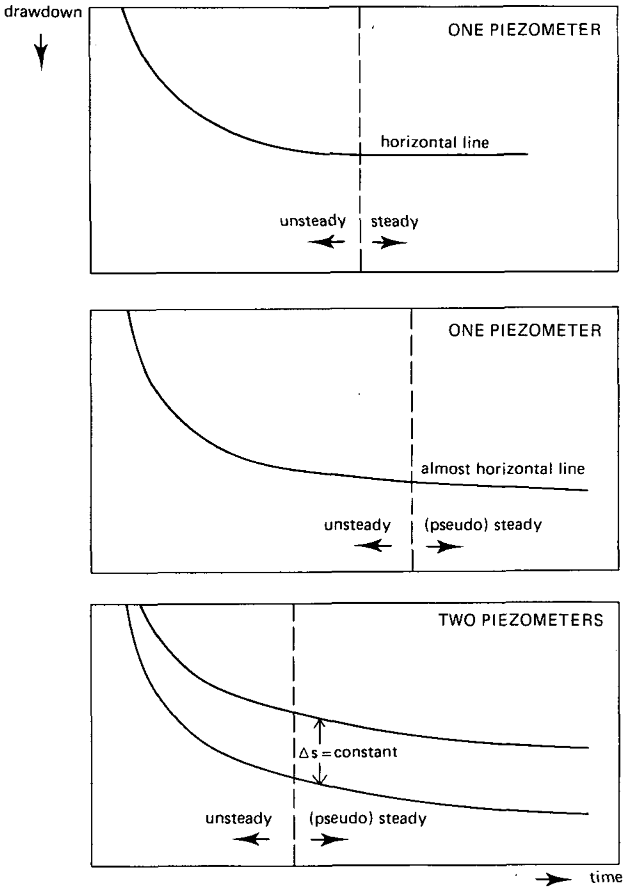

Figure 10.5 Time-drawdown plots showing the changes in drawdown during an aquifer test and their interpretations

To establish whether unsteady or (pseudo) steady-state conditions prevail, the changes in head during the pumping test should be plotted. Figure 10.5 shows the different plots and their interpretations.

Under average conditions, steady-state flow is generally reached in semi-confined aquifers after 15 to 20 hours of pumping. In unconfined aquifers, pseudo steady-state conditions may take several days. Preliminary plotting of data during the test will often show what is happening and may indicate whether or not the test should be continued.

10.4 Methods of Analysis

As already stated, the principle of a single-well or aquifer test is that a well is pumped and the effect of this pumping on the aquifer's hydraulic head is measured, in the well itself and/or in a number of observation wells in the vicinity. The change in water level induced by the pumping is known as the drawdown. In literature, tests based on the analysis of drawdowns during pumping are commonly referred to as 'pumping tests'.

The hydraulic characteristics can also be found from a recovery test. In such a test, a well that has been discharging for some time is shut down, after which the recovery of the aquifer's hydraulic head is measured, in the well and/or in the observation wells. Figure 10.6A gives the time-drawdown relationships during a pumping test, followed by a recovery test.

Analyses based on time-drawdown and time-recovery relationships can be applied to both single-well tests and aquifer tests. With aquifer tests, it is possible to make these analyses for each well separately and then to compare their results. Aquifer tests that provide drawdown data from two or more observation wells also make it possible to include the distance-drawdown relationship in the analysis (Figure 10.6B).

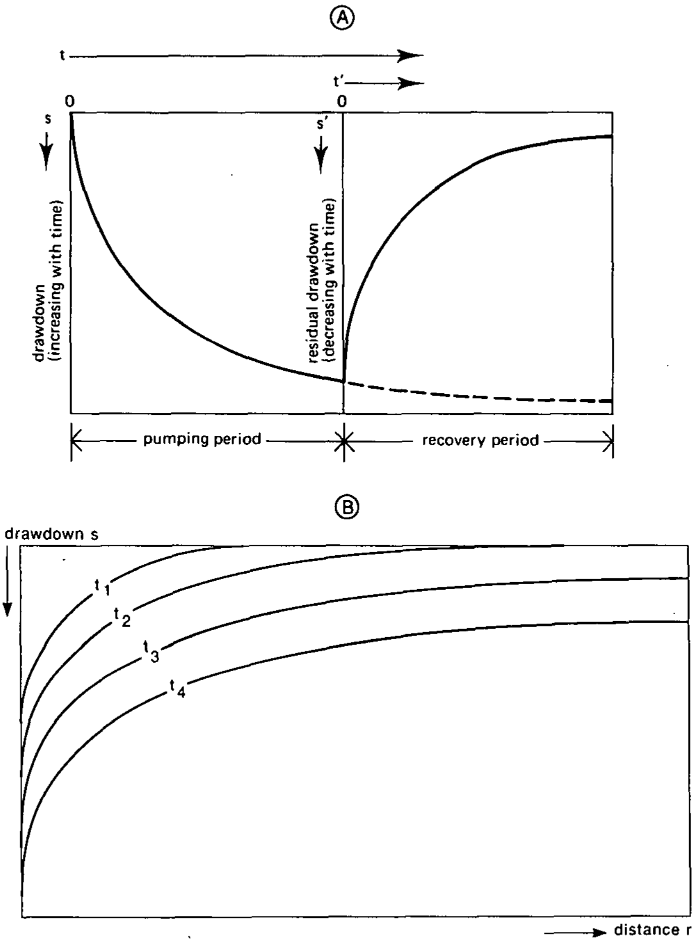

Figure 10.6 A: Time-drawdown relationship during a pumping test, followed by a recovery test; B: Distance-drawdown relationship during a pumping test

Altogether, the results of aquifer tests are more accurate than the results of single-well tests, and, moreover, are representative of a larger volume of the aquifer.

The methods presented in this section can be used to analyze single-well and aquifer tests conducted in unconfined and semi-confined aquifers. Although the methods used for unconfined aquifers were initially developed for confined aquifers, the latter will not be discussed, since they are not relevant for subsurface drainage.

All the methods presented were developed under the following common assumptions and conditions:

- The aquifer has a seemingly infinite areal extent;
- The aquifer is homogeneous, isotropic, and of uniform thickness over the area influenced by the test;
- Prior to pumping, the hydraulic head is horizontal (or nearly so) over the area that will be influenced by the test;
- The pumped well penetrates the entire thickness of the aquifer and thus receives water by horizontal flow;
- The aquifer is pumped at a constant-discharge rate;
- The water removed from storage is discharged instantaneously with decline of head (see Section 10.5.1);
- The diameter of the pumped well is small (i.e. the storage inside the well can be neglected).

Additional assumptions and limiting conditions are mentioned where the individual methods are discussed separately.

10.4.1 Time-Drawdown Analysis of Unconfined Aquifers

Theis (1935) was the first to develop an equation for unsteady-state flow which introduced the time factor and the storativity. He noted that when a fully-penetrating well pumps an extensive confined aquifer at a constant rate, the influence of the discharge extends outward with time. The rate of decline of head, multiplied by the storativity and summed over the area of influence, equals the discharge.

Jacob (1950) showed that if the drawdowns in an unconfined aquifer are small compared with the initial saturated thickness of the aquifer, the condition of horizontal flow towards the well is approximately satisfied, so that the Theis equation, which was originally developed for confined aquifers, can be applied to unconfined aquifers as well.

For unconfined aquifers, the Theis equation, which was derived from the analogy between the flow of groundwater and the conduction of heat, is written as

$$s = \frac{Q}{4\pi KH} \int_u^\infty \frac{1}{y} \exp(-y)\, dy = \frac{Q}{4\pi KH} W(u) \qquad (10.1)$$

and

$$u = \frac{r^2 \mu}{4KHt} \qquad (10.2)$$

where

s	=	drawdown measured in a well (m)
Q	=	constant well discharge (m^3/d)
KH	=	transmissivity of the aquifer (m^2/d)
r	=	distance from the pumped well (m)
μ	=	specific yield of the aquifer (–)
t	=	time since pumping started (d)
u	=	help parameter (–)
W(u)	=	Theis well function (–)

Values of the Theis well function can be found in Appendix 10.1.

In Figure 10.7, the Theis well function W(u) is plotted versus 1/u on semi-log paper. It can be seen in this figure that, for large values of 1/u, the Theis well function exhibits a straight-line segment. The Jacob method is based on this phenomenon. Cooper and Jacob (1946) showed that, for the straight-line segment, Equation 10.1 can be approximated by

$$s = \frac{2.30Q}{4\pi KH} \log \frac{2.25KHt}{r^2\mu} \tag{10.3}$$

with an error of less than 10, 5, 2, and 1 per cent for 1/u values larger than 7, 10, 20, and 30, respectively. For all practical purposes, Equation 10.3 can thus be used for 1/u values larger than 10.

If the time of pumping is long enough, a plot of the drawdown s observed at a particular distance r from the pumped well versus the logarithm of time t will show a straight line. If the slope of this straight-line segment is expressed as the drawdown difference ($\Delta s = s_1 - s_2$) per log cycle of time ($\log t_2/t_1 = 1$), rearranging Equation 10.3 gives

$$KH = \frac{2.3Q}{4\pi\Delta s} \tag{10.4}$$

If the straight line is extended until it intercepts the time-axis where s = 0, the interception point has the coordinates s = 0 and t = t_0. Substituting these values

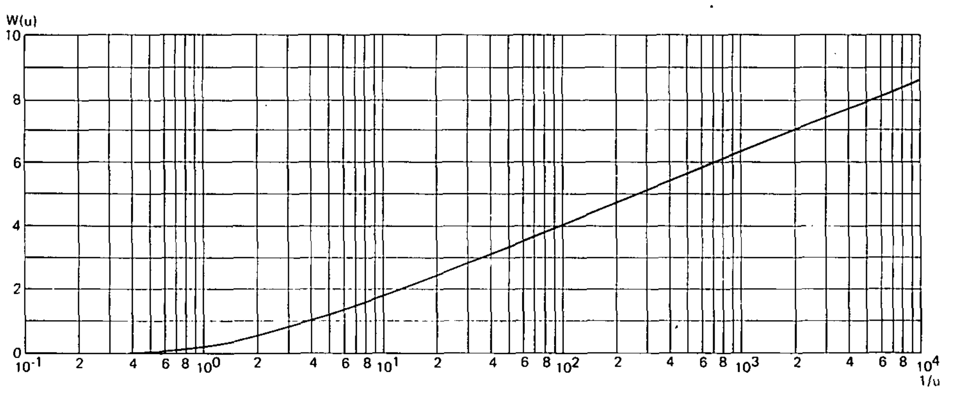

Figure 10.7 Theis well function W(u) versus 1/u for fully-penetrating wells in unconfined aquifers

into Equation 10.3 gives $\log [2.25KHt_0/r^2\mu] = 0$ or $[2.25KHt_0/r^2\mu] = 1$ or

$$\mu = \frac{2.25KHt_o}{r^2} \tag{10.5}$$

Jacob's straight-line method is based on the assumptions listed in Section 10.4 and on the limiting condition that the pumping time is sufficiently long for a straight-line segment to be distinguished in a time-drawdown plot on semi-log paper.

Procedure 1

- For one of the observation wells, plot the drawdown values s versus the corresponding time t on semi-log paper (t on logarithmic scale);
- Select a time range and draw a best-fitting straight line through that part of the plotted points;
- Determine the slope of the straight line (i.e. the drawdown difference Δs per log cycle of time);
- Substitute the values of Q and Δs into Equation 10.4 and solve for KH;
- Extend the straight line until it intercepts the time-axis where $s = 0$, and read the value of t_0;
- Substitute the values of KH, t_0, and r into Equation 10.5 and solve for μ;
- Substitute the values of KH, μ, and r into Equation 10.2, together with $1/u = 10$, and solve for t. This t value should be less than the time range for which the straight-line segment was selected (see Example 10.1);
- If drawdown values are available for more than one well, apply the above procedure to the other wells also.

Remark 1

When the drawdowns in an unconfined aquifer are large compared with the aquifer's original saturated thickness, the analysis should be based on corrected drawdown data. Jacob (1944) proposed the following correction

$$s_c = s - \frac{s^2}{2H} \tag{10.6}$$

where

s_c = corrected drawdown (m)
s = observed drawdown (m)
H = original saturated thickness of the aquifer (m)

Remark 2

With single-well tests, basically the same procedure can be applied. The r value now represents the effective radius of the single well. This is difficult to determine under field conditions; as a 'best' estimate, the outer radius of the well screen is often used.

A complicating factor is the phenomenon that due to non-linear well losses the water levels inside the well can be considerably lower than those directly outside the well screen.

This implies that drawdown data from the pumped well can, in general, only be used for the analysis when corrected for these non-linear well losses using the results of so-called step-drawdown tests. For information on this subject, see Kruseman and De Ridder (1990).

With the above procedure, however, we can use the uncorrected drawdown data and still determine accurate transmissivity values, because the slope of the straight-line segment in the time-drawdown plot on semi-log paper is not affected by this phenomenon (the non-linear well loss is constant with time). Specific-yield values, even when based on the corrected data, should be treated with caution, because they are highly sensitive to the value of the effective radius of the pumped well.

Example 10.1

A single-well test was made in an unconfined aquifer. The well was pumped at a constant rate of 3853 m^3/d for 10 hours. The outer radius of the well screen was 0.20 m. Table 10.2 shows the observed drawdowns as a function of time.

Calculate KH and μ, using Jacob's straight-line method.

The first step is to determine whether the observed drawdown values are small compared with the aquifer's thickness (see Remark 1). The depth of the pumped borehole was 271 m. Substituting this value and the last observed drawdown value into Equation 10.6 gives a maximum correction value of 0.05 m. It can thus be concluded that uncorrected drawdown values can be used in the analysis.

Figure 10.8 shows the time-drawdown plot on semi-logarithmic paper. The slope of the straight line shows that Δs is 0.50 m. Substituting the appropriate values into Equation 10.4 gives

$$KH = \frac{2.3Q}{4\pi\Delta s} = \frac{2.3 \times 3853}{4 \times 3.14 \times 0.50} = 1410 \, m^2/d$$

It can be seen from Figure 10.8 that the intersection of the straight line with the ordinate where $s = 0$ cannot be determined directly. When such a situation occurs, the following procedure can be followed:

- Within the range of plotted drawdowns, determine a drawdown value which is a multiple of the Δs-value;
- Determine the corresponding t value using the straight line;
- The t_0 value is then equal to $t \times 10^{-x}$, where x denotes the multiple of the first step.

Table 10.2 Time-drawdown values of a single-well test

Time (min)	Drawdown (m)	Time (min)	Drawdown (m)	Time (min)	Drawdown (m)
15	4.161	50	4.486	240	4.808
20	4.283	56	4.471	300	4.776
25	4.257	60	4.474	360	4.885
30	4.357	80	4.534	420	4.960
36	4.358	105	4.618	480	4.906
40	4.399	120	4.672	540	4.972
46	4.456	180	4.748	600	5.016

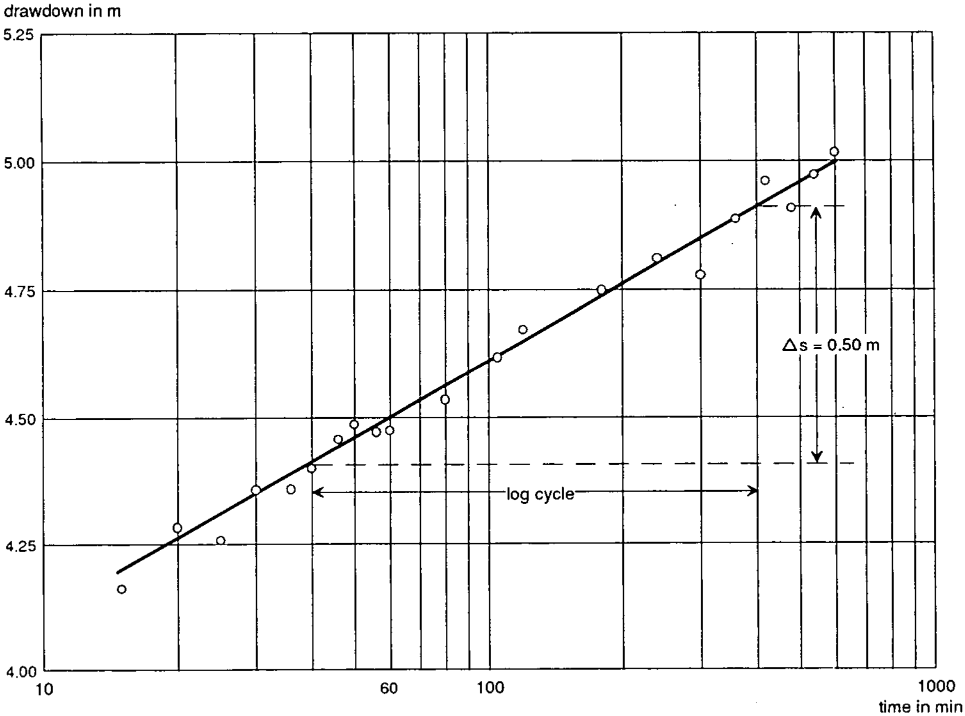

Figure 10.8 Time-drawdown plot of field data of a single-well test in an unconfined aquifer

The Δs value was calculated as 0.5 m. Figure 10.8 shows that for a drawdown of 4.5 m – being 9 times the Δs value – the corresponding time is 60 minutes. The t_0 value is then equal to 60×10^{-9} min $= 4.2 \times 10^{-11}$ d. Substituting the appropriate values into Equation 10.5 yields

$$\mu = \frac{2.25KHt_o}{r^2} = \frac{2.25 \times 1410 \times 4.2 \times 10^{-11}}{0.20^2} = 3.3 \times 10^{-6}$$

It is obvious that the above specific-yield value is not correct. Specific-yield values range from 5×10^{-3} to 5×10^{-1}.

At the same site, a step-drawdown test was also made. From the analysis of this test, the non-linear well loss was calculated to be 1.93 m. When the drawdown data of Table 10.2 are corrected with this value (see Remark 2), the intersection point where $s = 0$ then has the time value $t_0 = 4.4 \times 10^{-4}$ min $= 3.1 \times 10^{-7}$ d. Substituting the appropriate values into Equation 10.5 yields

$$\mu = \frac{2.25KHt_0}{r^2} = \frac{2.25 \times 1410 \times 3.1 \times 10^{-7}}{0.20^2} = 0.025$$

This specific-yield value looks better. It seems a reasonable estimate for the specific yield of this aquifer.

Finally, the condition that the 1/u value is larger than 10 should be verified. Substituting this condition into Equation 10.2 gives

$$t > \frac{10\, r^2 \mu}{4\, KH} \rightarrow t > \frac{2.5 \times 0.2^2 \times 0.025}{1410} \rightarrow t > 1.8 \times 10^{-6}\, d \text{ or } t > 2.6 \times 10^{-3}\, \text{min}$$

So, theoretically, all the observed drawdown values with t values larger than 2.6×10^{-3} minutes, can be expected to lie on a straight line. In other words, all the observed drawdown data as plotted in Figure 10.8 can be used to determine the slope of the straight line.

It should be noted that the condition $1/u > 10$ is usually fulfilled in single-well tests because of the small r value.

10.4.2 Time-Drawdown Analysis of Semi-Confined Aquifers

When a semi-confined aquifer is pumped (Figure 10.9), the piezometric level of the aquifer in the well is lowered. This lowering spreads radially outward as pumping continues, creating a difference in hydraulic head between the aquifer and the aquitard. Consequently, the groundwater in the aquitard will start moving vertically downward to join the water in the aquifer. The aquifer is thus recharged by downward percolation from the aquitard. As pumping continues, the percentage of the total discharge derived from this percolation increases. After a certain period of pumping, equilibrium will be established between the discharge rate of the pump and the recharge rate by vertical flow through the aquitard. This steady state will be maintained as long as the watertable in the aquitard is kept constant.

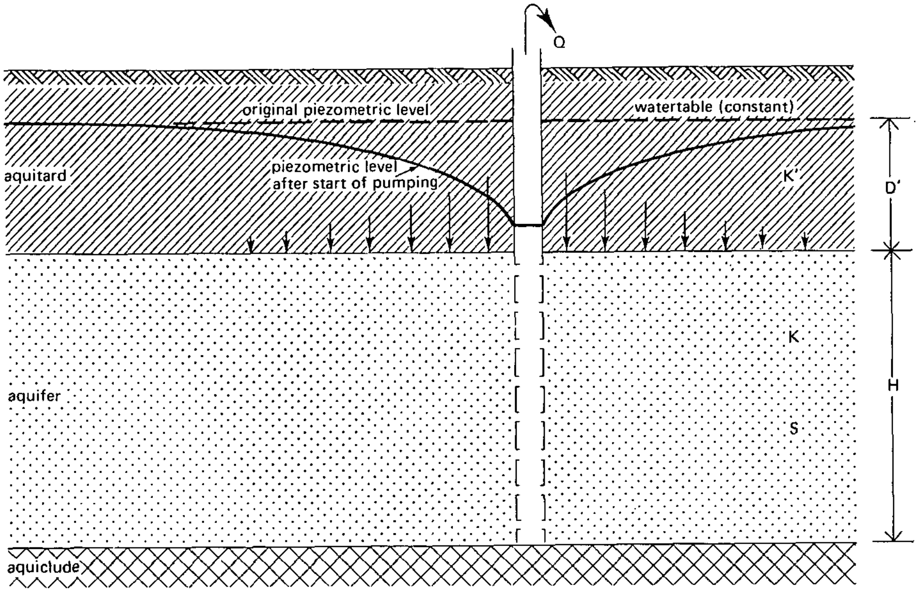

Figure 10.9 Cross-section of a pumped semi-confined aquifer

Attention is drawn to the assumption that the leakage from the aquitard is proportional to the drawdown of the piezometric level of the aquifer. A consequence of this assumption is that the watertable in the aquitard should be constant or, in practice, that the drawdown of the watertable is less than five per cent of the thickness of the saturated part of the aquitard. When pumping tests of long duration are performed, this assumption is generally not satisfied unless the aquitard is replenished by an outside source, say from surface water distributed over the aquitard via a system of narrowly-spaced ditches.

According to Hantush and Jacob (1955), the drawdown due to pumping a semi-confined aquifer can be described by the following equation

$$s = \frac{Q}{4\pi KH} \int_u^\infty \frac{1}{y} \exp\left(-y - \frac{r^2}{4L^2y}\right) dy = \frac{Q}{4\pi KH} W(u,r/L) \qquad (10.7)$$

and

$$u = \frac{r^2S}{4KHt} \qquad (10.8)$$

where

$L = \sqrt{KHc}$ = leakage factor (m)
$c = D'/K'$ = hydraulic resistance of the aquitard (d); D' and K' are indicated in Figure 10.9
$W(u,r/L)$ = Hantush well function (Appendix 10.2)
S = storativity of the aquifer (–)

Equation 10.7 has the same form as the Theis equation (Equation 10.1), but there are two parameters in the integral: u and r/L. Equation 10.7 approaches the Theis equation for large values of L, when the exponential term $r^2/4L^2y$ approaches zero.

In Figure 10.10, the Hantush well function W(u,r/L) is plotted versus 1/u on semi-log paper for an arbitrary value of r/L. This figure shows that the Hantush well function exhibits an S shape and, for large values of 1/u, a horizontal straight-line segment indicating steady-state flow. It is on these phenomena that the inflection-point method

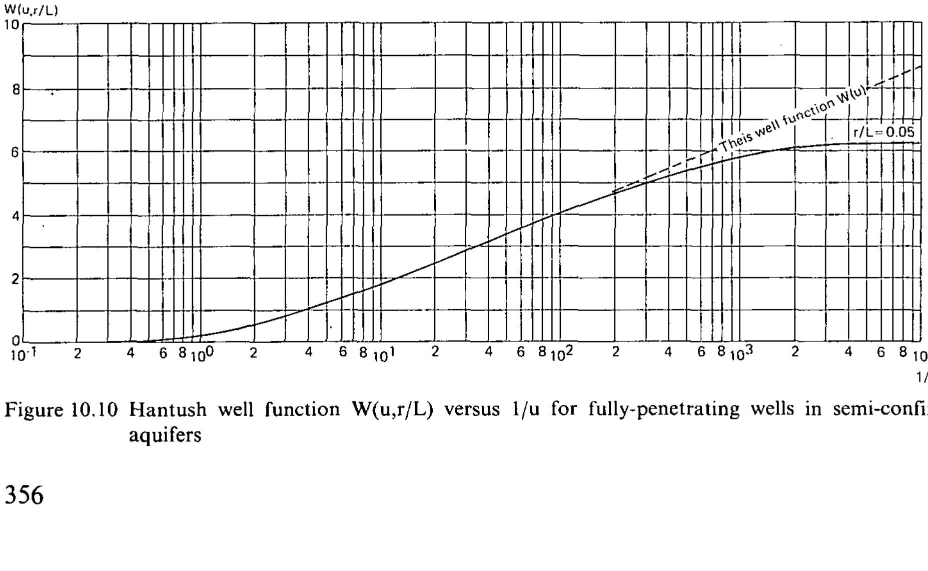

Figure 10.10 Hantush well function W(u,r/L) versus 1/u for fully-penetrating wells in semi-confined aquifers

was based. Hantush (1956) showed that for the inflection point the following relationships hold:

a)

$$s_p = 0.5\, s_m = \frac{Q}{4\pi KH} K_0\left(\frac{r}{L}\right) \qquad (10.9)$$

where K_0 (r/L) is the modified Bessel function of the second kind and zero order and s_m is the steady-state drawdown;

b)

$$u_p = \frac{r^2S}{4KHt_p} = \frac{r}{2L} \qquad (10.10)$$

c)
At the inflection point the slope of the curve, Δs_p, is given by

$$\Delta s_p = \frac{2.3Q}{4\pi KH} e^{-r/L} \qquad (10.11)$$

d)
At the inflection point, the relation between the drawdown and the slope of the curve is given by

$$2.3 \frac{s_p}{\Delta s_p} = e^{r/L} K_0 (r/L) \qquad (10.12)$$

In Equations 10.9 to 10.12, the index p means 'at the inflection point'. Further, Δs stands for the slope of a straight line, taking the time interval as a log cycle.

The Hantush inflection-point method is based on the assumptions listed in Section 10.4 and on the following limiting conditions:
- The watertable in the aquitard remains constant so that leakage through the aquitard takes place in proportion to the drawdown of the piezometric level;
- The pumping time is sufficiently long so that the steady-state drawdown can be estimated from extrapolation of a time-drawdown curve plotted on semi-log paper.

Procedure 2
- For one of the wells, plot the drawdown values s versus the corresponding time t on semi-log paper (t on logarithmic scale) and draw a curve that fits best through the plotted points;
- Determine from this plot the value of the steady-state drawdown s_m;
- Substitute the value of s_m into Equation 10.9 and solve for s_p. The value of s_p on the curve locates the inflection point p;
- Read the value of t_p at the inflection point from the time-axis;
- Determine the slope Δs_p of the curve at the inflection point. This can be closely approximated by reading the drawdown difference per log cycle of time over the straight portion of the curve on which the inflection point lies (= the tangent to

the curve at the inflection point);

- Substitute the values of s_p and Δs_p into Equation 10.12 and solve for $e^{r/L} K_0(r/L)$. By interpolation between values of this product, which can be found numerically, the value of r/L is found (see Appendix 10.3);
- Calculate the leakage factor L from the r/L value determined in the previous step, and the r value of the well;
- Substitute Q, Δs_p, and r/L into Equation 10.11 and solve for KH;
- Substitute r, KH, t_p, and L into Equation 10.10 and solve for S;
- Calculate the hydraulic resistance c from the relation $c = L^2/KH$;
- If drawdown values are available for more than one well, apply the above procedure to the other wells also.

Remark 3

The accuracy of the calculated hydraulic characteristics depends on the accuracy of the value of the (extrapolated) steady-state drawdown s_m.

The calculations should therefore be checked by substituting the different values into Equations 10.7 and 10.8. Calculations of s should be made for the observed values of t. If the values of t are not too small, the values of s should fall on the observed data curve. If the calculated data deviate from the observed data, the (extrapolated) value of s_m should be adjusted. Sometimes, the observed data curve can be drawn somewhat steeper or flatter through the plotted points, and so Δs_p can be adjusted too. With the new values of s_m and/or Δs_p, the calculation is repeated.

With the computer program SATEM (Boonstra 1989), the above time-consuming work can be avoided. This program follows the same procedure as described above. In addition, it displays the drawdowns calculated with Equations 10.7 and 10.8 on the monitor, together with the data observed in the field. This makes it easy to check whether the correct (extrapolated) steady-state drawdown s_m has been selected.

Remark 4

With single-well tests, basically the same procedure can be applied. The r value again represents the effective radius of the pumped well. Because of non-linear well losses, the water levels inside the well can be considerably lower than those directly outside the well screen. This implies that if we follow the above procedure, we can only determine accurate transmissivity values by using the uncorrected drawdown data, as was also mentioned in Section 10.4.1.

Example 10.2

An aquifer test was made in a semi-confined aquifer. The well was pumped at a constant rate of 545 m^3/d for 48 hours. One of the observation wells was located 20 m away from the pumped well. Table 10.3 shows the observed drawdowns as a function of time.

Using the Hantush inflection-point method, calculate KH, S, and c.

Figure 10.11 shows the time-drawdown plot on semi-logarithmic paper. As can be seen from this figure, steady-state conditions occurred at the end of the pumping test (s_m = 2.44 m). According to Equation 10.9, the s_p value is then 1.22 m. The figure

Table 10.3 Time-drawdown values of an aquifer test

Time (min)	Drawdown (m)	Time (min)	Drawdown (m)	Time (min)	Drawdown (m)
1	0.265	60	1.615	720	2.320
2	0.347	70	1.675	840	2.337
3	0.490	80	1.725	960	2.355
4	0.548	90	1.767	1080	2.377
5	0.635	120	1.895	1200	2.373
6	0.707	150	1.977	1320	2.385
7	0.793	180	2.020	1440	2.400
8	0.839	210	2.075	1620	2.410
9	0.882	240	2.113	1800	2.420
10	0.930	300	2.170	1980	2.440
15	1.079	360	2.210	2160	2.440
20	1.187	420	2.245	2340	2.450
25	1.275	480	2.270	2520	2.450
30	1.345	540	2.281	2700	2.440
40	1.457	600	2.298	2880	2.440
50	1.554	660	2.310		

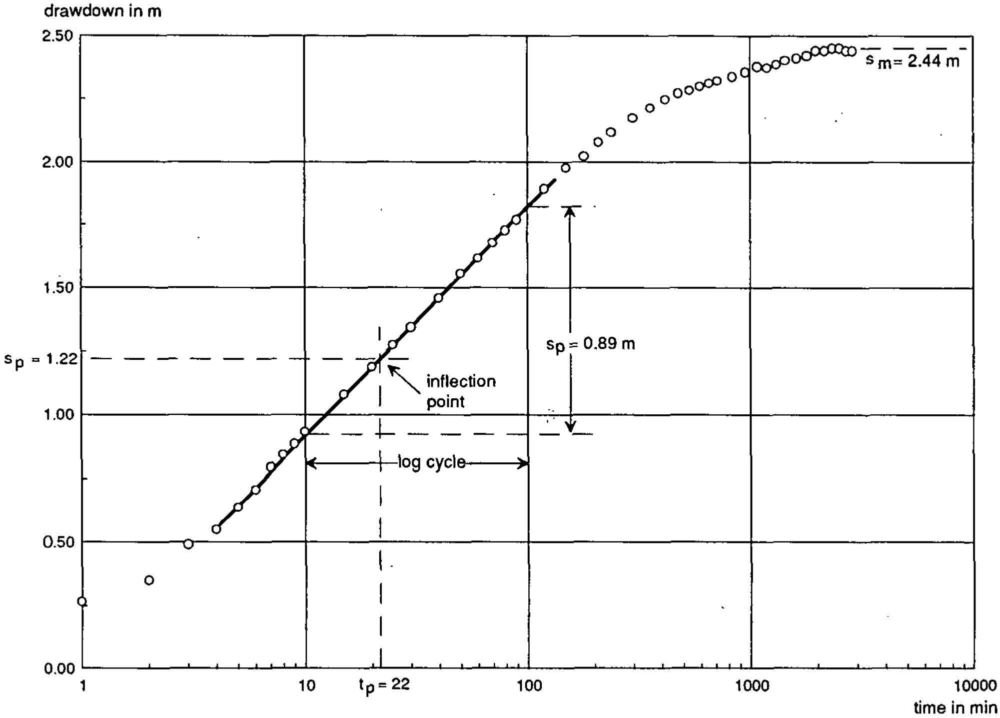

Figure 10.11 Time-drawdown plot of field data of an aquifer test in a semi-confined aquifer

shows that the t value of the inflection point is equal to 22 min = 1.5×10^{-2} d.

The slope of the straight line in Figure 10.11 shows that Δs_p is 0.89 m. Substituting the appropriate values into Equation 10.12 yields

$$e^{r/L} K_0 (r/L) = 2.3 \frac{s_p}{\Delta s_p} = 2.3 \frac{1.22}{0.89} = 3.15$$

According to Appendix 10.3, the r/L value is 0.0575. Substituting the r value yields

$$L = \frac{20}{0.0575} = 348 \text{ m}$$

Substituting the appropriate values into Equation 10.11 yields

$$KH = \frac{2.3\, Q}{4\pi \Delta s_p} e^{-r/L} = \frac{2.3 \times 545}{4 \times 3.14 \times 0.89} e^{-0.0575} = 106 \text{ m}^2/\text{d}$$

Substituting the appropriate values into Equation 10.10 yields

$$S = \frac{4KHt_p}{r^2} \frac{r}{2L} = \frac{4 \times 106 \times 1.5 \times 10^{-2}}{20^2} \times \frac{20}{2 \times 348} = 4.6 \times 10^{-4}$$

The c value is then

$$c = \frac{L^2}{KH} = \frac{348^2}{106} = 1142 \text{ d}$$

We checked the results of the above analysis by calculating theoretical drawdown values with Equations 10.7 and 10.8, and Appendix 10.2. These values were almost identical to the observed drawdown values, so we can regard the results of the analysis as being correct.

10.4.3 Time-Recovery Analysis

After a well has been pumped for a certain period of time, t_{pump}, and is shut down, the water level in the pumped well and in the piezometers – if any – will stop falling and will start to rise again to its original position (Figure 10.12). This recovery of the water level can be measured as residual drawdown s′ (i.e. as the difference between the original water level prior to pumping and the actual water level measured at a certain moment t′ since pumping stopped).

This residual drawdown s′ at time t′ is also equal to the difference between the drawdown that results from pumping the well at a discharge Q for the hypothetical time $t_{pump} + t'$ and the recovery that results from injecting water at the same point at the same rate Q through a hypothetical injection well for time t′

$$s'(t') = s(t_{pump} + t') - s(t') \tag{10.13}$$

On the basis of this principle, the recovery values s(t′) can be calculated if the hypothetical drawdown values $s(t_{pump} + t')$ can be estimated. This can be done if the drawdown data during pumping were analyzed with one of the methods given in the previous sections. By back-substituting the hydraulic characteristics that were

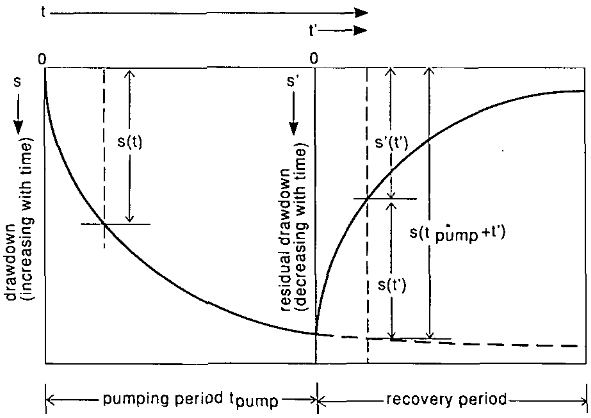

Figure 10.12 Time-drawdown behaviour during pumping tests and recovery tests

obtained into the appropriate equations, we can calculate hypothetical values of $s(t_{pump} + t')$. When, from these, we subtract the observed residual-drawdown data $s'(t')$, we obtain synthetic recovery values $s(t')$.

An analysis of recovery data is identical to that of drawdown data, so any of the methods discussed in the previous sections can be used. This time-consuming work can be avoided with the computer program SATEM (Boonstra 1989).

Unconfined Aquifers

Instead of using synthetic recovery data, we can also use the residual-drawdown data directly in the analysis.

On the basis of Equation 10.13, Theis developed his recovery method for confined aquifers. For unconfined aquifers, after a constant-rate pumping test, the residual drawdown s′ during the recovery period is given by

$$s' = \frac{Q}{4\pi KH}\{W(u) - W(u')\} \tag{10.14}$$

and

$$u = \frac{r^2\mu}{4KHt} \quad \text{or} \quad u' = \frac{r^2\mu'}{4KHt'} \tag{10.15}$$

where

s' = residual drawdown (m)
μ' = specific yield during recovery (–)
t = $t_{pump} + t'$ = time since pumping started (d)
t' = time since pumping stopped (d)

In Figure 10.13, the expression W(u)-W(u′) is plotted versus u′/u on semi-log paper. This shows that, for small values of u′/u, the expression exhibits a straight-line segment. If we take $u' < 0.1$ – the same value as in Section 10.4.1 – it will result,

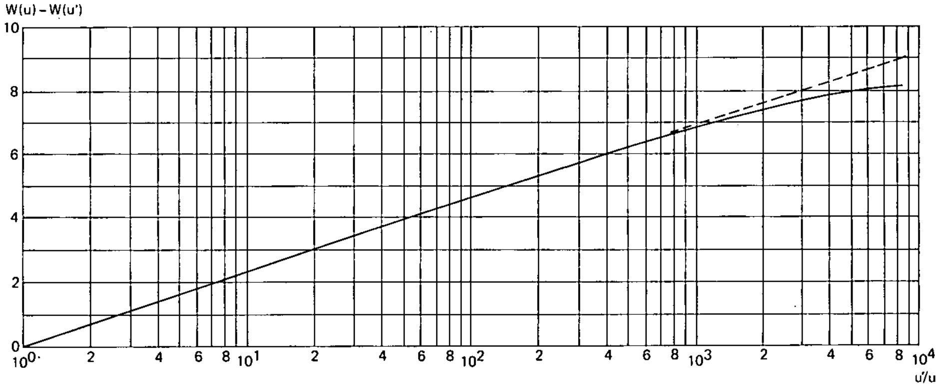

Figure 10.13 Theis recovery well function W(u) – W(u′) versus u′/u for fully-penetrating wells in unconfined aquifers

with $t/t' = u'\mu/u\mu'$ and $t = t_{pump} + t'$ in the following limiting condition

$$\frac{t}{t'} < 1 + \frac{KH\, t_{pump}}{2.5\, r^2 \mu'} \tag{10.16}$$

When u′ is sufficiently small ($u' < 0.1$) – the value of u is then also smaller than 0.1 – Equation 10.14 can be approximated by

$$s' = \frac{2.3Q}{4\pi KH} \log \frac{\mu' t}{\mu t'} \tag{10.17}$$

If we use residual-drawdown observations at a particular distance r from the pumped well and plot the residual drawdown s′ versus the logarithm of the time ratio t/t′, we obtain a straight line, provided that the time of pumping t_{pump} was long enough. If we express the slope of this straight-line segment as the drawdown difference ($\Delta s' = s_1' - s_2'$) per log cycle of the time ratio, rearranging Equation 10.17 gives

$$KH = \frac{2.3Q}{4\pi \Delta s'} \tag{10.18}$$

If we extend the straight line until it intercepts the time-axis where $s' = 0$, the interception point has the coordinates $s' = 0$ and $t/t' = (t/t')_0$. Substituting these values into Equation 10.17 gives $\log [\mu' t/\mu t'] = 0$ or $[\mu' t/\mu t'] = 1$ or

$$\mu' = \frac{\mu}{(t/t')_0} \tag{10.19}$$

The Theis recovery method is based on the assumptions listed in Section 10.4 and on the limiting condition that the pumping time is sufficiently long for a straight-line segment to be distinguished in a time-residual drawdown plot on semi-log paper.

Procedure 3

- For one of the wells, plot the residual-drawdown values s′ versus the corresponding time ratio t/t′ on semi-log paper (t/t′ on logarithmic scale);

- Select a time-ratio range and draw a best-fitting straight line through that part of the plotted points;
- Determine the slope of the straight line (i.e. the drawdown difference $\Delta s'$ per log cycle of time ratio t/t');
- Substitute the values of Q and $\Delta s'$ into Equation 10.18 and solve for KH;
- Extend the straight line until it intercepts the time-ratio axis where $s' = 0$, and read the value of $(t/t')_0$;
- Substitute this value and that of the specific yield obtained from analyzing the drawdown data into Equation 10.19 and solve for μ';
- Substitute the values of t_{pump}, KH, r, and μ' into Equation 10.16 and solve for t/t'. This t/t' value should be greater than the time-ratio range for which the straight-line segment was selected;
- If residual-drawdown values are available for more than one well, apply the above procedure to the other wells also.

Remark 5

When, for a semi-confined aquifer, the residual-drawdown data are plotted versus t/t' on semi-log paper, the plot will usually show an S curve like the one in Figure 10.10. If we analyze these data with the Theis recovery method, using the slope at the inflection point, we overestimate the transmissivity (compare Equations 10.4 and 10.11) and underestimate the specific yield μ', because the $(t/t')_0$ value is greater than one. The Theis recovery method can only be used for semi-confined aquifers when the r/L value is small. This is usually the case when the residual-drawdown data of the pumped well itself are being analyzed (i.e. in single-well tests).

Example 10.3

This example concerns the same single-well test as in Example 10.1. Table 10.4 shows the observed residual drawdowns as a function of time.

Calculate KH and μ, using the Theis recovery method.

Figure 10.14 shows the plot of residual drawdown versus time on semi-logarithmic

Table 10.4 Time-residual drawdown values of a single-well test

Time t' (min)	Residual drawdown s'(t') (m)	Time t' (min)	Residual drawdown s'(t') (m)	Time t' (min)	Residual drawdown s'(t') (m)
5	0.888	46	0.588	240	0.226
10	0.847	50	0.563	330	0.158
15	0.817	56	0.517	450	0.193
20	0.760	60	0.514	570	0.060
25	0.683	75	0.460	630	0.075
30	0.648	105	0.393	750	0.015
34	0.636	120	0.321	870	0.015
40	0.588	180	0.310	990	0.001

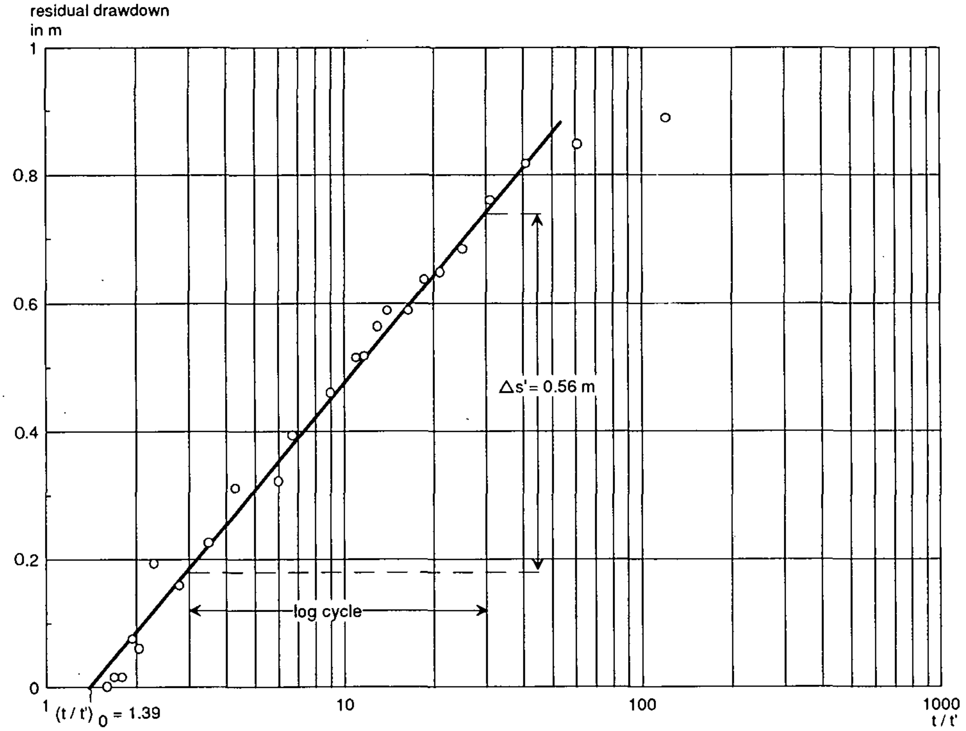

Figure 10.14 Time-residual drawdown plot of field data of a single-well test in an unconfined aquifer

paper. The slope of the straight line in this figure shows that $\Delta s'$ is 0.56 m. Substituting the appropriate values into Eq. 10.18 gives

$$KH = \frac{2.3Q}{4\pi\Delta s'} = \frac{2.3 \times 3853}{4 \times 3.14 \times 0.56} = 1260\ m^2/d$$

From Figure 10.14, we can see that the straight line intersects the axis where $s' = 0$, at $(t/t')_0 = 1.39$. Substituting the appropriate values into Equation 10.19 yields

$$\mu' = \frac{\mu}{1.39} = 0.7\,\mu$$

The above implies that the specific-yield value during recovery is less than during pumping. This phenomenon is often encountered because of air entrapment when the pores are again filled with water.

The above transmissivity value corresponds well with that found from the time-drawdown analysis (see Example 10.1).

The *application of time-drawdown and time-recovery* analyses thus enables us to check the calculated transmissivity value. When the two values are close to each other, it implies that the data are consistent (i.e. that the results of the test are reliable).

Remark 6

The residual drawdown data of Table 10.4 seem completely different from the drawdown data of Table 10.2. This difference can be explained by the non-linear well loss. Using the value of 1.93 m for the calculated non-linear well loss mentioned in Section 10.4.1, Example 10.1, the corrected drawdown at the end of the pumping period is

$$s(t_{pump}) = 5.016 - 1.93 = 3.09 \text{ m}$$

Assume, for the sake of simplicity, that, if pumping had been continued after 600 minutes, the drawdown at $t = 615$ min could be taken equal to that at $t = 600$ min. Then, according to Equation 10.13, the synthetic recovery value for $t' = 15$ min is

$$s(t') = s(t_{pump} + t') - s'(t') = 3.09 - 0.817 = 2.27 \text{ m}$$

The corrected drawdown after 15 minutes of pumping (see Table 10.2) is

$$s(t) = 4.161 - 1.93 = 2.23 \text{ m}$$

Theoretically, these two values should have been the same, as was discussed at the beginning of this section.

10.4.4 Distance-Drawdown Analysis of Unconfined Aquifers

When an unconfined aquifer is pumped, the cone of depression will continuously deepen and expand. Even at late pumping times, the water levels in the observation wells will never stabilize to a real steady state, as was illustrated theoretically in Figure 10.7. Although the water levels continue to drop, the cone of depression will eventually deepen uniformly over the area influenced by the pumping. At that stage, the hydraulic gradient has become constant; this phenomenon is called pseudo-steady state.

For this situation, Thiem (1906), using two or more observation wells, developed an equation to determine the transmissivity of an aquifer. This equation is known as the Thiem-Dupuit equation and is written as

$$s_1 - s_2 = \frac{2.3Q}{2\pi KH} \log\frac{r_2}{r_1} \tag{10.20}$$

If the time of pumping is long enough, a plot of the drawdown s observed at a particular time, versus the logarithm of distance r, will show a straight line. If the slope of this straight line is expressed as the drawdown difference ($\Delta s = s_1 - s_2$) per log cycle of distance ($\log r_2/r_1 = 1$), rearranging Equation 10.20 gives

$$KH = \frac{2.3Q}{2\pi\Delta s} \tag{10.21}$$

Thiem's straight-line method is based on the assumptions listed in Section 10.4 and on the limiting condition that the late-time-drawdown graphs of the observation wells run parallel, thus indicating a constant hydraulic gradient.

Procedure 4

- Plot the pseudo-steady state drawdown values s of each observation well versus the corresponding distance r on semi-log paper (r on logarithmic scale);
- Draw the best-fitting straight line through the plotted points;

- Determine the slope of the straight line (i.e. the drawdown difference Δs per log cycle of distance);
- Substitute the values of Q and Δs into Equation 10.21 and solve for KH.

Remark 7
When the drawdowns in an unconfined aquifer are large compared with the aquifer's original saturated thickness, the above analysis should be based on corrected drawdown data. The same correction should be made as was discussed in Section 10.4.1 (see Equation 10.6).

Remark 8
When the water levels in the pumped well are also recorded in addition to those in two or more observation wells, its pseudo-steady state drawdown can be affected by non-linear well losses. When not corrected, its value may deviate from the straight line.

Remark 9
When the pumped well only partially penetrates the aquifer, all pseudo-steady-state drawdowns observed in the wells within a distance approximately equal to the thickness of the aquifer – the pumped well included – will have an extra drawdown due to the effect of partial penetration. Because this effect decreases with increasing distance from the pumped well, the slope of the straight line will be affected and, with that, the transmissivity value of the aquifer.

Example 10.4
An aquifer test was made in a shallow unconfined aquifer; prior to pumping, the aquifer's saturated thickness was only 6.5 m. The well was pumped at a constant rate of 167 m^3/d for 520 minutes. The watertable was observed in seven observation wells. Figure 10.15 shows the corrected time-drawdown graphs of these seven wells (see Remark 7). As can be seen from this figure, the curves run parallel in the last hours of the test. Table 10.5 shows the drawdown values in the seven observation wells at the end of the test; these were regarded as representing the pseudo-steady-state drawdowns.

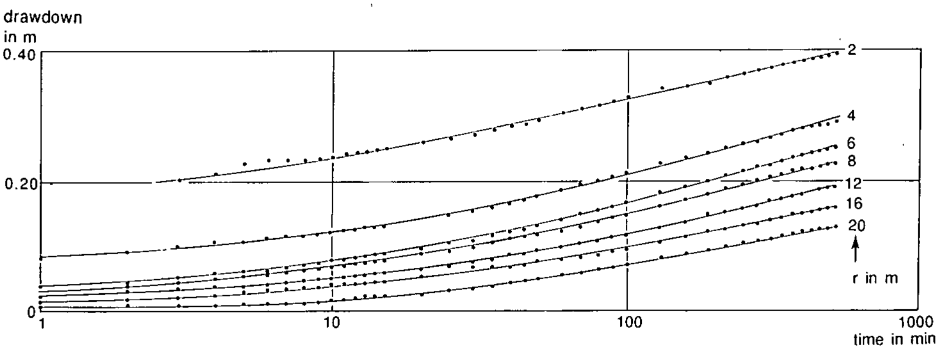

Figure 10.15 Time-drawdown curves of seven observation wells showing field data of an aquifer test in an unconfined aquifer

Table 10.5 Pseudo-steady-state drawdown values

Distance to pumped well (m)	Pseudo-steady-state drawdown Uncorrected (m)	Corrected (m)
2	0.407	0.394
4	0.294	0.287
6	0.252	0.247
8	0.228	0.224
12	0.193	0.190
16	0.161	0.159
20	0.131	0.130

Figure 10.16 shows the distance-drawdown plot on semi-logarithmic paper. As can be seen, all points in the plot lie on a straight line, except that of the observation well at 2 m distance. This phenomenon can be explained by an additional head loss which usually occurs near the well because of the relatively strong curvature of the watertable. The slope of the straight line through the remaining six points shows that Δs is 0.21 m. Substituting the appropriate values into Equation 10.21 yields

$$KH = \frac{2.3Q}{2\pi\Delta s} = \frac{2.3 \times 167}{2 \times 3.14 \times 0.21} = 291 \text{ m}^2/\text{d}$$

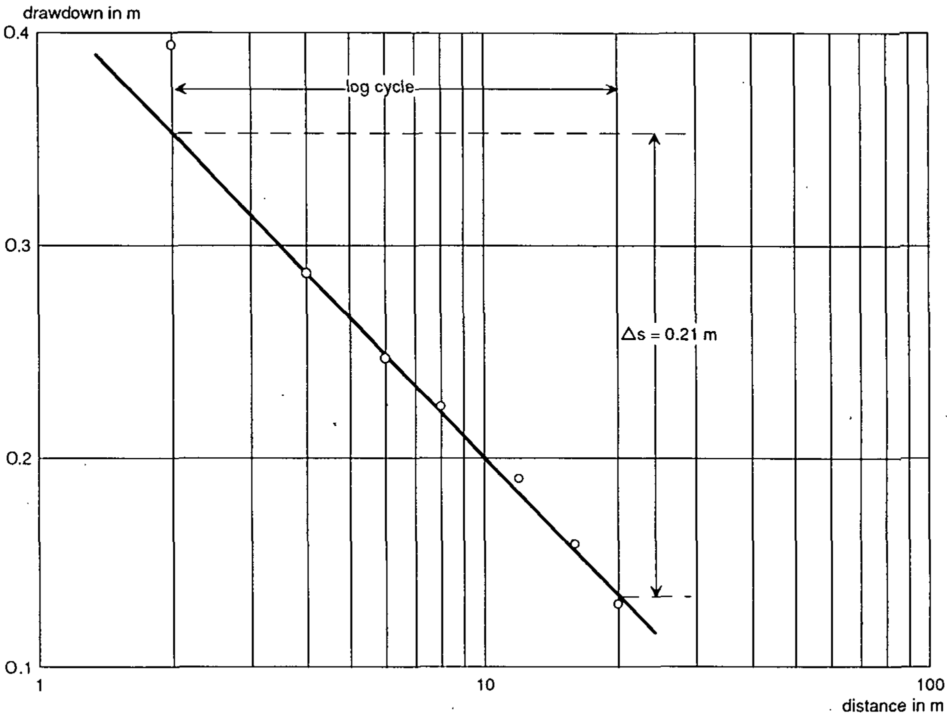

Figure 10.16 Distance-drawdown plot of field data of an aquifer test in an unconfined aquifer

It should be noted that, because of the very shallow aquifer, the observation wells were drilled at relatively short distances from the pumped well.

10.4.5 Distance-Drawdown Analysis of Semi-Confined Aquifers

In semi-confined aquifers, a steady-state situation will develop when the recharge from the overlying aquitard by downward percolation equals the discharge rate of the pump. For this situation, De Glee (1930, 1951) developed the following equation

$$s_m = \frac{Q}{2\pi KH} K_0\left(\frac{r}{L}\right) \qquad (10.22)$$

Unaware of the work done many years earlier by De Glee, Hantush and Jacob (1955) also derived Equation 10.22. Hantush (1956, 1964) noted that, if r/L is small, Equation 10.22 can, for all practical purposes, be approximated by

$$s_m = \frac{2.3Q}{2\pi KH} \log\left(1.123\frac{L}{r}\right) \qquad (10.23)$$

with an error of 10, 5, 2, and 1 per cent for r/L values smaller than 0.45, 0.33, 0.22, and 0.16, respectively. In practice, the above approximation is satisfactory up to values of r equal to 0.2 L.

For two observation wells at small distances r_1 and r_2 from the pumped well, Equation 10.23 reads

$$s_{m1} - s_{m2} = \frac{2.3Q}{2\pi KH} \log\left(1.123\frac{L}{r_1}\right) - \frac{2.3Q}{2\pi KH} \log\left(1.123\frac{L}{r_2}\right)$$

or

$$s_{m1} - s_{m2} = \frac{2.3Q}{2\pi KH} \log\frac{r_2}{r_1}$$

which is the Thiem-Dupuit equation presented in Section 10.4.4.

It is important to note that the flow system in a pumped semi-confined aquifer consists of a vertical component in the overlying aquitard and a horizontal component in the aquifer. In reality, the flow lines in the aquifer are not horizontal but curved (i.e. there are both vertical and horizontal flow components in the aquifer). The above equations can therefore only be used when the vertical-flow component in the aquifer is so small compared to the horizontal-flow component that it can be neglected. In practice, this condition is fulfilled when $L > 3H$.

A plot of the steady-state drawdown s_m versus the logarithm of the distance r will thus also show a straight line. If the slope of this straight line is expressed as the drawdown difference ($\Delta s_m = s_{m1} - s_{m2}$) per log cycle of distance ($\log r_2/r_1 = 1$), the transmissivity value of the aquifer can be calculated as follows

$$KH = \frac{2.3Q}{2\pi\Delta s_m}$$

If the straight line is extended until it intercepts the distance-axis where $s_m = 0$, the interception point has the coordinates $s_m = 0$ and $r = r_0$. Substituting these values into Equation 10.23 gives

$$\log\left[1.123\frac{L}{r_0}\right] = 0 \quad \text{or} \quad \left[1.123\frac{L}{r_0}\right] = 1 \quad \text{or} \quad L = \frac{r_0}{1.123} \qquad (10.25)$$

The Hantush-Jacob method is based on the assumptions listed in Section 10.4 and on the following limiting conditions:
- The flow to the pumped well is in steady state;
- $L > 3H$;
- $r/L < 0.2$.

Procedure 5
- Plot the steady-state drawdown values s_m of each observation well versus the corresponding distance r on semi-log paper (r on logarithmic scale);
- Draw the best-fitting straight line through the plotted points;
- Determine the slope of the straight line (i.e. the drawdown difference Δs_m per log cycle of distance);
- Substitute the values of Q and Δs_m into Equation 10.24 and solve for KH;
- Extend the straight line until it intercepts the distance axis where $s_m = 0$, and read the value of r_0;
- Substitute this value into Equation 10.25 and solve for L;
- Calculate c, using $L = \sqrt{KHc}$.

Remark 10
When the water level in the pumped well is recorded in addition to those in two or more observation wells, its steady-state drawdown can be affected by non-linear well losses. If not corrected, its plotting position may deviate from the straight line.

Remark 11
When the pumped well only partially penetrates the aquifer, all steady-state drawdowns observed in the wells within a distance approximately equal to the thickness of the aquifer – the pumped well included – will have an extra drawdown due to the effect of partial penetration. Because this effect decreases with increasing distance from the pumped well, the slope of the straight line will be affected and, with that, the transmissivity value of the aquifer and the hydraulic resistance of the aquitard.

Example 10.5
An aquifer test was made in a semi-confined aquifer with a thickness of some 10 m. This test (Dalem) is described in detail by Kruseman and De Ridder (1990). The well was pumped at a constant rate of 761 m^3/d for 480 minutes. Table 10.6 shows the extrapolated steady-state drawdowns in the six observation wells.

Using the Hantush-Jacob method, calculate KH and c.

Figure 10.17 shows the distance-drawdown plot on semi-logarithmic paper. The slope

Table 10.6 Extrapolated steady-state drawdowns

Distance from pumped well (m)	Steady-state drawdown (m)
10	0.282
30	0.235
60	0.170
90	0.147
120	0.132
400	0.059

of the straight line shows that Δs_m is 0.14 m. Substituting the appropriate values into Equation 10.24 yields

$$KH = \frac{2.3Q}{2\pi\Delta s_m} = \frac{2. \times 761}{2 \times 3.14 \times 0.14} = 1990\ m^2/d$$

The intersection point where $s_m = 0$ has the distance value $r = 1000$ m. Substituting this value into Equation 10.25 gives

$$L = \frac{r_0}{1.123} = \frac{1000}{1.123} = 890\ m$$

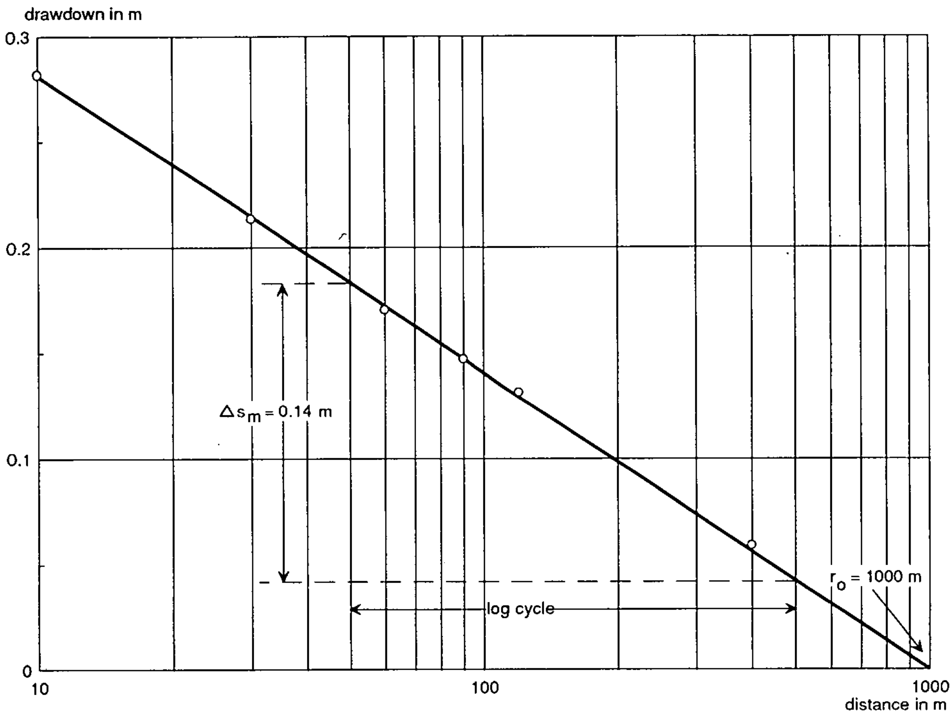

Figure 10.17 Distance-drawdown plot of field data of an aquifer test in a semi-confined aquifer

The c value can now be calculated as follows

$$c = \frac{L^2}{KH} = \frac{890^2}{1990} = 398 \text{ d}$$

Finally, the limiting conditions should be checked. Substituting the above L value into the condition $L > 3H$ gives $H < 296$ m, so this condition is fulfilled.

Substituting the appropriate values into the condition $r/L < 0.2$ gives

$$r < 0.2\,L \rightarrow r < 0.2 \times 890 \rightarrow r < 178 \text{ m}$$

According to this condition, the drawdown value of the observation well at a distance of 400 m should be eliminated from the analysis. Figure 10.17, however, shows that this point, too, lies on the straight line, so in this case this condition is not a limiting factor.

10.5 Concluding Remarks

The diagnostic plots of time-drawdown data presented in the previous sections are theoretical curves. The time-drawdown curves based on field data will often deviate from these theoretical shapes. These deviations can stem from the fact that one or more of the general assumptions and conditions listed in Section 10.4 are not met in the field, or that the method selected is not the correct one for the test site.

It should be realised that all the methods we have discussed are based on highly simplified representations of the natural aquifer. No real aquifers conform fully to these assumed geological or hydrological conditions. In itself, it is quite surprising that these methods so often produce such good results!

Some of the common departures from the theoretical curves will now be discussed.

10.5.1 Delayed-Yield Effect in Unconfined Aquifers

The general assumption that water removed from storage is discharged instantaneously with decline of head is not always met. Drawdown data in an unconfined aquifer often show a 'delayed-yield' effect. The delayed yield is caused by a time lag between the early elastic response of the aquifer and the subsequent downward movement of the watertable. When the time-drawdown curve is plotted on semi-log paper, it shows a typical shape: a relatively steep early-time segment, a flat intermediate segment, and a relatively steep segment again at later times (Figure 10.18).

During the early stage of a test – a stage that may last for only a few minutes – the discharge of the pumped well is derived uniquely from the elastic storage within the aquifer. Hence, the reaction of the unconfined aquifer immediately after the start of pumping is similar to the reaction of a confined aquifer as described by the flow equation of Theis.

Only after some time will the watertable start to fall and the effect of the delayed yield will become apparent. The influence of the delayed yield is comparable to that

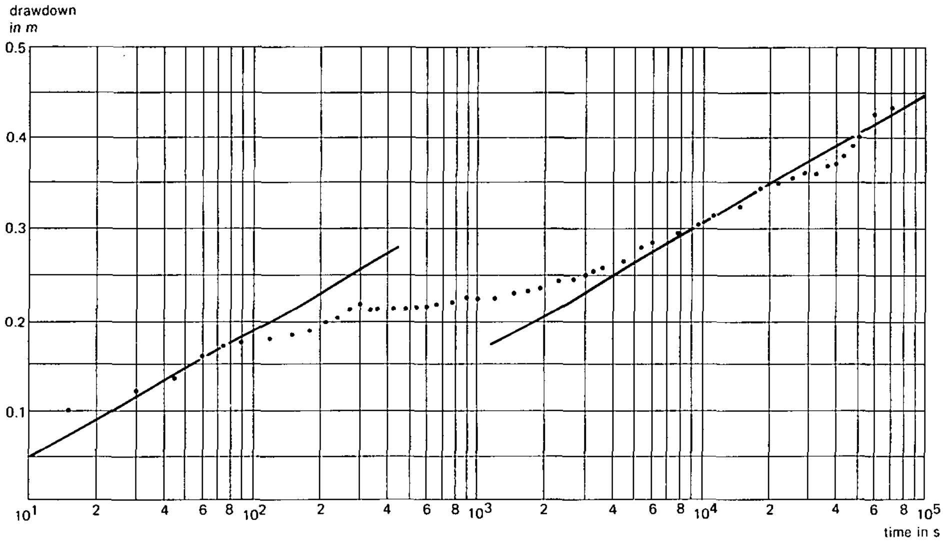

Figure 10.18 Time-drawdown plot of an unconfined aquifer showing delayed-yield effect

of leakage: the drawdown slows down with time and no longer conforms to the Theis curve. After a few minutes to a few hours of pumping, the time-drawdown curve approaches a horizontal position.

The late-time segment of the time-drawdown curve may start from several minutes to several days after the start of pumping. The declining watertable can now keep pace with the increase in the average drawdown. The flow in the aquifer is essentially horizontal again and, as in the early pumping time, the time-drawdown curve approaches the Theis curve.

The above phenomenon means that when a time-drawdown plot shows an S shape as depicted in Figure 10.18, both straight-line segments, which theoretically should run parallel, can be used to determine the transmissivity of the aquifer according to Jacob's straight-line method (Section 10.4.1). With the same method, but using only the straight line through the late-time drawdown data, the specific-yield value can also be found.

It should be noted that, for observation wells relatively close to the pumped well, usually only the right-hand side of the curve of Figure 10.18 will be present in a time-drawdown plot of field data. This phenomenon is thus also encountered with single-well test data.

10.5.2 Partially-Penetrating Effect in Unconfined Aquifers

Some aquifers are so thick that it is not justified to install a fully-penetrating well. Instead, the aquifer has to be pumped by a partially-penetrating well. Because partial penetration induces vertical-flow components in the vicinity of the pumped well, the assumption that the well receives water from horizontal flow is not valid. Hence, the standard methods of analysis cannot be used unless allowance is made for partial penetration.

Partial penetration causes the flow velocity in the immediate vicinity of the well to be higher than it would be otherwise, leading to an extra loss of head. This effect is strongest at the well face, and decreases with increasing distance from the well. It is negligible if measured at a distance that is one to two times greater than the saturated thickness of the aquifer, depending on the degree of penetration.

Hantush (1962) presented a solution for partially-penetrating wells in confined aquifers. Because of the large aquifer thickness, the induced drawdowns are usually relatively small, so Hantush's solution can also be applied to unconfined aquifers. Figure 10.19 shows the typical time-drawdown shape of a confined or unconfined aquifer pumped by a partially-penetrating well. The curve shows a curved-line segment, an inflection point, a second curved-line segment, and finally a straight-line segment under a slope. This last segment can be used to determine the transmissivity of the aquifer according to Jacob's straight-line method (Section 10.4.1). An estimate of the specific-yield value, however, is not possible. This can be done with the log-log procedure (see Kruseman and De Ridder 1990) or with the computer program SATEM (Boonstra 1989). If SATEM is used, the saturated thickness of the aquifer can be determined in a trial-and-error fashion.

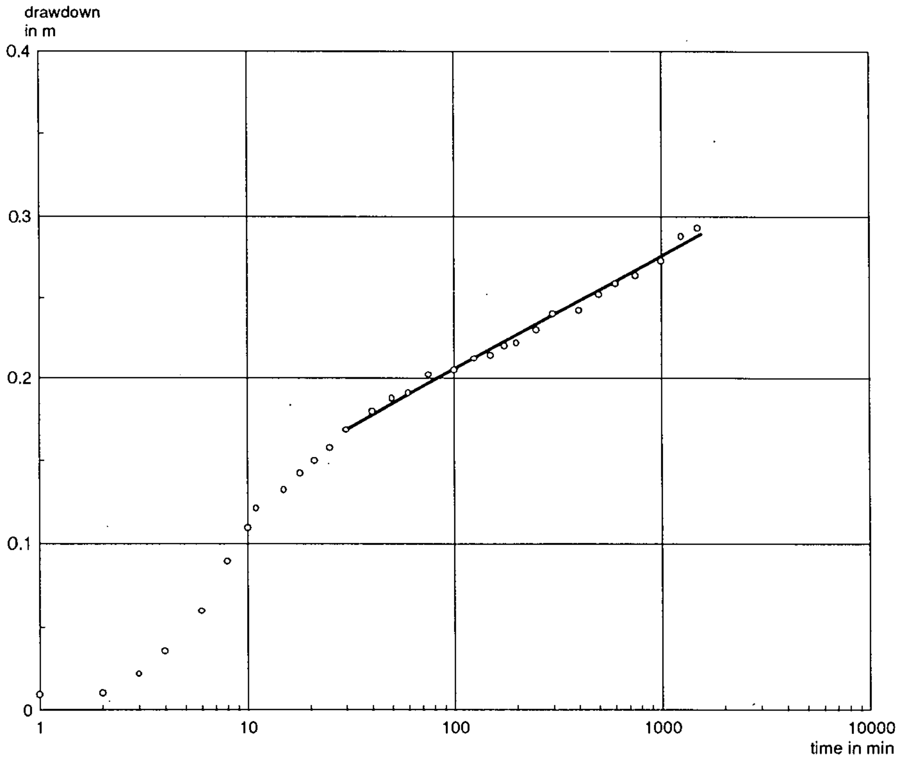

Figure 10.19 Time-drawdown plot of an unconfined aquifer when the pumped well only partially penetrates the aquifer

It should be noted that, for observation wells relatively close to the pumped well, usually only the second curved-line segment and the straight-line segment of the curve in Figure 10.19 will be present in a time-drawdown plot of field data. This phenomenon is thus also encountered with single-well-test data.

10.5.3 Deviations in Late-Time Drawdown Data

Steepening of Late-Time Slope

All real aquifers are limited by geological or hydrological boundaries. If, however, at the end of the pumping period, no such boundaries have been met within the cone of depression, it is said that the aquifer has a seemingly infinite areal extent. When the cone of depression intersects an impervious boundary (e.g. a fault or an impermeable valley wall), it can expand no farther in that direction. The cone must expand and deepen more rapidly at the fault or valley wall to maintain the yield of the well.

All the methods we have presented also assume that the tested aquifer is homogeneous within the area influenced by the pumping. This condition is never fully met, but it depends on the variations in hydraulic conductivity whether these variations will cause deviations from the theoretical time-drawdown curves. When, in one of the directions, the sediments become finer and the hydraulic conductivity decreases, the slope of the time-drawdown curve will become steeper when the cone of depression spreads into that area. The typical shape resulting from this phenomenon is identical to that of an impervious boundary. Well interference will also result in a similar phenomenon.

Flattening of Late-Time Slope

An opposite phenomenon is encountered when the cone of depression intersects an open water body. If the open water body is hydraulically connected with the aquifer, the aquifer is recharged at an increasing rate as the cone of depression spreads with time. This results in a flattening of the slope of the time-drawdown curve at later times (Figure 10.20). As a phenomenon, it resembles the recharge that occurs in a

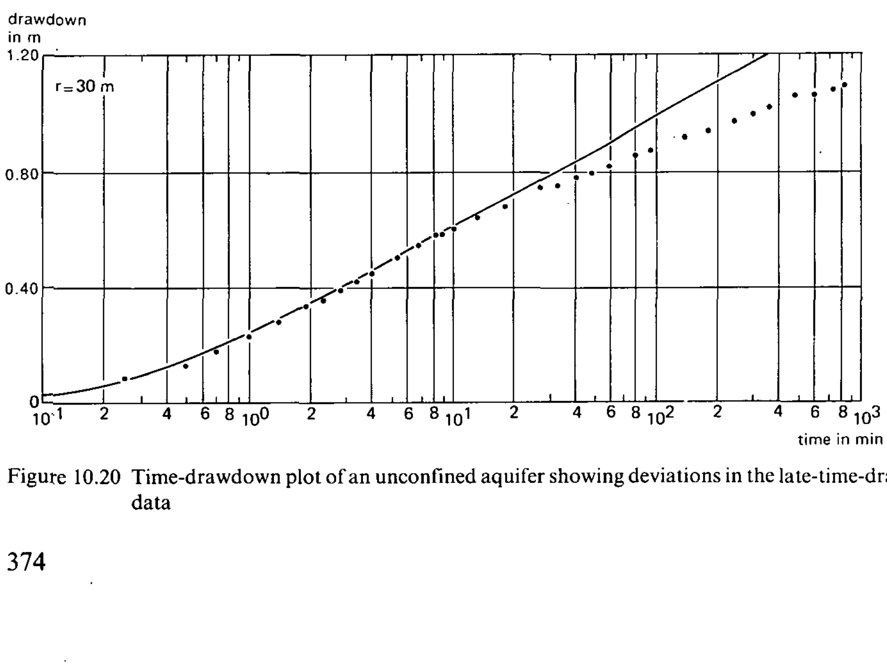

Figure 10.20 Time-drawdown plot of an unconfined aquifer showing deviations in the late-time-drawdown data

semi-confined aquifer. The same phenomenon occurs when, in one of the directions, the hydraulic conductivity or the aquifer thickness increases.

The above cases will result in time-drawdown plots in which the last part of the late-time drawdown data will deviate from a straight line under a slope. This part of the plot should be disregarded when the slope of the straight-line segment is being determined.

10.5.4 Conclusions

It will be clear that there are various reasons why time-drawdown data depart from the theoretical curves. It will also be clear that different phenomena can cause identical anomalies. So, if one is to make a correct analysis, one must have a proper knowledge of the geology of the test site. Because, unfortunately, this knowledge is often fragmentary, determining hydraulic characteristics is more an art than a science. This is one of the main reasons why it is strongly recommended to continue to monitor the watertable behaviour during the recovery period. This allows a second estimate of the aquifer's transmissivity to be made, which can then be compared with the one found during the pumping period. Even with single well tests, this second estimate is possible.

Finally, a few remarks on the difference between aquifer tests and single-well tests. The results of aquifer tests are more reliable and more accurate than those of single-well tests. Another advantage is that aquifer tests allow estimates to be made of both the aquifer's transmissivity and its specific yield or storativity, which is not possible with single-well tests. Further, if an aquifer test uses more than one observation well, separate estimates of the hydraulic characteristics can be made for each well, allowing the various values to be compared. Moreover, one can make yet another estimate of the hydraulic characteristics by using not only the time-drawdown relationship, but also the distance-drawdown relationships.

References

Boonstra, J. 1989. SATEM : Selected aquifer test evaluation methods : a microcomputer program. ILRI Publication 48, Wageningen, 80 p.

Bos, M.G. (ed.) 1989. Discharge measurement structures. 3rd rev. ed. ILRI Publication 20, Wageningen, 401 p.

Cooper, H.H. and C.E. Jacob 1946. A generalized graphical method for evaluating formation constants and summarizing well field history. Am. Geophys. Union Trans., 27, pp. 526-534.

De Glee, G.J. 1930. Over grondwaterstromingen bij wateronttrekking door middel van putten. Waltman, Delft, 175 p.

De Glee, G.J. 1951. Berekeningsmethoden voor de winning van grondwater. In: Winning van grondwater: derde vacantiecursus in drinkwatervoorziening. Delft University of Technology. pp. 38-80.

Driscoll, F.G. 1986. Groundwater and wells. 2nd ed. Johnson Division, St. Paul, 1089 p.

Hantush, M.S. 1956. Analysis of data from pumping tests in leaky aquifers. Am. Geophys. Union Trans., 37, pp. 702-714.

Hantush, M.S. 1962. Aquifer tests on partially-penetrating wells. Am. Soc. Civ. Eng. Trans., 127, Part I, pp. 284-308.

Hantush, M.S. 1964. Hydraulics of wells. In: V.T. Chow (ed.). Adv. Hydroscience 1, Academic Press, New York. pp. 281-432.

Hantush, M.S. and C.E. Jacob 1955. Non-steady radial flow in an infinite leaky aquifer. Am. Geophys. Union Trans., 36, pp. 95-100.

Jacob, C.E. 1944. Notes on determining permeability by pumping tests under watertable conditions. U.S. Geol. Surv. Open File Rept.

Jacob, C.E. 1950. Flow of groundwater. In: H. Rouse (ed.). Engineering Hydraulics. Wiley, New York. pp. 321-386.

Kruseman, G.P. and N.A. de Ridder 1990. Analysis and evaluation of pumping test data. ILRI Publication 47, Wageningen, 377 p.

Theis, C.V. 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. Am. Geophys. Union Trans., 16, pp. 519-524.

Thiem, G. 1906. Hydrologische Methoden. Gebhardt, Leipzig, 56 p.

Appendix 10.1 Values of the Theis well function W(u) as a function of 1/u

1/u	$x10^{-1}$	$x10^{0}$	$x10^{1}$	$x10^{2}$	$x10^{3}$	$x10^{4}$	$x10^{5}$	$x10^{6}$	$x10^{7}$	$x10^{8}$	$x10^{9}$
1.0	4.16(−6)	2.19(−1)	1.82	4.04	6.33	8.63	1.09(1)	1.32(1)	1.55(1)	1.78(1)	2.01(1)
1.2	2.60(−5)	2.93(−1)	1.99	4.22	6.51	8.82	1.11(1)	1.34(1)	1.57(1)	1.80(1)	2.03(1)
1.5	1.68(−4)	3.98(−1)	2.20	4.44	6.74	9.04	1.13(1)	1.36(1)	1.59(1)	1.82(1)	2.06(1)
2.0	1.15(−3)	5.60(−1)	2.47	4.73	7.02	9.33	1.16(1)	1.39(1)	1.62(1)	1.85(1)	2.08(1)
2.5	3.78(−3)	7.02(−1)	2.68	4.95	7.25	9.55	1.19(1)	1.42(1)	1.65(1)	1.88(1)	2.11(1)
3.0	8.57(−3)	8.29(−1)	2.86	5.13	7.43	9.73	1.20(1)	1.43(1)	1.66(1)	1.89(1)	2.12(1)
3.5	1.57(−2)	9.42(−1)	3.01	5.28	7.58	9.89	1.22(1)	1.45(1)	1.68(1)	1.91(1)	2.14(1)
4.0	2.49(−2)	1.04	3.14	5.42	7.72	1.00(1)	1.23(1)	1.46(1)	1.69(1)	1.92(1)	2.15(1)
4.5	3.61(−2)	1.14	3.25	5.53	7.83	1.01(1)	1.24(1)	1.47(1)	1.70(1)	1.93(1)	2.17(1)
5.0	4.89(−2)	1.22	3.35	5.64	7.94	1.02(1)	1.25(1)	1.48(1)	1.72(1)	1.95(1)	2.18(1)
6.0	7.83(−2)	1.37	3.53	5.82	8.12	1.04(1)	1.27(1)	1.50(1)	1.73(1)	1.96(1)	2.19(1)
7.0	1.11(−1)	1.51	3.69	5.98	8.28	1.06(1)	1.29(1)	1.52(1)	1.75(1)	1.98(1)	2.21(1)
8.0	1.46(−1)	1.62	3.82	6.11	8.41	1.07(1)	1.30(1)	1.53(1)	1.76(1)	1.99(1)	2.22(1)
9.0	1.83(−1)	1.73	3.93	6.23	8.53	1.08(1)	1.31(1)	1.54(1)	1.77(1)	2.00(1)	2.23(1)

Note: 1.15(−3) means 1.15×10^{-3} or 0.00115

Example: $1/u = 5 \times 10^{5}$

$W(u) = 12.5$

Appendix 10.2 Values of the Hantush well function W(u,r/L) as function of 1/u and r/L

1/u	r/L=.005	r/L=.01	r/L=.02	r/L=.03	r/L=.04	r/L=.05	r/L=.06	r/L=.07	r/L=.08	r/L=.09	r/L=.1
1.0											2.19(−1)
1.5										3.98(−1)	3.97(−1)
2.5		W(u,r/L)	= W(u)				7.02(−1)	7.01(−1)	7.01(−1)	7.00(−1)	7.00(−1)
4.0							1.04	1.04	1.04	1.04	1.04
6.5							1.44	1.44	1.44	1.43	1.43
1.0(1)							1.82	1.81	1.81	1.81	1.80
1.5(1)			2.20	2.19	2.19	2.19	2.19	2.18	2.18	2.17	2.17
2.5(1)			2.68	2.68	2.67	2.67	2.66	2.66	2.65	2.64	2.63
4.0(1)		3.14	3.13	3.13	3.12	3.11	3.10	3.09	3.08	3.07	3.05
6.5(1)		3.61	3.61	3.60	3.59	3.58	3.56	3.54	3.52	3.49	3.47
1.0(2)		4.04	4.03	4.02	4.00	3.98	3.95	3.93	3.89	3.86	3.82
1.5(2)	4.44	4.44	4.43	4.41	4.38	4.35	4.31	4.27	4.22	4.17	4.11
2.5(2)	4.95	4.94	4.92	4.89	4.85	4.80	4.74	4.67	4.59	4.51	4.42
4.0(2)	5.41	5.41	5.38	5.33	5.27	5.19	5.09	4.99	4.88	4.76	4.64
6.5(2)	5.90	5.89	5.84	5.76	5.66	5.54	5.40	5.25	5.09	4.93	4.77
1.0(3)	6.33	6.31	6.23	6.12	5.97	5.80	5.61	5.41	5.21	5.01	4.83
1.5(3)	6.73	6.70	6.59	6.43	6.22	5.98	5.74	5.50	5.27	5.05	4.85
2.5(3)	7.23	7.19	7.01	6.76	6.45	6.14	5.83	5.55	5.29	5.06	
4.0(3)	7.69	7.62	7.35	6.99	6.59	6.20	5.86	5.56			
6.5(3)	8.16	8.05	7.65	7.14	6.65	6.22	5.87				
1.0(4)	8.57	8.40	7.84	7.21	6.67	6.23					
1.5(4)	8.95	8.70	7.96	7.24							
2.5(4)	9.40	9.01	8.03	7.25							
4.0(4)	9.78	9.22	8.05								
6.5(4)	1.01(1)	9.36	8.06								
1.0(5)	1.04(1)	9.42						$W(u,r/L) = 2 K_0(r/L)$			
1.5(5)	1.06(1)	9.44									
2.5(5)	1.07(1)										
4.0(5)	1.08(1)										
∞	1.08(1)	9.44	8.06	7.25	6.67	6.23	5.87	5.56	5.29	5.06	4.85

Appendix 10.2 (cont.)

1/u	r/L = .2	r/L = .3	r/L = .4	r/L = .6	r/L = .8	r/L = 1	r/L = 2	r/L = 3	r/L = 4	r/L = 5	r/L = 6
1.0(−1)	4.15(−6)	4.15(−6)	4.14(−6)	4.12(−6)	4.10(−6)	4.06(−6)	3.79(−6)	3.36(−6)	2.84(−6)	2.29(−6)	1.80(−6)
1.5(−1)	1.68(−4)	1.68(−4)	1.67(−4)	1.66(−4)	1.65(−4)	1.63(−4)	1.47(−4)	1.25(−4)	9.86(−5)	7.30(−5)	5.03(−5)
2.5(−1)	3.77(−3)	3.76(−3)	3.75(−3)	3.71(−3)	3.65(−3)	3.58(−3)	3.06(−3)	2.35(−3)	1.63(−3)	1.02(−3)	5.79(−4)
4.0(−1)	2.48(−2)	2.47(−2)	2.46(−2)	2.42(−2)	2.37(−2)	2.30(−2)	1.82(−2)	1.23(−2)	7.22(−3)	3.66(−3)	1.69(−3)
6.5(−1)	9.40(−2)	9.35(−2)	9.27(−2)	9.05(−2)	8.75(−2)	8.39(−2)	5.90(−2)	3.35(−2)	1.57(−2)	6.42(−3)	2.38(−3)
1.0	2.18(−1)	2.16(−1)	2.14(−1)	2.06(−1)	1.97(−1)	1.85(−1)	1.14(−1)	5.34(−2)	2.07(−2)	7.27(−3)	2.48(−3)
1.5	3.95(−1)	3.90(−1)	3.84(−1)	3.66(−1)	3.44(−1)	3.17(−1)	1.66(−1)	6.48(−2)	2.21(−2)	7.38(−3)	2.49(−3)
2.5	6.93(−1)	6.81(−1)	6.65(−1)	6.21(−1)	5.65(−1)	5.02(−1)	2.10(−1)	6.91(−2)	2.23(−2)		
4.0	1.02	9.99(−1)	9.65(−1)	8.77(−1)	7.70(−1)	6.57(−1)	2.25(−1)	6.95(−2)			
6.5	1.40	1.35	1.29	1.13	9.46(−1)	7.68(−1)	2.28(−1)				
1.0(1)	1.75	1.67	1.56	1.31	1.05	8.19(−1)					
1.5(1)	2.08	1.95	1.79	1.44	1.10	8.37(−1)					
2.5(1)	2.48	2.27	2.02	1.52	1.13	8.42(−1)					
4.0(1)	2.81	2.49	2.14	1.55							
6.5(1)	3.10	2.64	2.21	1.55							
1.0(2)	3.29	2.71	2.23	1.56							
1.5(2)	3.41	2.74				W(u,r/L)	$= 2\ K_0(r/L)$				
2.5(2)	3.48										
4.0(2)	3.50										
6.5(2)	3.51										
∞	3.51	2.74	2.23	1.56	1.13	8.42(−1)	2.28(−1)	6.95(−2)	2.23(−2)	7.38(−3)	2.49(−3)

Note: 6.5(2) means 6.5×10^2 or 650

Example: $1/u = 1.5(3) = 1.5 \times 10^3 = 1500$ and $r/L = 0.1$

$W(u,r/L) = 4.85$

Appendix 10.3 Values of K_o (r/L) and $e^{r/L}$ K_0(r/L) as function of r/L

r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)	r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)	r/L	K_0(r/L)	$e^{r/l}$ K_0(r/L)	r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)
1.0(−2)	4.72	4.77	3.8(−2)	3.39	3.52	6.6(−2)	2.84	3.03	9.4(−2)	2.49	2.73
1.1(−2)	4.63	4.68	3.9(−2)	3.36	3.50	6.7(−2)	2.82	3.02	9.5(−2)	2.48	2.72
1.2(−2)	4.54	4.59	4.0(−2)	3.34	3.47	6.8(−2)	2.81	3.01	9.6(−2)	2.47	2.72
1.3(−2)	4.46	4.52	4.1(−2)	3.31	3.45	6.9(−2)	2.79	2.99	9.7(−2)	2.46	2.71
1.4(−2)	4.38	4.45	4.2(−2)	3.29	3.43	7.0(−2)	2.78	2.98	9.8(−2)	2.45	2.70
1.5(−2)	4.32	4.38	4.3(−2)	3.26	3.41	7.1(−2)	2.77	2.97	9.9(−2)	2.44	2.69
1.6(−2)	4.25	4.32	4.4(−2)	3.24	3.39	7.2(−2)	2.75	2.96	1.0(−1)	2.43	2.68
1.7(−2)	4.19	4.26	4.5(−2)	3.22	3.37	7.3(−2)	2.74	2.95	1.1(−1)	2.33	2.60
1.8(−2)	4.13	4.21	4.6(−2)	3.20	3.35	7.4(−2)	2.72	2.93	1.2(−1)	2.25	2.53
1.9(−2)	4.08	4.16	4.7(−2)	3.18	3.33	7.5(−2)	2.71	2.92	1.3(−1)	2.17	2.47
2.0(−2)	4.03	4.11	4.8(−2)	3.15	3.31	7.6(−2)	2.70	2.91	1.4(−1)	2.10	2.41
2.1(−2)	3.98	4.06	4.9(−2)	3.13	3.29	7.7(−2)	2.69	2.90	1.5(−1)	2.03	2.36
2.2(−2)	3.93	4.02	5.0(−2)	3.11	3.27	7.8(−2)	2.67	2.89	1.6(−1)	1.97	2.31
2.3(−2)	3.89	3.98	5.1(−2)	3.09	3.26	7.9(−2)	2.66	2.88	1.7(−1)	1.91	2.26
2.4(−2)	3.85	3.94	5.2(−2)	3.08	3.24	8.0(−2)	2.65	2.87	1.8(−1)	1.85	2.22
2.5(−2)	3.81	3.90	5.3(−2)	3.06	3.22	8.1(−2)	2.64	2.86	1.9(−1)	1.80	2.18
2.6(−2)	3.77	3.87	5.4(−2)	3.04	3.21	8.2(−2)	2.62	2.85	2.0(−1)	1.75	2.14
2.7(−2)	3.73	3.83	5.5(−2)	3.02	3.19	8.3(−2)	2.61	2.84	2.1(−1)	1.71	2.10
2.8(−2)	3.69	3.80	5.6(−2)	3.00	3.17	8.4(−2)	2.60	2.83	2.2(−1)	1.66	2.07
2.9(−2)	3.66	3.76	5.7(−2)	2.98	3.16	8.5(−2)	2.59	2.82	2.3(−1)	1.62	2.04
3.0(−2)	3.62	3.73	5.8(−2)	2.97	3.14	8.6(−2)	2.58	2.81	2.4(−1)	1.58	2.01
3.1(−2)	3.59	3.70	5.9(−2)	2.95	3.13	8.7(−2)	2.56	2.80	2.5(−1)	1.54	1.98
3.2(−2)	3.56	3.67	6.0(−2)	2.93	3.11	8.8(−2)	2.55	2.79	2.6(−1)	1.50	1.95
3.3(−2)	3.53	3.65	6.1(−2)	2.92	3.10	8.9(−2)	2.54	2.78	2.7(−1)	1.47	1.93
3.4(−2)	3.50	3.62	6.2(−2)	2.90	3.09	9.0(−2)	2.53	2.77	2.8(−1)	1.44	1.90
3.5(−2)	3.47	3.59	6.3(−2)	2.88	3.07	9.1(−2)	2.52	2.76	2.9(−1)	1.40	1.88
3.6(−2)	3.44	3.57	6.4(−2)	2.87	3.06	9.2(−2)	2.51	2.75	3.0(−1)	1.37	1.85
3.7(−2)	3.41	3.54	6.5(−2)	2.85	3.04	9.3(−2)	2.50	2.74	3.1(−1)	1.34	1.83

Appendix 10.3 (cont.)

r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)	r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)	r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)	r/L	K_0(r/L)	$e^{r/L}$ K_0(r/L)
3.2(−1)	1.31	1.81	6.0(−1)	7.78(−1)	1.42	8.8(−1)	5.01(−1)	1.21	2.6	5.54(−2)	7.46(−1)
3.3(−1)	1.29	1.79	6.1(−1)	7.65(−1)	1.41	8.9(−1)	4.94(−1)	1.20	2.7	4.93(−2)	7.33(−1)
3.4(−1)	1.26	1.77	6.2(−1)	7.52(−1)	1.40	9.0(−1)	4.87(−1)	1.20	2.8	4.38(−2)	7.21(−1)
3.5(−1)	1.23	1.75	6.3(−1)	7.40(−1)	1.39	9.1(−1)	4.80(−1)	1.19	2.9	3.90(−2)	7.09(−1)
3.6(−1)	1.21	1.73	6.4(−1)	7.28(−1)	1.38	9.2(−1)	4.73(−1)	1.19	3.0	3.47(−2)	6.98(−1)
3.7(−1)	1.18	1.71	6.5(−1)	7.16(−1)	1.37	9.3(−1)	4.66(−1)	1.18	3.1	3.10(−2)	6.87(−1)
3.8(−1)	1.16	1.70	6.6(−1)	7.04(−1)	1.36	9.4(−1)	4.59(−1)	1.18	3.2	2.76(−2)	6.77(−1)
3.9(−1)	1.14	1.68	6.7(−1)	6.93(−1)	1.35	9.5(−1)	4.52(−1)	1.17	3.3	2.46(−2)	6.67(−1)
4.0(−1)	1.11	1.66	6.8(−1)	6.82(−1)	1.35	9.6(−1)	4.46(−1)	1.16	3.4	2.20(−2)	6.58(−1)
4.1(−1)	1.09	1.65	6.9(−1)	6.71(−1)	1.34	9.7(−1)	4.40(−1)	1.16	3.5	1.96(−2)	6.49(−1)
4.2(−1)	1.07	1.63	7.0(−1)	6.61(−1)	1.33	9.8(−1)	4.33(−1)	1.15	3.6	1.75(−2)	6.40(−1)
4.3(−1)	1.05	1.62	7.1(−1)	6.50(−1)	1.32	9.9(−1)	4.27(−1)	1.15	3.7	1.56(−2)	6.32(−1)
4.4(−1)	1.03	1.60	7.2(−1)	6.40(−1)	1.31	1.0	4.21(−1)	1.14	3.8	1.40(−2)	6.24(−1)
4.5(−1)	1.01	1.59	7.3(−1)	6.30(−1)	1.31	1.1	3.66(−1)	1.10	3.9	1.25(−2)	6.17(−1)
4.6(−1)	9.94(−1)	1.57	7.4(−1)	6.20(−1)	1.30	1.2	3.19(−1)	1.06	4.0	1.12(−2)	6.09(−1)
4.7(−1)	9.76(−1)	1.56	7.5(−1)	6.11(−1)	1.29	1.3	2.78(−1)	1.02	4.1	9.98(−3)	6.02(−1)
4.8(−1)	9.58(−1)	1.55	7.6(−1)	6.01(−1)	1.29	1.4	2.44(−1)	9.88(−1)	4.2	8.93(−3)	5.95(−1)
4.9(−1)	9.41(−1)	1.54	7.7(−1)	5.92(−1)	1.28	1.5	2.14(−1)	9.58(−1)	4.3	7.99(−3)	5.89(−1)
5.0(−1)	9.24(−1)	1.52	7.8(−1)	5.83(−1)	1.27	1.6	1.88(−1)	9.31(−1)	4.4	7.15(−3)	5.82(−1)
5.1(−1)	9.08(−1)	1.51	7.9(−1)	5.74(−1)	1.26	1.7	1.65(−1)	9.06(−1)	4.5	6.40(−3)	5.76(−1)
5.2(−1)	8.92(−1)	1.50	8.0(−1)	5.65(−1)	1.26	1.8	1.46(−1)	8.83(−1)	4.6	5.73(−3)	5.70(−1)
5.3(−1)	8.77(−1)	1.49	8.1(−1)	5.57(−1)	1.25	1.9	1.29(−1)	8.61(−1)	4.7	5.13(−3)	5.64(−1)
5.4(−1)	8.61(−1)	1.48	8.2(−1)	5.48(−1)	1.25	2.0	1.14(−1)	8.42(−1)	4.8	4.60(−3)	5.59(−1)
5.5(−1)	8.47(−1)	1.47	8.3(−1)	5.40(−1)	1.24	2.1	1.01(−1)	8.25(−1)	4.9	4.12(−3)	5.53(−1)
5.6(−1)	8.32(−1)	1.46	8.4(−1)	5.32(−1)	1.23	2.2	8.93(−2)	8.06(−1)	5.0	3.69(−3)	5.48(−1)
5.7(−1)	8.18(−1)	1.45	8.5(−1)	5.24(−1)	1.23	2.3	7.91(−2)	7.89(−1)			
5.8(−1)	8.04(−1)	1.44	8.6(−1)	5.16(−1)	1.22	2.4	7.02(−2)	7.74(−1)			
5.9(−1)	7.91(−1)	1.43	8.7(−1)	5.09(−1)	1.21	2.5	6.23(−2)	7.60(−1)			

Note: 3.7(−2) means 3.7×10^{-2} or 0.037

Example: r/L = 5.0(−1) = 5×10^{-1} = 0.5 $Ko_0(0.5) = 0.924$ $e^{0.5}K_0(0.5) = 1.52$