratio $(t_p + t'_i)/t'_i$ by substituting the values of the total pumping time t_p, the calculated KD, the known value of r_w, and an assumed (or known) value of S into Equation 14.27;

– Read the value of s'_{wi} corresponding to the calculated value of $(t_p + t'_i)/t'_i$ from the extrapolated straight line of the data plot s'_w versus t/t';

– Substitute the observed value of $s_w(t_p)$ corresponding to pumping time $t = t_p$, and the known values of s'_{wi}, Q, and KD into Equation 14.28 and solve for the skin factor.