Discharge measurement structures

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Third revised edition

Edited by
M.G. Bos

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Represented in the Working Group on Small Hydraulic Structures are the following institutions:

International Institute for Land Reclamation and Improvement/ILRI, Wageningen

Delft Hydraulics Laboratory, Delft

University of Agriculture, Departments of Hydraulics and Irrigation, Wageningen

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Preface to the first edition

The Working Group on Small Hydraulic Structures was formed in September 1971 and charged with the tasks of surveying current literature on small structures in open channels and of conducting additional research as considered necessary.

The members of the Working Group are all engaged in irrigation engineering, hydrology, or hydraulics, and are employed by the Delft Hydraulics Laboratory (DHL), the University of Agriculture (LU) at Wageningen, or the International Institute for Land Reclamation and Improvement (ILRI) at Wageningen.

The names of those participating in the Group are:
Ing. W. Boiten (DHL)
Ir. M.G. Bos (ILRI)
Prof.Ir. D.A. Kraijenhoff van de Leur (LU)
Ir. H. Oostinga (DHL) during 1975
Ir. R.H. Pitlo (LU)
Ir. A.H. de Vries (DHL)
Ir. J. Wijdieks (DHL)

The Group lost one of its initiators and most expert members in the person of Professor Ir. J. Nugteren (LU), who died on April 20, 1974.

The manuscripts for this publication were written by various group members. Ing. W. Boiten prepared the Sections 4.3, 4.4, and 7.4; Ir. R.H. Pitlo prepared Section 7.5; Ir. A.H. de Vries prepared the Sections 7.2, 7.3, 9.2, and 9.7, and the Annexes 2 and 3. The remaining manuscripts were written by Ir. M.G. Bos. All sections were critically reviewed by all working group members, after which Ir. M.G. Bos prepared the manuscripts for publication.

Special thanks are due to Ir. E. Stamhuis and Ir. T. Meijer for their critical review of Chapter 3, to Dr. P.T. Stol for his constructive comments on Annex 2 and to Dr. M.J. Hall of the Imperial College of Science and Technology, London, for proof-reading the entire manuscript.

This book presents instructions, standards, and procedures for the selection, design, and use of structures, which measure or regulate the flow rate in open channels. It is intended to serve as a guide to good practice for engineers concerned with the design and operation of such structures. It is hoped that the book will serve this purpose in three ways: (i) by giving the hydraulic theory related to discharge measurement structures; (ii) by indicating the major demands made upon the structures; and (iii) by providing specialized and technical knowledge on the more common types of structures now being used throughout the world.

The text is addressed to the designer and operator of the structure and gives the hydraulic dimensions of the structure. Construction methods are only given if they influence the hydraulic performance of the structure. Otherwise, no methods of construction nor specifications of materials are given since they vary greatly from country to country and their selection will be influenced by such factors as the availability of materials, the quality of workmanship, and by the number of structures that need to be built.

The efficient management of water supplies, particularly in the arid regions of the world, is becoming more and more important as the demand for water grows even greater with the world's increasing population and as new sources of water become harder to find. Water resources are one of our most vital commodities and they must be conserved by reducing the amounts of water lost through inefficient management. An essential part of water conservation is the accurate measurement and regulation of discharges.

We hope that this book will find its way, not only to irrigation engineers and hydrologists, but also to all others who are actively engaged in the management of water resources. Any comments which may lead to improved future editions of this book will be welcomed.

Wageningen, October 1975

M.G.Bos
Editor

Preface to the second edition

The second edition of this book is essentially similar to the first edition in 1976, which met with such success that all copies have been sold. The only new material in the second edition is found in Chapter 7, Sections 1 and 5. Further all known errors have been corrected, a number of graphs has been redrawn and, where possible, changes in the lay-out have been made to improve the readability.

Remarks and criticism received from users and reviewers of the first edition have been very helpful in the revision of this book.

Wageningen, July 1978

M.G.Bos
Editor

Preface to the third edition

This third edition retains the concept of the two previous editons, of which some 6700 copies have been sold. Nevertheless, major revisions have been made: in Sections 1.9, 4.1, 4.3, and 7.1 (which all deal with broad-crested weirs and long-throated flumes); in Sections 1.5, 1.16, and 3.2.2; and in Annex 4.

Minor classifications have been added and errors corrected. Further, the typeface and lay-out have been changed to improve the legibility of the text and accomodate some additional information.

Wageningen, January 1989

Dr. M.G. Bos
Editor

Contents

1 Basic principles of fluid flow as applied to measuring structures

1.1 General

The purpose of this chapter is to explain the fundamental principles involved in evaluating the flow pattern in weirs, flumes, orifices and other measuring structures, since it is the flow pattern that determines the head-discharge relationship in such structures.

Since the variation of density is negligible in the context of these studies, we shall regard the mass density (ρ) of water as a constant. Nor shall we consider any flow except time invariant or steady flow, so that a streamline indicates the path followed by a fluid particle.

The co-ordinate system, used to describe the flow phenomena at a point P of a streamline in space, has the three directions as illustrated in Figure 1.1.

Before defining the co-ordinate system, we must first explain some mathematical concepts. A tangent to a curve is a straight line that intersects the curve at two points which are infinitely close to each other. An osculating plane intersects the curve at three points which are infinitely close to each other. In other words, the curvature at a point P exists in the local osculating plane only. Hence the tangent is a line in the osculating plane. The normal plane to a curve at P is defined as the plane perpendicular to the tangent of the curve at P. All lines through P in this normal plane are called normals, the normal in the osculating plane being called the principal normal,

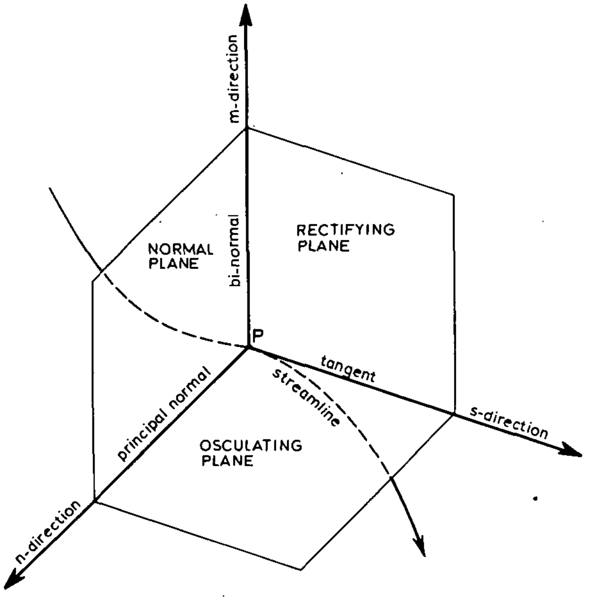

Figure 1.1 The co-ordinate system

and the one perpendicular to the osculating plane being called the bi-normal.
The three co-ordinate directions are defined as follows:

s-direction: The direction of the velocity vector at point P. By definition, this vector coincides with the tangent to the streamline at P ($v_s = v$);

n-direction: The normal direction towards the centre of curvature of the streamline at P. By definition, both the s- and n-direction are situated in the osculating plane;

m-direction: The direction perpendicular to the osculating plane at P as indicated in Figure 1.1.

It should be noted that, in accordance with the definition of the osculating plane, the acceleration of flow in the m-direction equals zero ($a_m = 0$).
Metric units (SI) will be used throughout this book, although sometimes for practical purposes, the equivalent Imperial units will be used in addition.

1.2 Continuity

An elementary flow passage bounded by streamlines is known as a stream tube. Since there is, per definition, no flow across these boundaries and since water is assumed here to be incompressible, fluid must enter one cross-section of the tube at the same volume per unit time as it leaves the other.

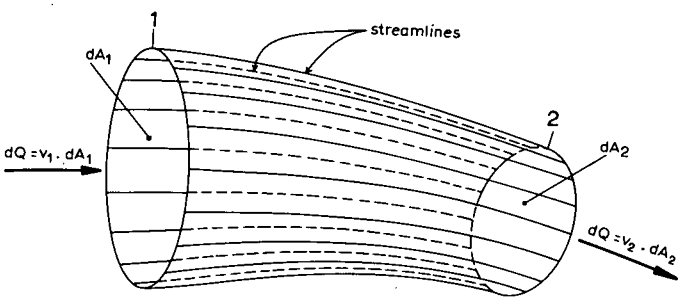

Figure 1.2 The stream tube

From the assumption of steady flow, it follows that the shape and position of the stream tube do not change with time. Thus the rate at which water is flowing across a section equals the product of the velocity component perpendicular to the section and the area of this section. If the subscripts 1 and 2 are applied to the two ends of the elementary stream tube, we can write:

$$\text{Discharge} = dQ = v_1 dA_1 = v_2 dA_2 \qquad (1\text{-}1)$$

This continuity equation is valid for incompressible fluid flow through any stream tube. If Equation 1-1 is applied to a stream tube with finite cross-sectional area, as in an open channel with steady flow (the channel bottom, side slopes, and water surface being the boundaries of the stream tube), the continuity equation reads:

$$Q = \int^A v dA = \bar{v}A = \text{constant}$$

or

$$\bar{v}_1 A_1 = \bar{v}_2 A_2 \tag{1-2}$$

where $\bar{v}$ is the average velocity component perpendicular to the cross-section of the open channel.

1.3 Equation of motion in the s-direction

Since we do not regard heat and sound as being types of energy which influence the liquid flow in open channels, an elementary fluid particle has the following three interchangeable types of energy per unit of volume:

$^1/_2 \rho v^2$ = kinetic energy per unit of volume
$\rho g z$ = potential energy per unit of volume
P = pressure energy per unit of volume.

Consider a fluid particle moving in a time interval Δt from Point 1 to Point 2 along a streamline, there being no loss of energy due to friction or increased turbulence. (See Fig. 1.3.) Since, on the other hand, there is no gain of energy either, we can write

$$(^1/_2 \rho v^2 + \rho g z + P)_1 = (^1/_2 \rho v^2 + \rho g z + P)_2 = \text{constant} \tag{1-3}$$

This equation is valid for points along a streamline only if the energy losses are negligible and the mass density (ρ) is a constant. According to Equation 1-3

$$^1/_2 \rho v^2 + \rho g z + P = \text{constant} \tag{1-4}$$

or

$$v^2/2g + P/\rho g + z = H = \text{constant} \tag{1-5}$$

where, as shown in Figure 1.3,

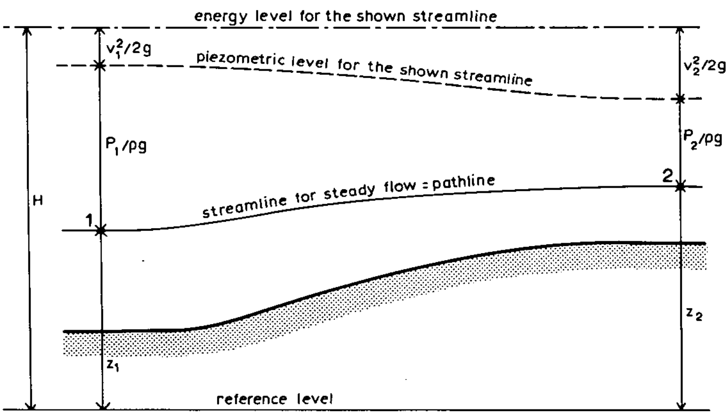

Figure 1.3 The energy of a fluid particle

$v^2/2g$ = the velocity head
$P/\rho g$ = the pressure head
z = the elevation head
$P/\rho g + z$ = the piezometric head
H = the total energy head.

The last three heads all refer to the same reference level. The reader should note that each individual streamline may have its own energy head. Equations 1-3, 1-4, and 1-5 are alternative forms of the well-known Bernoulli equation, of which a detailed derivation is presented in Annex 1.

1.4 Piezometric gradient in the n-direction

On a particle (ds, dn, dm) following a curved streamline, a force F is acting towards the centre of curvature in order to accelerate the particle in a sense perpendicular to its direction of motion. Since in Section 1.1 the direction of motion and the direction towards the centre of curvature have been defined as the s- and n-direction respectively, we consider here the movement of a particle along an elementary section of a streamline in the osculating plane.

By Newton's second law of motion

$$F = ma \tag{1-6}$$

the centripetal acceleration (a) in consequence of the passage along a circle with a radius (r) with a velocity (v), according to mechanics, equals:

$$a = \frac{v^2}{r} \tag{1-7}$$

Since the mass (m) of the particle equals ρ(ds dn dm), the force (F) can be expressed as

$$F = \rho \, ds \, dn \, dm \frac{v^2}{r} \tag{1-8}$$

This force (F) is due to fluid pressure and gravitation acting on the fluid particle. It can be proved (see Annex 1) that the negative energy gradient in the n-direction equals the centripetal force per unit of mass (equals centripetal acceleration). In other words:

$$-\frac{d}{dn}\left(\frac{P}{\rho} + gz\right) = \frac{v^2}{r} \tag{1-9}$$

or

$$d\left(\frac{P}{\rho g} + z\right) = -\frac{1}{g}\frac{v^2}{r}dn \tag{1-10}$$

After integration of this equation from Point 1 to Point 2 in the n-direction we obtain the following equation for the fall of piezometric head in the n-direction (see Fig.1.4)

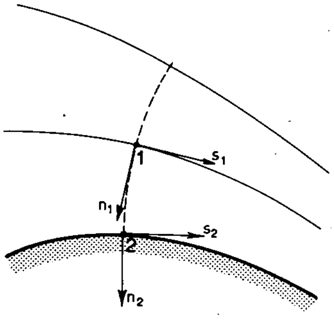

Figure 1.4 Key to symbols

$$\left[\frac{P}{\rho g} + z\right]_1 - \left[\frac{P}{\rho g} + z\right]_2 = \frac{1}{g}\int_1^2 \frac{v^2}{r} dn \qquad (1\text{-}11)$$

In this equation

$\left[\frac{P}{\rho g} + z\right]_1$ = the piezometric head at Point 1

$\left[\frac{P}{\rho g} + z\right]_2$ = the piezometric head at Point 2

$\int_1^2 \frac{v^2}{gr} dn$ = the difference between the piezometric heads at Points 1 and 2 due to the curvature of the streamlines

From this equation it appears that, if the streamlines are straight ($r = \infty$), the integral has zero value, and thus the piezometric head at Point 1 equals that at Point 2, so that

$$\left[\frac{P}{\rho g} + z\right]_1 = \left[\frac{P}{\rho g} + z\right]_2 = \text{constant} \qquad (1\text{-}12)$$

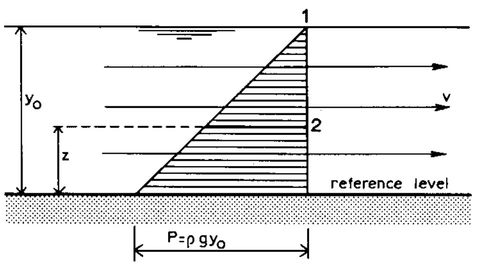

Figure 1.5 Hydrostatic pressure distribution

At the water surface in an open channel, $P_1 = 0$; hence

$$\frac{P_2}{\rho g} = y_o - z$$

or

$$P_2 = \rho g(y_o - z) \quad (1\text{-}13)$$

Thus, if $r = \infty$ there is what is known as a hydrostatic pressure distribution. If the streamlines are curved, however, and there is a significant flow velocity, the integral may reach a considerable value.

At a free overfall with a fully aerated air pocket underneath the nappe, there is atmospheric pressure at both Points 1 and 2, while a certain distance upstream there is a hydrostatic pressure distribution. The deviation from the hydrostatic pressure distribution at the end of the weir is mainly caused by the value of the integral (see Fig.1.6). A decrease of piezometric head, which is due to the centripetal acceleration, necessarily induces a corresponding increase of velocity head:

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \int_1^2 \frac{v^2}{gr}\, dn \quad (1\text{-}14)$$

To illustrate the effect of streamline curvature on the velocity distribution in the n-direction, Figure 1.6 shows the velocity distribution over a cross-section where a hydrostatic pressure distribution prevails and the velocity distribution at the free overfall.

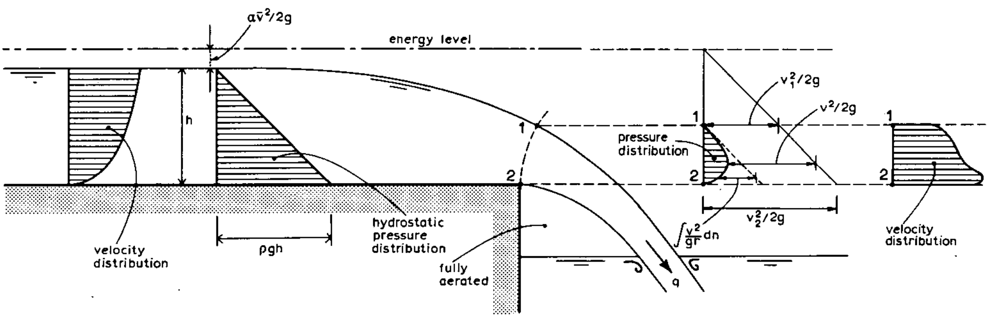

Figure 1.6 Pressure and velocity distribution at a free overfall

1.5 Hydrostatic pressure distribution in the m-direction

As mentioned in Section 1.1, in the direction perpendicular to the osculating plane, not only $v_m = 0$, but also

$$a_m \doteq \frac{dv_m}{dt} = 0$$

Consequently, there is no net force acting in the m-direction, and therefore the pressure distribution is hydrostatic. Hence

$$P + \rho gz = \text{constant} \tag{1-15}$$

or

$$\frac{P}{\rho g} + z = \text{constant} \tag{1-16}$$

1.6 The total energy head of an open channel cross-section

According to Equation 1-4, the total energy per unit of volume of a fluid particle can be expressed as the sum of the three types of energy

$$\tfrac{1}{2}\rho v^2 + \rho gz + P \tag{1-17}$$

We now want to apply this expression to the total energy which passes through the entire cross-section of a channel. We therefore need to express the total kinetic energy of the discharge in terms of the average flow velocity of the cross-section.

In this context, the reader should note that this average flow velocity is not a directly measurable quantity but a derived one, defined by

$$\bar{v} = \frac{Q}{A} \tag{1-18}$$

Due to the presence of a free water surface and the friction along the solid channel boundary, the velocities in the channel are not uniformly distributed over the channel cross-section (Fig.1.7). Owing to this non-uniform velocity distribution, the true average kinetic energy per unit of volume across the section, $(\tfrac{1}{2}\rho v^2)_{average}$ will not necessarily be equal to $\tfrac{1}{2}\rho\bar{v}^2$.
In other words:

$$(\tfrac{1}{2}\rho v^2)_{average} = \alpha\,\tfrac{1}{2}\rho\bar{v}^2 \tag{1-19}$$

The velocity distribution coefficient (α) always exceeds unity. It equals unity when the flow is uniform across the entire cross-section and becomes greater the further flow departs from uniform.

For straight open channels with steady turbulent flow, α-values range between 1.03 and 1.10. In many cases the velocity head makes up only a minor part of the total

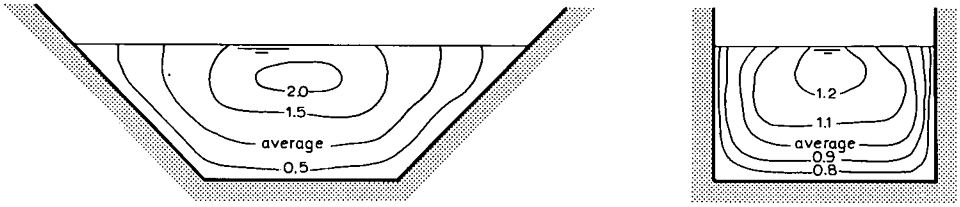

Figure 1.7 Examples of velocity profiles in a channel section

energy head and $\alpha = 1$ can then be used for practical purposes. Thus, the average kinetic energy per unit of volume of water passing a cross-section equals $\alpha\ {}^1/_2\ \rho\bar{v}^2$.

The variation of the remaining two terms over the cross-section is characterized by Equations 1-9 and 1-15. If we consider an open channel section with steady flow, where the streamlines are straight and parallel, there is no centripetal acceleration and, therefore, both in the n- and m-direction, the sum of the potential and pressure energy at any point is constant. In other words;

$$\rho gz + P = \text{constant} \tag{1-20}$$

for all points in the cross-section. Since at the water surface $P = 0$, the piezometric level of the cross-section coincides with the local water surface. For the considered cross-section the expression for the average energy per unit of volume passing through the cross-section reads:

$$E = \alpha\ {}^1/_2\ \rho\bar{v}^2 + P + \rho gz \tag{1-21}$$

or if expressed in terms of head

$$\alpha \frac{\bar{v}^2}{2g} + \frac{P}{\rho g} + z = H \tag{1-22}$$

where H is the total energy head of a cross-sectional area of flow. We have now reached the stage that we are able to express this total energy head in the elevation of the water surface $(P/\rho g + z)$ plus the velocity head $\alpha\bar{v}^2/2g$.

In the following sections it will be assumed that over a short reach of accelerated flow, bounded by channel cross-sections perpendicular to straight and parallel streamlines, the loss of energy head is negligible with regard to the interchangeable types of energy (Figure 1.8). Hence:

$$\alpha \frac{\bar{v}_1^2}{2g} + \left[\frac{P}{\rho g} + z\right]_1 = H = \alpha \frac{\bar{v}_2^2}{2g} + \left[\frac{P}{\rho g} + z\right]_2 \tag{1-23}$$

Thus, if we may assume the energy head (H) in both cross-sections to be the same, we have an expression that enables us to compare the interchange of velocity head and piezometric head in a short zone of acceleration.

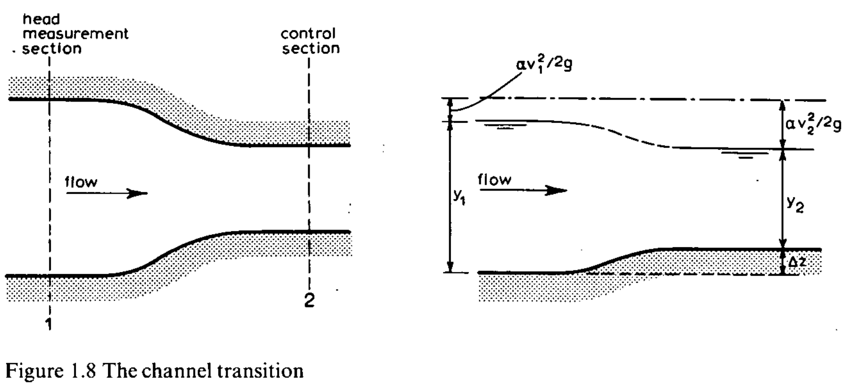

Figure 1.8 The channel transition

1.7 Recapitulation

For a short zone of acceleration bounded by cross-sections perpendicular to straight and parallel streamlines, the following two equations are valid:

Continuity equation (1-2)

$$Q = \bar{v}_1 A_1 = \bar{v}_2 A_2$$

Bernoulli's equation (1-23)

$$H = \alpha \frac{\bar{v}_1^2}{2g} + \left[\frac{P}{\rho g} + z\right]_1 = \alpha \frac{\bar{v}_2^2}{2g} + \left[\frac{P}{\rho g} + z\right]_2$$

In both cross-sections the piezometric level coincides with the water surface and the latter determines the area A of the cross-section. We may therefore conclude that if the shapes of the two cross-sections are known, the two unknowns $\bar{v}_1$ and $\bar{v}_2$ can be determined from the two corresponding water levels by means of the above equations.

It is evident, however, that collecting and handling two sets of data per measuring structure is an expensive and time-consuming enterprise which should be avoided if possible. It will be shown that under critical flow conditions one water level only is sufficient to determine the discharge. In order to explain this critical condition, the concept of specific energy will first be defined.

1.8 Specific energy

The concept of specific energy was first introduced by Bakhmeteff in 1912, and is defined as the average energy per unit weight of water at a channel section as expressed with respect to the channel bottom. Since the piezometric level coincides with the water surface, the piezometric head with respect to the channel bottom is

$$\frac{P}{\rho g} + z = y, \text{ the water depth} \tag{1-24}$$

so that the specific energy head can be expressed as:

$$H_o = y + \alpha \bar{v}^2/2g \tag{1-25}$$

We find that the specific energy at a channel section equals the sum of the water depth (y) and the velocity head, provided of course that the streamlines are straight and parallel. Since $\bar{v} = Q/A$, Equation 1-25 may be written

$$H_o = y + \alpha \frac{Q^2}{2gA^2} \tag{1-26}$$

where A, the cross-sectional area of flow, can also be expressed as a function of the water depth, y. From this equation it can be seen that for a given channel section and a constant discharge (Q), the specific energy in an open channel section is a function of the water depth only. Plotting this water depth (y) against the specific energy (H_o) gives a specific energy curve as shown in Figure 1.9.

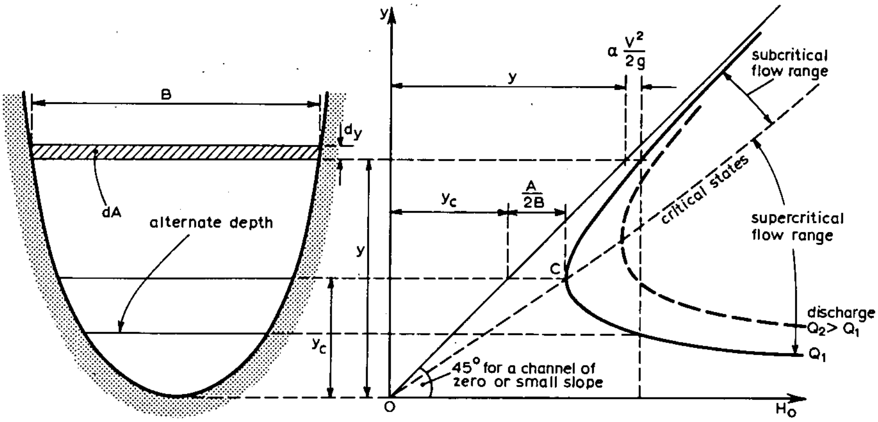

Figure 1.9 The specific energy curve

The curve shows that, for a given discharge and specific energy there are two 'alternate depths' of flow. At Point C the specific energy is a minimum for a given discharge and the two alternate depths coincide. This depth of flow is known as 'critical depth' (y_c).

When the depth of flow is greater than the critical depth, the flow is called subcritical; if it is less than the critical depth, the flow is called supercritical. The curve illustrates how a given discharge can occur at two possible flow regimes; slow and deep on the upper limb, fast and shallow on the lower limb, the limbs being separated by the critical flow condition (Point C).

When there is a rapid change in depth of flow from a high to a low stage, a steep depression will occur in the water surface; this is called a 'hydraulic drop'. On the other hand, when there is a rapid change from a low to a high stage, the water surface will rise abruptly; this phenomenon is called a 'hydraulic jump' or 'standing wave'. The standing wave shows itself by its turbulence (white water), whereas the hydraulic drop is less apparent. However, if in a standing wave the change in depth is small, the water surface will not rise abruptly but will pass from a low to a high level through a series of undulations (undular jump), and detection becomes more difficult. The normal procedure to ascertain whether critical flow occurs in a channel contraction – there being subcritical flow upstream and downstream of the contraction – is to look for a hydraulic jump immediately downstream of the contraction.

From Figure 1.9 it is possible to see that if the state of flow is critical, i.e. if the specific energy is a minimum for a given discharge, there is one value for the depth of flow only. The relationship between this minimum specific energy and the critical depth is found by differentiating Equation 1-26 to y, while Q remains constant.

$$\frac{dH_o}{dy} = 1 - \alpha \frac{Q^2}{gA^3}\frac{dA}{dy} = 1 - \alpha \frac{\bar{v}^2}{gA}\frac{dA}{dy} \qquad (1\text{-}27)$$

Since dA = B dy, this equation becomes

$$\frac{dH_o}{dy} = 1 - \alpha \frac{\bar{v}^2 B}{g\,A} \qquad (1\text{-}28)$$

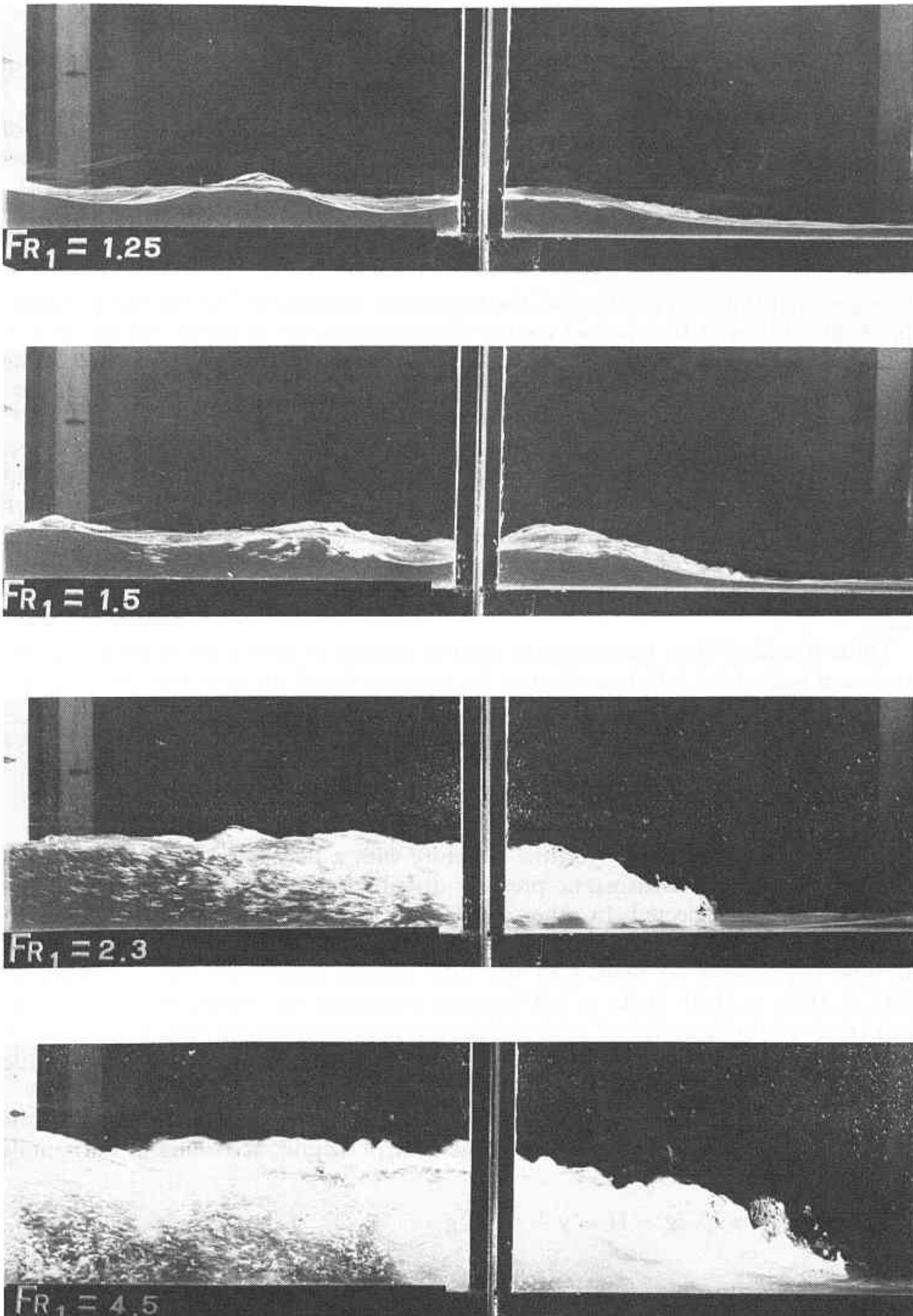

Photo 1 Hydraulic jumps