12 Variable-discharge tests and tests in well fields

Aquifers are sometimes pumped at variable discharge rates. This may be done deliberately, or it may be due to the characteristics of the pump. Sometimes, aquifers are pumped step-wise (i.e. at a certain discharge from t_0 to t_1, then at another discharge from t_1 to t_2, and so on), or they may be pumped intermittently at different discharge rates. For confined aquifers that are pumped at variable discharge rates, Birsoy and Summers (1980) devised the method presented in Section 12.1.1.

It may happen that the discharge decreases with the decline of head in the well. If so, the sharpest decrease will occur soon after the start of pumping. For confined aquifers, the Aron-Scott and the Birsoy-Summers methods take this phenomenon into account. These are presented in Sections 12.1.2 and 12.1.1.

Although, strictly speaking, free-flowing wells are not pumped, the methods of analysis applied to them are very similar to those for pumped wells. Hantush's method for unsteady-state flow to a free-flowing well in a confined aquifer can be found in Section 12.2.1, and the Hantush-De Glee method for steady-state flow in a leaky aquifer in Section 12.2.2. Both methods are based on the condition that the decline of head in the well is constant and that the discharge decreases with time.

The methods presented in the previous chapters are based on analytical solutions for the drawdown response in an aquifer that is pumped by a single well. If two or more wells pump the same aquifer, the drawdown will be influenced by the combined effects of these wells. The Cooper-Jacob method (Section 12.3.1) takes such effects into account.

The principle of superposition, which was discussed in Chapter 6, is used in some of the methods in this chapter. According to this principle, two or more drawdown solutions, each for a given set of conditions for the aquifer and the well, can be summed algebraically to obtain a solution for the combined conditions.

12.1 Variable discharge

12.1.1 Confined aquifers, Birsoy-Summers's method

Birsoy and Summers (1980) present an analytical solution for the drawdown response in a confined aquifer that is pumped step-wise or intermittently at different discharge rates (Figure 12.1). Applying the principle of superposition to Jacob's approximation of the Theis equation (Equation 3.7), they obtain the following expression for the drawdown in the aquifer at time t during the nth pumping period of intermittent pumping

$$s_n = \frac{2.30Q_n}{4\pi KD} \log \left\{ \left(\frac{2.25KD}{r^2S} \right) \beta_{t(n)}(t-t_n) \right\} \tag{12.1}$$

where

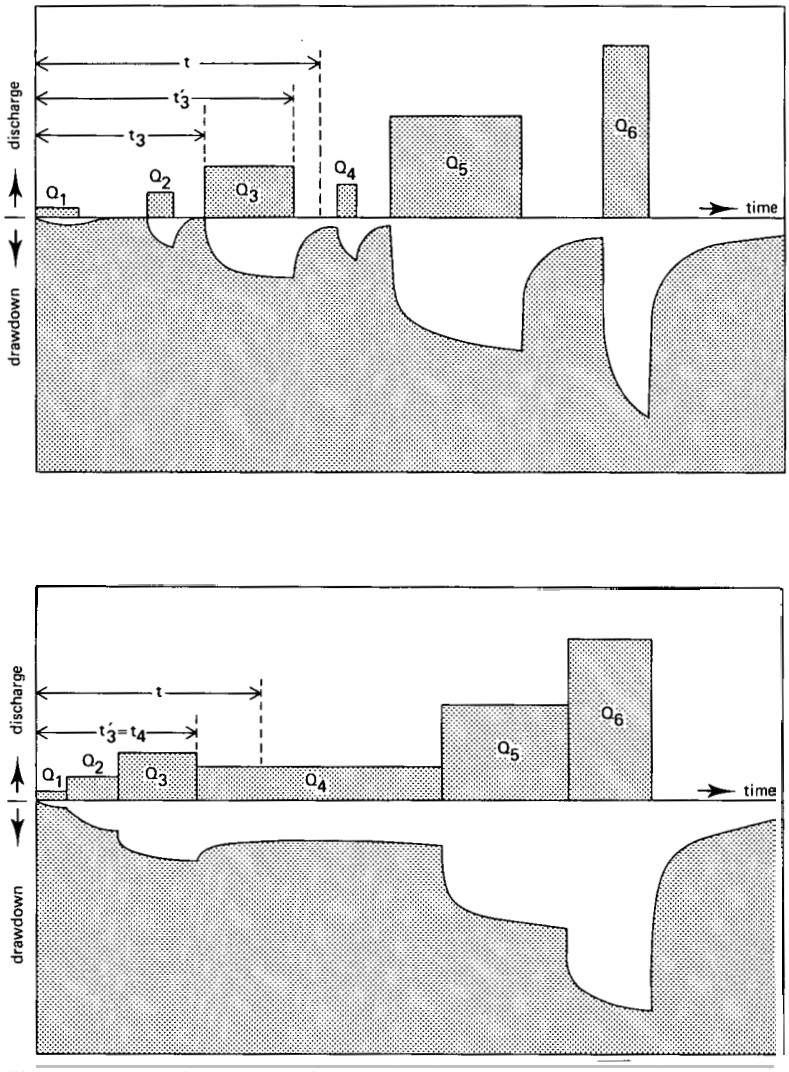

Figure 12.1 Step-wise and intermittently changing discharge rates and the resulting drawdown responses (after Birsoy and Summers 1980)

$$\beta_{t(n)} = \prod_{i=1}^{n-1} \left(\frac{t-t_i}{t-t'_i}\right)^{Q_i/Q_n}$$

$$= \left(\frac{t-t_1}{t-t'_1}\right)^{Q_1/Q_n} \times \left(\frac{t-t_2}{t-t'_2}\right)^{Q_2/Q_n} \times \cdots \times \left(\frac{t-t_{n-1}}{t-t'_{n-1}}\right)^{Q_{n-1}/Q_n} \qquad (12.2)$$

where

t_i = time at which the i-th pumping period started

$t-t_i$ = time since the i-th pumping period started
t'_i = time at which the i-th pumping period ended
$t-t'_i$ = time since the i-th pumping period ended
Q_i = constant well discharge during the i-th pumping period

For step-wise or uninterrupted pumping, $t'_{(i-1)} = t_i$, and the 'adjusted time' $\{\beta_{t(n)}(t-t_n)\}$ becomes

$$\beta_{t(n)}(t-t_n) = \prod_{i=1}^{n} (t-t_i)^{\Delta Q_i/Q_n}$$

$$= (t-t_1)^{\Delta Q_1/Q_n} \times (t-t_2)^{\Delta Q_2/Q_n} \times \ldots \times (t-t_n)^{\Delta Q_n/Q_n} \qquad (12.3)$$

where $\Delta Q_i = Q_i - Q_{i-1}$ = discharge increment beginning at time t_i.

If the intermittent pumping rate is constant (i.e. $Q = Q_1 = Q_2 = \ldots = Q_n$), the adjusted time becomes

$$\beta_{t(n)}(t-t_n) = \frac{t_1}{t'_1}\frac{t_2}{t'_2}\ldots\frac{t_{n-1}}{t'_{n-1}}t_n \qquad (12.4)$$

Dividing both sides of Equation 12.1 by Q_n gives an expression for the specific drawdown

$$\frac{s_n}{Q_n} = \frac{2.30}{4\pi KD}\log\left\{\frac{2.25KD}{r^2S}\beta_{t(n)}(t-t_n)\right\} \qquad (12.5)$$

The Birsoy-Summers method can be used if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the fifth assumption, which is replaced by:
  - The aquifer is pumped step-wise or intermittently at a variable discharge rate or is intermittently pumped at a constant discharge rate.

The following conditions are added:

- The flow to the well is in an unsteady state;
- $\frac{r^2S}{4KD} \times \frac{1}{\beta_{t(n)}(t-t_n)} < 0.01$ (see also Section 3.2.2)

Procedure 12.1

- For a single piezometer, calculate the adjusted time $\beta_{t(n)}(t-t_n)$ from Equations 12.2, 12.3, or 12.4 (whichever is applicable), using all the observed discharges and the appropriate values of time;
- On semi-log paper, plot the observed specific drawdown s_n/Q_n versus the corresponding values of $\beta_{t(n)}(t-t_n)$ (the adjusted time on the logarithmic scale), and draw a straight line through the plotted points;
- Determine the slope of the straight line, $\Delta(s_n/Q_n)$, which is the difference of s_n/Q_n per log cycle of adjusted time;
- Calculate KD from $\Delta(s_n/Q_n) = 2.30/4\pi KD$;
- Extend the straight line until it intersects the $s_n/Q_n = 0$ axis and determine the value of the interception point $\{\beta_{t(n)}(t-t_n)\}_0$;

– Knowing r, KD, and $\{\beta_{t(n)}(t-t_n)\}_0$, calculate S from

$$S = \frac{2.25KD}{r^2}\{\beta_{t(n)}(t-t_n)\}_0 \tag{12.6}$$

Remarks

– Procedure 12.1 can also be applied when the well discharge changes uninterruptedly with time. In that case, however, Q versus t for a single piezometer should be plotted on arithmetic paper. The time axis is then divided into appropriate equal time intervals $t'_i - t_i$ and the average discharge Q_i for each time interval is calculated;

– Calculating the adjusted time $\beta_{t(n)}(t-t_n)$ by hand is a tedious process. Birsoy and Summers (1980) give a program for an HP-25 pocket calculator that computes $\beta_{t(n)}$ for n < 4 for step-wise pumping.

Example 12.1

We use drawdown data from a hypothetical pumping test conducted in a fully penetrated confined aquifer. During the test, the discharge rates changed step-wise (Table 12.1). For a piezometer at r = 5 m, the adjusted time $\beta_{t(n)}(t-t_n)$ can be calculated with Equation 12.3.

For example, for n = 3 and t = 100 min., the adjusted time is calculated as follows

$$\beta_{t(3)}(t-t_3) = (t-t_1)^{\Delta Q_1/Q_3} \times (t-t_2)^{\Delta Q_2/Q_3} \times (t-t_3)^{\Delta Q_3/Q_3}$$
$$= (100\text{-}0)^{500/600} \times (100\text{-}30)^{200/600} \times (100\text{-}80)^{-100/600} = 116 \text{ min}$$

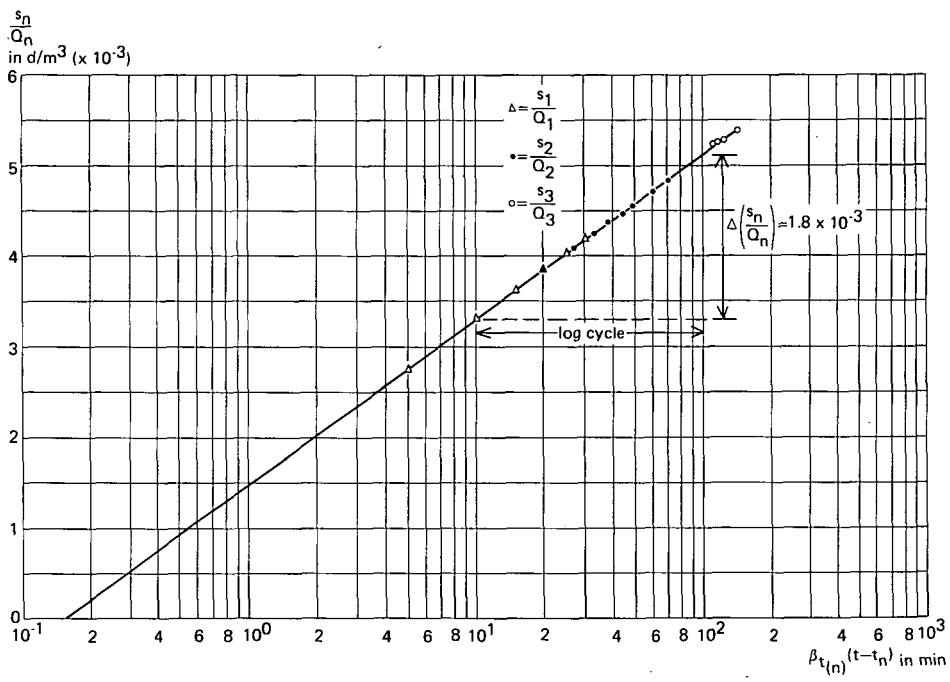

Figure 12.2 Analysis of data with the Birsoy-Summers method for variable discharge

Table 12.1 gives the results of the calculations.

The specific drawdown data (Table 12.1) are plotted against the calculated adjusted time on semi-log paper (Figure 12.2). The slope of the straight line through the plotted points $\Delta(s_n/Q_n) = 1.8 \times 10^{-3}$.
The transmissivity is

$$KD = \frac{2.30}{4\pi\Delta(s_n/Q_n)} = \frac{2.30}{4 \times 3.14 \times 1.8 \times 10^{-3}} = 102\ m^2/d$$

The straight line intersects the $s_n/Q_n = 0$ axis at $\{\beta_{t(n)}(t\text{-}t_n)\}_o = 1.5 \times 10^{-1}$ min.
Hence

$$S = \frac{2.25KD}{r^2}\{\beta_{t(n)}(t-t_n)\}_o = \frac{2.25 \times 102}{25} \times \frac{1.5 \times 10^{-1}}{1440} = 9.6 \times 10^{-4}$$

In each step, the condition $u < 0.01$ is fulfilled after $t = 8.5$ min. The less restrictive condition $u < 0.05$ (Section 3.2.2) is already fulfilled after 1.7 min., i.e. all drawdown data can be used in the analysis.

Table 12.1 Data from a pumping test with step-wise changing discharge rates

n	t min	s_n (m)	Q_n m^3/d	s_n/Q_n d/m^2	$\beta_{t(n)}(t\text{-}t_n)$ min
1	5	1.38	500	2.76×10^{-3}	5
1	10	1.65	500	3.30×10^{-3}	10
1	15	1.81	500	3.62×10^{-3}	15
1	20	1.93	500	3.86×10^{-3}	20
1	25	2.02	500	4.04×10^{-3}	25
1	30	2.09	500	4.18×10^{-3}	30
2	35	2.68	700	3.83×10^{-3}	20
2	40	2.85	700	4.07×10^{-3}	27
2	45	2.96	700	4.23×10^{-3}	33
2	50	3.05	700	4.36×10^{-3}	38
2	55	3.12	700	4.46×10^{-3}	44
2	60	3.18	700	4.54×10^{-3}	49
2	70	3.29	700	4.70×10^{-3}	60
2	80	3.38	700	4.83×10^{-3}	70
3	90	3.13	600	5.22×10^{-3}	113
3	100	3.15	600	5.25×10^{-3}	116
3	110	3.17	600	5.28×10^{-3}	123
3	130	3.23	600	5.38×10^{-3}	140

12.1.2 Confined aquifers, Aron-Scott's method

In a confined aquifer, when the head in the well declines as a result of pumping, many pumps decrease their discharge, the sharpest decrease taking place soon after the start of pumping (Figure 12.3).

An appropriate method that takes this phenomenon into account has been developed by Aron and Scott (1965). They show that when $r^2S/4KDt_n < 0.01$, the draw-

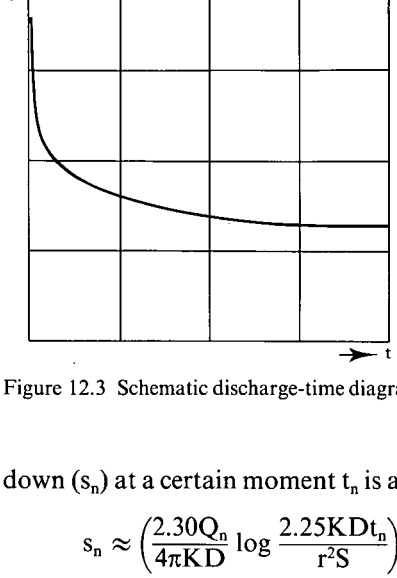

Figure 12.3 Schematic discharge-time diagram of a pump with decreasing discharge rate

down (s_n) at a certain moment t_n is approximately equal to

$$s_n \approx \left(\frac{2.30Q_n}{4\pi KD} \log \frac{2.25KDt_n}{r^2S}\right) + s_e \tag{12.7}$$

where Q_n is the discharge at time t_n, and s_e is the excess drawdown caused by the earlier higher discharge.

If $\bar{Q}_n$ is the average discharge from time 0 to t_n, the excess volume pumped is $(\bar{Q}_n - Q_n)t_n$. If the fully developed drawdown is considered to extend to the distance r_i at which $\log(2.25KDt_n/r_i^2S) = 0$, the excess drawdown s_e can be approximated by

$$s_e = \frac{(\bar{Q}_n - Q_n)t_n}{A_iS} = \frac{(\bar{Q}_n - Q_n)t_n}{S} \times \frac{S}{2.25\pi KDt_n} = \frac{\bar{Q}_n - Q_n}{2.25\pi KD} \tag{12.8}$$

where $A_i = \pi r_i^2$ = area influenced by the pumping.

If $r^2S/4KDt_n < 0.01$, a semi-log plot of s_n/Q_n versus t_n will yield a straight line. KD can then be determined by introducing the slope of the straight line, $\Delta(s_n/Q_n)$, i.e. the specific drawdown difference per log cycle of time, into

$$KD \approx \frac{2.30}{4\pi\Delta(s_n/Q_n)} \tag{12.9}$$

and S can be determined from

$$S \approx \frac{2.25KDt_0}{r^2} \tag{12.10}$$

where t_0 is the intercept of the straight line with the absciss $s_n/Q_n = \overline{s_e/Q_n}$, the latter being the average of several values of s_e/Q_n calculated from

$$\frac{s_e}{Q_n} = \frac{(\bar{Q}_n/Q_n) - 1}{2.25\pi KD} \tag{12.11}$$

The Aron-Scott method, which is analogous to the Jacob method (Section 3.2.2), can be used if the following assumptions and conditions are met:

- The assumptions listed at the beginning of Chapter 3, with the exception of the fifth assumption, which is replaced by:
  - The discharge rate decreases with time, the sharpest decrease occurring soon after the start of pumping.

The following conditions are added:
- The flow to the well is in an unsteady state;
- $r^2S/4KDt_n < 0.01$ (see also Section 3.2.2).

Procedure 12.2
- For one of the piezometers, plot s_n/Q_n versus t_n on semi-log paper (t_n on logarithmic scale). Fit a straight line through the plotted points (Figure 12.4);
- Determine the slope of the straight line, $\Delta(s_n/Q_n)$;
- Calculate KD from Equation 12.9;
- Calculate s_e/Q_n from Equation 12.11 for several values of t_n and determine the average value, $\overline{s_e/Q_n}$;
- Determine the interception point of the straight line with the absciss $s_n/Q_n = \overline{s_e/Q_n}$. The t value of this point is t_0;
- Calculate S from Equation 12.10;
- Repeat this procedure for all piezometers that satisfy the conditions. The results should show a close agreement.

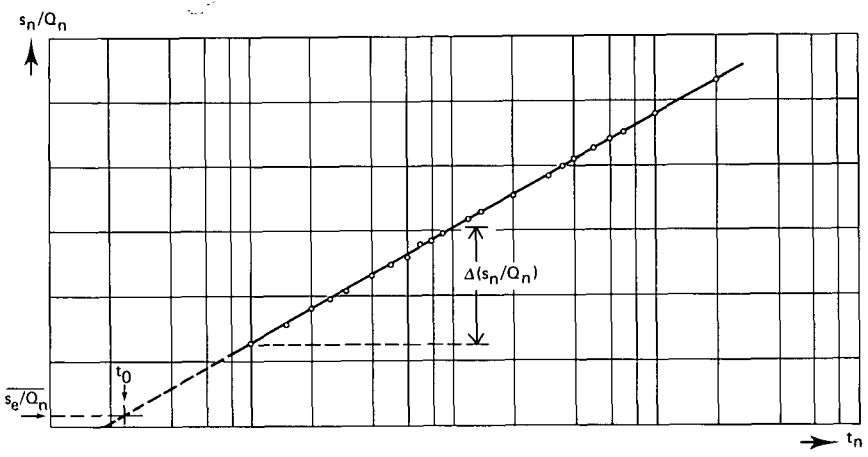

Figure 12.4 Illustration of the application of the Aron-Scott method

12.2 Free-flowing wells

The methods for free-flowing wells are based on the conditions that the drawdown in the well is constant and that the discharge decreases with time. To satisfy these conditions, the well is shut down for a period long enough for the pressure to have become static. When the well is opened up again at time t = 0, the water level in

the well drops instantaneously to a constant drawdown level, which is equal to the outflow opening of the well, while the well starts discharging at a decreasing rate.

12.2.1 Confined aquifer, unsteady-state flow, Hantush's method

The unsteady-state drawdown induced by a free-flowing well in a confined aquifer is given by Hantush (1964) (see also Reed 1980) as

$$s = s_w A(u_w, r/r_{ew}) \tag{12.12}$$

where

$A(u_w, r/r_{ew})$ = Hantush's free-flowing-well function for confined aquifers

$$u_w = \frac{r_{ew}^2 S}{4KDt} \tag{12.13}$$

r_{ew} = effective radius of flowing well

s_w = constant drawdown in flowing well = difference between static head measured during shutdown of the well and the outflow opening of the well

Annex 12.1 presents values of $A(u_w, r/r_{ew})$ in tabular form for different values of $1/u_w$ and r/r_{ew}.

The Hantush method can be used if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 3, with the exception of the fifth assumption, which is replaced by:
  - At the start of the test ($t = 0$), the water level in the free-flowing well drops instantaneously. At $t > 0$, the drawdown in the well is constant, and its discharge is variable.

The following condition is added:

- The flow to the well is in an unsteady state.

Procedure 12.3

- Using Annex 12.1, draw on log-log paper the family of type curves by plotting $A(u_w, r/r_{ew})$ versus $1/u_w$ for a range of values of r/r_{ew};
- On another sheet of log-log paper of the same scale, prepare the data curve by plotting s/s_w against the corresponding t for a single piezometer at r from the well;
- Match the data curve with one of type curves and note the r/r_{ew} value of the type curve;
- Select an arbitrary point A on the overlapping portion of the two sheets and note for this point the values of t and $1/u_w$;
- Substitute the values of $1/u_w$, r/r_{ew}, r, and t into Equation 12.13, now written as

$$\frac{KD}{S} = \frac{1}{4}\left(\frac{1}{u_w}\right)\left(\frac{r_{ew}}{r}\right)^2\left(\frac{r^2}{t}\right)$$

and calculate the diffusivity KD/S.

Remark

- If the value of r_{ew} is known, one type curve of $A(u_w, r/r_{ew})$ versus $1/u_w$ for the known value of r/r_{ew} can be used.

12.2.2 Leaky aquifers, steady-state flow, Hantush-De Glee's method

The steady-state drawdown in a leaky aquifer tapped by a fully penetrating free-flowing well is given by Hantush (1959a) as

$$s_m = \frac{Q_m}{2\pi KD} K_0(r/L) \tag{12.14}$$

where

s_m = steady-state drawdown in a piezometer at r from the well
Q_m = steady-state discharge (= minimum discharge) of the well

The data obtained during the steady-state phase of the free-flowing-well test can be analyzed with De Glee's method (Section 4.1.1), provided that the Hantush equation (Equation 12.14) is used instead of Equation 4.1. The following assumptions and conditions should be satisfied:

- The assumptions and conditions that underlie the standard methods for leaky aquifers (Chapter 4), with the exception of the fifth assumption, which is replaced by:
  - At the beginning of the test ($t = 0$), the water level in the well drops instantaneously. At $t > 0$, the drawdown in the well is constant, and its discharge is variable.

The following conditions are added:

- The flow to the well is in a steady state;
- $L > 3D$.

12.3 Well field

12.3.1 Cooper-Jacob's method

A modified version of the Jacob method, previously described in Section 3.2.2, can be used to resolve the effects of a well field on the drawdown (Cooper and Jacob 1946). By applying the principle of superposition and using values of specific drawdown ($s_n/\Sigma Q_i$) instead of drawdown (s), and values of the weighted logarithmic mean $\overline{(t_n/r_i^2)}$ instead of t/r^2, the same procedure as outlined for the Jacob method can be followed. The specific drawdown ($s_n/\Sigma Q_i$) is the drawdown (s_n) in a piezometer at a certain time t_n, divided by the sum of the discharges of the different pumped wells for the same time (ΣQ_i).

The assumptions and conditions underlying the Cooper-Jacob method are the same as those for the Jacob method (see Section 3.2.2) i.e.:

- The assumptions listed in Chapter 3;

– The flow to the well is in unsteady state;

$$-u \frac{S}{4KD(\overline{t/r_i^2})n} < 0.01.$$

Procedure 12.4 (see also Section 3.2.2)

– Calculate for one of the piezometers the value of the specific drawdown $(s_n/\Sigma Q_i)$ for each corresponding time t_n;
– Determine the weighted logarithmic mean, $\overline{(t/r_i^2)}_n$, corresponding to each value of t_n in the following way:
  • Divide the elapsed time t_n by the square of the distance from each pumped well to the piezometer, r_i^2, (t_n/r_i^2);
  • Multiply the logarithm of each of those values by the individual well discharge $[Q_i \log(t_n/r_i^2)]$;
  • Sum the products algebraically $[\Sigma\, Q_i \log(t_n/r_i^2)]$;
  • Divide that sum by the sum of the discharges of the different pumping wells $[\{\Sigma\, Q_i \log(t_n/r_i^2)\}/\Sigma Q_i] = (x)$;
  • Extract the antilogarithm of the quotient $(10^{(x)})$ which is the requested value of $\overline{(t/r_i^2)}_n$;
– Plot the values of $(s_n/\Sigma Q_i)$ versus $\overline{(t/r_i^2)}_n$ on semi-log paper ($\overline{(t/r_i^2}$ on the logarithmic axis). Draw a straight line through the plotted points;
– Extend the straight line till it intercepts the time-axis where $s_n/\Sigma Q_i = 0$, and read the value of $\overline{(t/r_i^2)}_0$;
– Determine the slope of the straight line, i.e. the drawdown difference $\Delta(s_n/\Sigma Q_i)$ per log cycle of $\overline{(t/r_i^2)}_n$;
– Substitute the values of $\Delta(s_n/\Sigma Q_i)$ into – a modified version of – Equation 3.13

$$KD = \frac{2.30}{4\pi\Delta(s_n/\Sigma Q_i)}$$

and solve for KD;
– With KD and $\overline{(t/r_i^2)}_0$ known, calculate S from Equation 3.12

$$S = 2.25\, KD\, \overline{(t/r_i^2)}_0$$

Remark

– The Cooper-Jacob method can also be applied if the individual wells are pumped at a variable discharge rate. Hence the discharge rate of each individual well is dependent on the elapsed time t_n, and the value of ΣQ_i will not be constant.

Example 12.2

In a hypothetical well field, the pumping started simultaneously in three wells (1, 2, 3) at constant discharge rates of $Q_1 = 150\ m^3/d$, $Q_2 = 200\ m^3/d$, and $Q_3 = 300\ m^3/d$. The drawdown was observed in a piezometer at a distance of $r_1 = 10$ m from Well 1, $r_2 = 20$ m from Well 2, and $r_3 = 30$ m from Well 3 (Table 12.2).

Table 12.2 gives the calculated values of $s_n/\Sigma\, Q_i$, and shows the step-by-step procedure to calculate the weighted logarithmic mean $\overline{(t/r_i^2)}_n$.

The values of $s_n/\Sigma\, Q_i$ and $\overline{(t/r_i^2)}_n$ are plotted on semi-log paper (Figure 12.5). The slope of the straight line through the plotted points $\Delta(s_n/\Sigma Q_i) = 4.75 \times 10^{-4}$. Hence

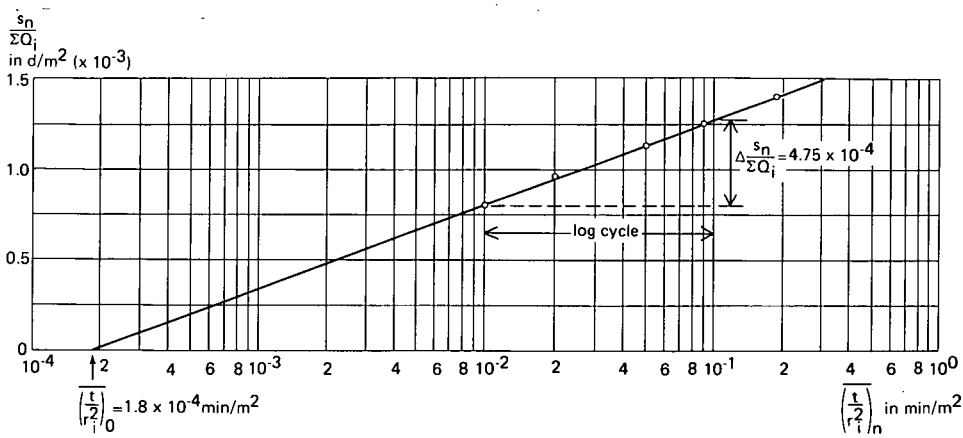

Figure 12.5 Analysis of data with the Cooper-Jacob method for well fields

$$KD = \frac{2.30}{4\pi\Delta(s_n/\Sigma Q_i)} = \frac{2.30}{4 \times 3.14 \times 4.75 \times 10^{-4}} = 386\ m^2/d$$

The interception point of the straight line with the $(s_n/\Sigma Q_i) = 0$ axis is $\overline{(t/r_i^2)}_0 = 1.8 \times 10^{-4}\ min/m^2$.
S can be calculated from

$$S = 2.25\ KD\ \overline{(t/r_i^2)}_0 = 2.25 \times 386 \times 1.8 \times 10^{-4} \times \frac{1}{1440} = 10^{-4}$$

Table 12.2 Calculation of parameter $\overline{(t/r_i^2)}_n$ of the Cooper-Jacob method

	$r \to 1$	2	3	4	5
s_n (m)	0.53	0.62	0.74	0.82	0.91
ΣQ_i (m^3/d)	650	650	650	650	650
$s_n/\Sigma Q_i$ (d/m^2)	8.15×10^{-4}	9.54×10^{-4}	1.13×10^{-3}	1.26×10^{-3}	1.4×10^{-3}
t_n (min)	5	10	20	40	80
$t_n/r_1^2 = t_n/100$	0.05	0.10	0.20	0.40	0.80
$t_n/r_2^2 = t_n/400$	0.0125	0.025	0.05	0.10	0.20
$t_n/r_3^2 = t_n/900$	0.0056	0.0111	0.0222	0.0444	0.0889
$Q_1 \log(t_n/r_1^2)$	− 195.2	− 150	− 104.8	− 59.7	− 14.5
$Q_2 \log(t_n/r_2^2)$	− 380.6	− 320.4	− 260.2	− 200	− 139.8
$Q_3 \log(t_n/r_3^2)$	− 676.6	− 586.3	− 496.0	− 405.7	− 315.3
	+	+	+		+
$\Sigma Q_i \log(t_n/r_i^2)$	− 1252.4	− 1056.7	− 861.0	− 665.4	− 496
$\frac{\Sigma Q_i \log(t_n/r_i^2)}{\Sigma Q_i}$	− 1.927	− 1.626	− 1.325	− 1.024	− 0.722
$\overline{(t/r_i^2)}_n$ (min/m^2)	0.01	0.02	0.05	0.09	0.19