

Annex 3

Side weirs and oblique weirs

3.1 Introduction

Most of the weirs described in this book serve mainly to measure discharges. Some, however, such as those described in Chapters 4 and 6 can also be used to control upstream water levels. To perform this dual function, the weirs have to be installed according to the requirements given in the relevant chapters. Since these weirs are usually relatively wide with respect to the upstream head, the accuracy of their flow measurements is not very high. Sometimes the discharge measuring function of the weir is entirely superseded by its water level control function, resulting in a contravention in their installation rules. The following weirs are typical examples of water level control structures.

Side weir: This weir is part of the channel embankment, its crest being parallel to the flow direction in the channel. Its function is to drain water from the channel whenever the water surface rises above a predetermined level so that the channel water surface downstream of the weir remains below a maximum permissible level.

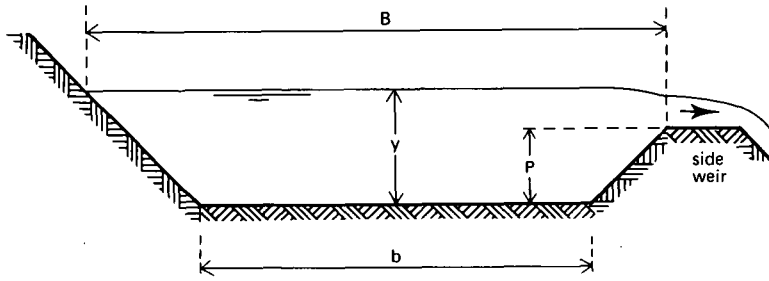
Oblique weir: The most striking difference between an oblique weir and other weirs is that the crest of the oblique weir makes an angle with the flow direction in the channel. The crest must be greater than the width of the channel so that with a change in discharge the water surface upstream of the weir remains between narrow limits. Some other weir types which can maintain such an almost constant upstream water level will also be described.

3.2 Side weirs

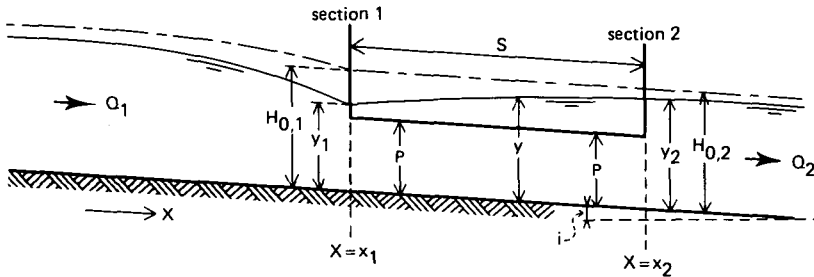
3.2.1 General

In practice, sub-critical flow will occur in almost all rivers and irrigation or drainage canals in which side weirs are constructed. Therefore, we shall restrict our attention to side weirs in canals where the flow remains subcritical. The flow profile parallel to the weir, as illustrated in Figure A3.1, shows an increasing depth of flow.

The side weir shown in Figure A3.1 is broad-crested and its crest is parallel to the channel bottom. It should be noted, however, that a side weir need not necessarily be broad-crested. The water depth downstream of the weir y_2 and also the specific energy head $H_{o,2}$ are determined by the flow rate remaining in the channel (Q_2) and the hydraulic characteristics of the downstream channel. This water depth is either controlled by some downstream construction or, in the case of a long channel, it will equal the normal depth in the downstream channel. Normal depth being the only water depth which remains constant in the flow direction at a given discharge (Q_2), hydraulic radius, bottom slope, and friction coefficient of the downstream channel.



CROSS SECTION



WATER SURFACE PROFILE

Figure A3.1 Dimension sketch of side weir.

3.2.2 Theory

The theory on flow over side weirs given below is only applicable if the area of water surface drawdown perpendicular to the centre line of the canal is small in comparison with the water surface width of this canal. In other words, if $y - p_1 < 0.1 B$.

For the analysis of spatially varied flow with decreasing discharge, we may apply the energy principle as introduced in Chapter 1, Sections 1.6 and 1.8. When water is being drawn from a channel as in Figure A3.1, energy losses in the overflow process are assumed to be small, and if we assume in addition that losses in specific energy head due to friction along the side weir equal the fall of the channel bottom, the energy line is parallel to this bottom. We should therefore be able to write

$$H_{o,1} = y_1 + \frac{Q_1^2}{2g A_1^2} = y_2 + \frac{Q_2^2}{2g A_2^2} = H_{o,2} \quad (A3.1)$$

If the specific energy head of the water remaining in the channel is (almost) constant

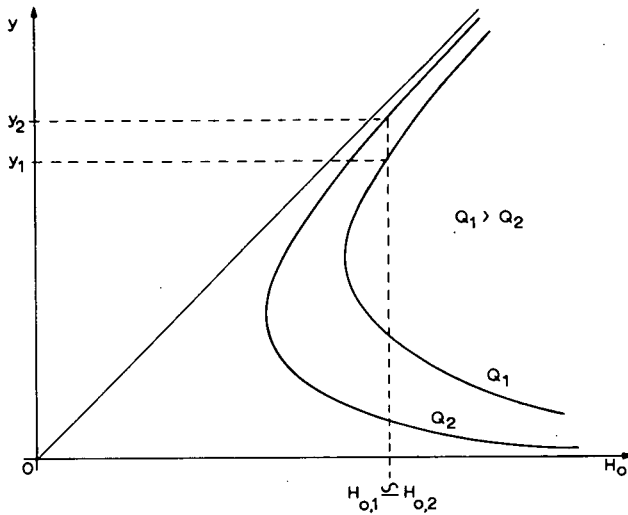


Figure A3.2 H_o - y diagram for the on-going channel

while at the same time the discharge decreases, the water depth y along the side weir should increase in downstream direction as indicated in Figures A3.1 and A3.2, which is the case if the depth of flow along the side weir is subcritical (see also Chapter 1, Figure 1.9).

Far upstream of the side weir, the channel water depth y equals the normal depth related to the discharge Q_1 and the water has a specific energy $H_{o,0}$, which is greater than $H_{o,2}$. Over a channel reach upstream of the weir, the water surface is drawn down in the direction of the weir. This causes the flow velocity to increase and results in an additional loss of energy due to friction expressed in the loss of specific energy head $H_{o,0} - H_{o,2}$. Writing Equation A3.1 as a differential equation we get

$$\frac{dH_o}{dx} = \frac{dy}{dx} + \frac{d}{dx} \frac{Q^2}{2gA^2} \quad (A3.2)$$

or

$$\frac{dH_o}{dx} = 0 = \frac{dy}{dx} + \frac{1}{2g} \left(\frac{2Q}{A^2} \frac{dQ}{dx} - \frac{2Q^2}{A^3} \frac{dA}{dx} \right) \quad (A3.3)$$

The continuity equation for this channel reach reads $dQ/dx = -q$, and the flow rate per unit of channel length across the side weir equals

$$q = C_s \frac{2}{3} \sqrt{\frac{2}{3}} g (y - p_1)^{1.5} \quad (A3.4)$$

The flow rate in the channel at any section is

$$Q = A \sqrt{2g(H_o - y)}$$

and finally

$$\frac{dA}{dx} = B \frac{dy}{dx}$$

so that Equation A3.3 can be written as follows

$$\frac{dy}{dx} = \frac{4C_s (H_o - y)^{0.5} (y - p)^{1.5}}{3^{1.5} B \left[\frac{A}{B} + 2y - 2H_o \right]} \quad (\text{A3.5})$$

where C_s denotes the effective discharge coefficient of the side weir. Equation A3.4 differs from Equation 1-36 (Chapter 1) in that, since there is no approach velocity towards the weir crest, y has been substituted for H_o . Equation A3.5, which describes the shape of the water surface along the side weir, can be further simplified by assuming a rectangular channel where B is constant and $A/B = y$, resulting in

$$\frac{dy}{dx} = \frac{4C_s (H_o - y)^{0.5} (y - p)^{1.5}}{3^{1.5} B (3y - 2H_o)} \quad (\text{A3.6})$$

For this differential equation De Marchi (1934) found a solution which was confirmed experimentally by Gentilini (1938) and Collinge (1957) and reads

$$x = \frac{3^{1.5} B}{2C_s} \left[\frac{2H_o - 3p}{H_o - p} \left(\frac{H_o - y}{y - p} \right)^{0.5} - 3 \arcsin \left(\frac{H_o - y}{H_o - p} \right)^{0.5} \right] + K \quad (\text{A3.7})$$

where K is an integration constant. The term in between the square brackets may be denoted as $\phi(y/H_o)$ and is a function of the dimensionless ratios $y/H_{o,2}$ and $p/H_{o,2}$ as shown in Figure A3.3. If p_1 , y_2 , and $H_{o,2}$ are known, the water surface elevation at any cross section at a distance $(x - x_2)$ along the side weir can be determined from the equation*

$$x - x_2 = \frac{3^{1.5} B}{2C_s} [\phi(y/H_{o,2}) - \phi(y_2/H_{o,2})] \quad (\text{A3.8})$$

If the simplifying assumptions made to write Equation A3.1 cannot be retained or in other words, if the statement

$$\int \frac{v^2}{C^2 R} - S \tan i \ll y_2 - y_1 \quad (\text{A3.9})$$

is not correct, the water surface elevation parallel to the weir can only be obtained by making a numerical calculation starting at the downstream end of the side weir (at $x = x_2$). This calculation also has to be made if the cross section of the channel is not rectangular.

For this procedure the following two equations can be used

$$y_u - y_d = - \frac{(v_u + v_d)(v_u - v_d)}{2g} + \left[\frac{v_d^2}{C^2 R_d} - i \right] \Delta x \quad (\text{A3.10})$$

* If the flow along the weir is supercritical and no hydraulic jump occurs along the weir and the same simplifying assumptions are retained, Equations A3.1 to A3.8 are also valid. Greater discrepancies, however, occur between theory and experimental results. Also, the water surface profile along the weir has a shape different from that shown in Figure A3.1.

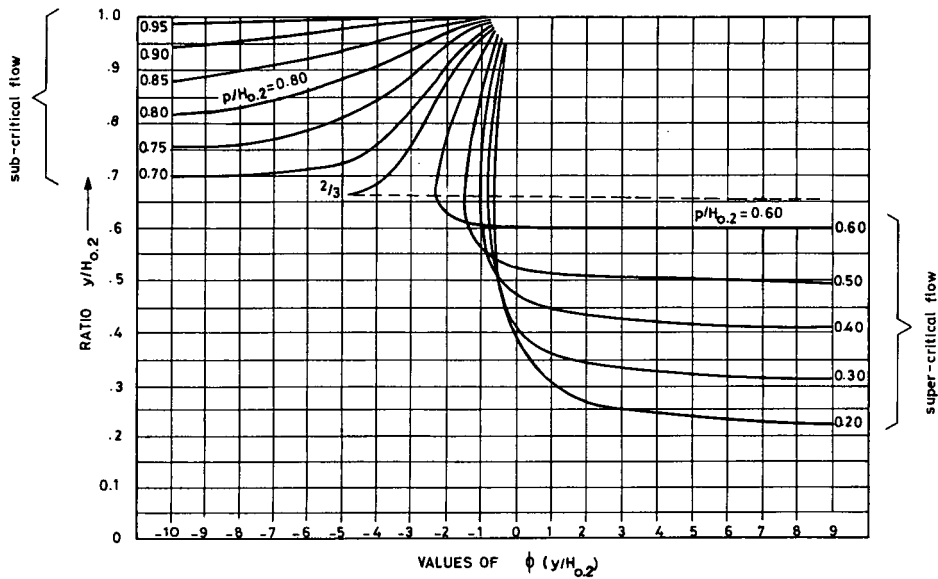


Figure A3.3 Values of $\phi(y/H_{0,2})$ for use in Equation A3.8

$$v_u A_u - v_d A_d = C_s \frac{2}{3} \sqrt{\frac{2}{3}} g (y_d - p_1)^{1.5} \Delta x \quad (\text{A3.11})$$

where; Δx = length of the considered channel section, u = subscript denoting upstream end of section, d = subscript denoting downstream end of section, C = coefficient of Chézy, R = hydraulic radius of channel.

It should be noted that before one can use Equations A3.10 and A3.11 sufficient information must be available on both A and R along the weir. The accuracy of the water surface elevation computation will depend on the length and the chosen number of elementary reaches Δx .

3.2.3 Practical C_s -value

The reader will have noted that in Equations A3.3 to A3.9 an effective discharge coefficient C_s is used. For practical purposes, a value

$$C_s = 0.95 C_d \quad (\text{A3.12})$$

may be used, where C_d equals the discharge coefficient of a standard weir of similar crest shape to those described in Chapters 4 and 6.

If Equations A3.4 to A3.11 are used for a sharp-crested side weir, the reader should be aware of a difference of $\sqrt{3}$ in the numerical constant between the head-discharge equations of broad-crested and sharp-crested weirs with rectangular control section. In addition it is proposed that the discharge coefficient (C_s) of a sharp-crested weir be reduced by about 10% if it is used as a side weir. This leads to the following C_s -value

to be used in the equations for sharp-crested side weirs

$$C_s \doteq 0.90 \sqrt{3} C_e \simeq 1.55 C_e \quad (\text{A3.13})$$

3.2.4 Practical evaluation of side weir capacity

Various authors proposed simplified equations describing the behaviour of sharp-crested side weirs along rectangular channels. However, discrepancies exist between the experimental results and the equations proposed, and it follows that each equation has only a restricted validity. In this Annex we shall only give the equations as proposed by Forchheimer (1930), which give an approximate solution to the Equations A3.3 and A3.4 assuming that the water surface profile along the side weir is a straight line. The Forchheimer equations read

$$\Delta Q = C_s \frac{2}{3} \sqrt{\frac{2}{3}} g S \left[\frac{y_1 + y_2}{2} - p_1 \right]^{1.5} \quad (\text{A3.14})$$

and

$$\xi_1 = \frac{y_2 - y_1}{v_1^2/2g - v_2^2/2g - \Delta H_o} \quad (\text{A3.15})$$

where ΔH_o is the loss of specific energy head along the side weir due to friction. ΔH_o can be estimated from

$$\Delta H_o = 2 \left(\frac{v_1 - v_2}{2g} \right)^2 \frac{S}{C^2 R} - i S \quad (\text{A3.16})$$

The most common problem is how to calculate the side weir length S , if $\Delta Q = Q_1 - Q_2$, y_2 and p_1 are known. To find S an initial value of y_1 has to be estimated, which is then substituted into the Equations A3.14 and A3.15. By trial and error y_1 (and thus S) should be determined in such a way that $\xi_1 = 1.0$.

The Equations A3.14 and A3.15 are applicable if

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} < 0.75 \quad (\text{A3.17})$$

and

$$y_1 - p \geq 0 \quad (\text{A3.18})$$

If the above limits do not apply, the water depth y_1 at the entrance of the side weir and the side weir length S required to discharge a flow $Q_1 - Q_2$ should be calculated by the use of Equation A3.1, which reads

$$H_{o,2} = y_1 + \frac{Q_1^2}{2gA_1^2} = y_2 + \frac{Q_2^2}{2gA_2^2} \quad (\text{A3.19})$$

In combination with the equation

$$-S = x_1 - x_2 = 2.73 \frac{B}{C_d} [\phi(y_1/H_{o,2}) - \phi(y_2/H_{o,2})] \quad (\text{A3.20})$$

The latter equation is a result of substituting Equation A3.12 into Equation A3.8. In using Equation A3.20 the reader should be aware that the term $x_1 - x_2$ is negative since $x_1 < x_2$. As mentioned before, values of $\phi(y/H_{o,2})$ can be read from Figure A3.3 as a function of the ratios $p_1/H_{o,2}$ and $y/H_{o,2}$.

3.3 Oblique weirs

3.3.1 Weirs in rectangular channels

According to Aichel (1953), the discharge q per unit width of crest across oblique weirs placed in a rectangular canal as shown in Figure A3.4 can be calculated by the equation

$$q = \left(1 - \frac{h_1}{p_1} \beta\right) q_n \quad (\text{A3.21})$$

where q_n is the discharge over a weir per unit width if the same type of weir had been placed perpendicular to the canal axis ($\epsilon = 90^\circ$) and β is a dimensionless empirical function of the angle of the weir crest (in degrees) with the canal axis.

Equation A3.21 is valid provided that the length of the weir crest L is small with respect to the weir width b and the upstream weir face is vertical. Values of the β coefficient are available (see Figure A3.5) for

$$h_1/p_1 < 0.62 \quad \text{and} \quad \epsilon > 30^\circ \quad (\text{A3.22})$$

or

$$h_1/p_1 < 0.46 \quad \text{and} \quad \epsilon < 30^\circ \quad (\text{A3.23})$$

3.3.2 Weirs in trapezoidal channels

Three weir types, which can be used to suppress water level variations upstream of the weir are shown in Figure A3.6. Provided that the upstream head over the weir crest does not exceed 0.20 m ($h_1 < 0.20$ m) the unit weir discharge can be estimated by the equation

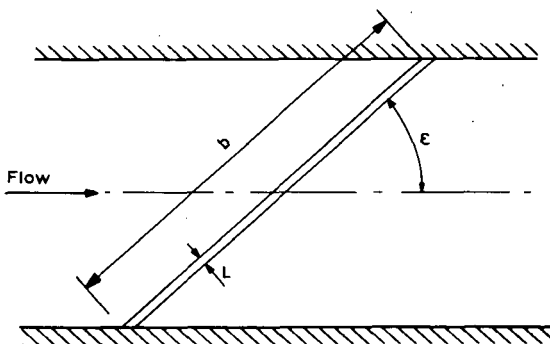


Figure A3.4 Oblique weir in channel having rectangular cross section

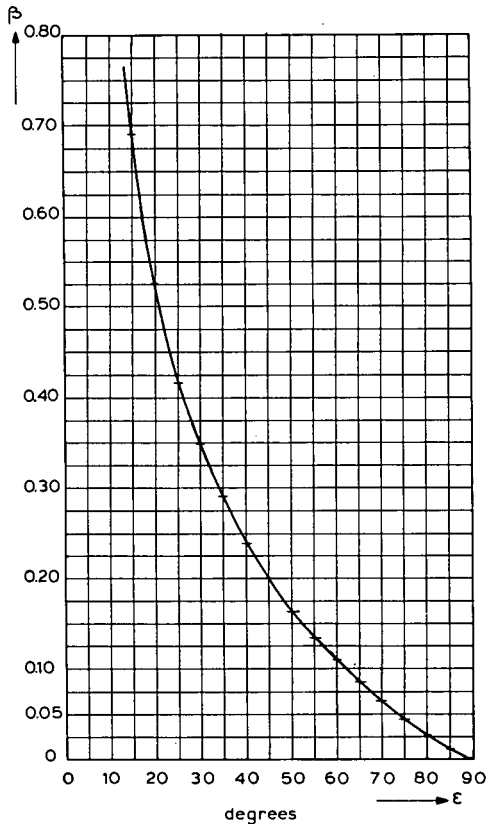


Figure A3.5 β -values as a function of ϵ

$$q = r q_n \quad (\text{A3.24})$$

where q_n is the discharge across a weir per unit width if the weir had been placed perpendicular to the canal axis (see Chapters 4 and 6) and r is a reduction factor as shown in Figure A3.6.

3.4 Selected list of references

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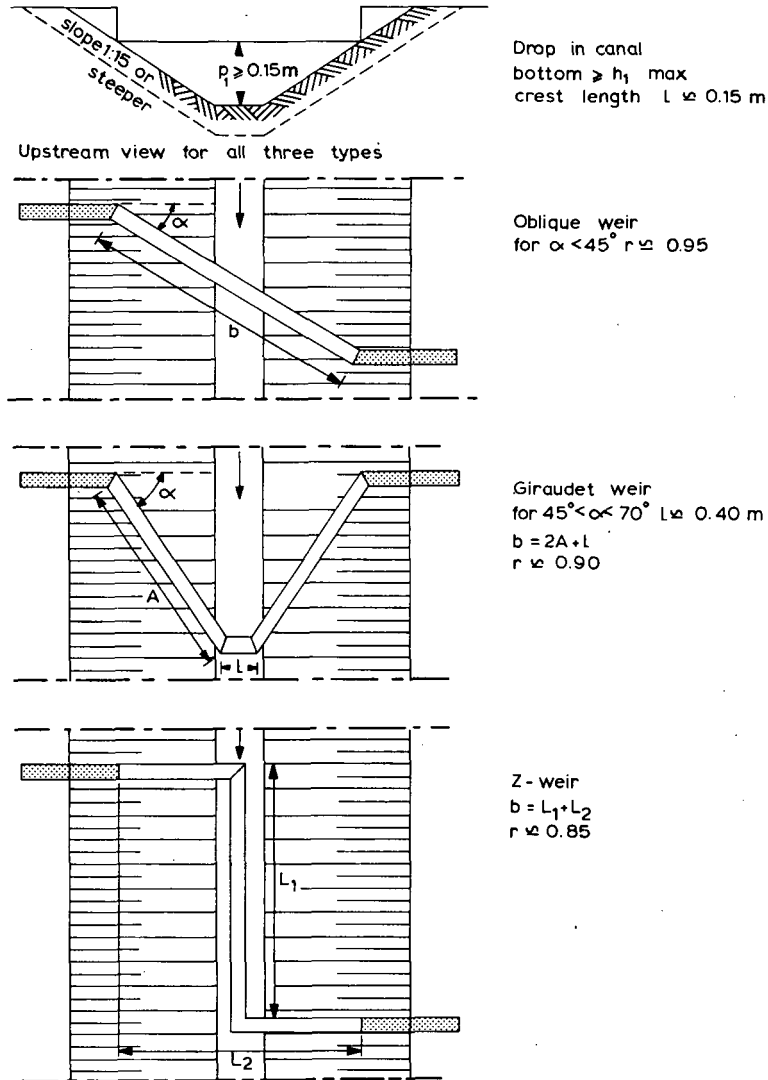


Figure A3.6 Weirs in trapezoidal channels