6. STRUCTURES: WEIRS

To complete this code, a short summary of the calculation of weirs is included.

For the situation of the weirs, indications are provided by the longitudinal section (see Section 2.1.).

No directions are given here to assist in the choice of type and the construction.

The backwater curve can be determined by the methods of calculation applicable to non-uniform flow. These are not dealt with in this Code. In practice, formula 9 gives an approximation; see Section 5.1. and also Note 7.

The flow over a weir with horizontal crest can be in principle calculated for simple cases from the formula: $Q = 1.7\ mbh^{3/2}$,

where Q = flow over the weir (m^3/sec.)
m = a coefficient, depending on the shape of the weir structure; for somewhat rounded crest and wing-walls the value 1.1 may be assumed
b = width of the weir (m)
h = upstream water level with respect to the crest of the weir.

Calculation may be by means of a graph[1]).

For less simple cases (e.g. approach velocity not negligible, broad-crested weir = standing wave flume, drowned weir, side-contraction, sharp crests), handbooks of hydraulics should be consulted or advice from a hydraulics laboratory should be sought.

0

[1]) See the collection of graphs published separately.

NOTE 1

DRAWING UP A PLAN FOR THE CONSTRUCTION OR IMPROVEMENT OF WATERCOURSES

In drawing up plans for drainage or water-supply, it is advisable to work according to a lucid system.

For improvements in drainage, the boundaries of the area draining to the outfall structure are first of all established. When there is more than one outfall structure, each area draining relative to a structure is denoted separately.

Inside each area, a separate notation is applied to each subsidiary catchment whose excess water discharges into the watercourses lying in the area. These component catchments are bounded by the natural and artificial watersheds found in the area. The natural watersheds are determined by the geohydrology of the area, artificial ones are due to the presence of roads, embankments etc.

In cases where several outfall structures are present it is advisable to leave some room for additions between the last symbol in the first area and the first symbol in a subsequent area, in connection with possible later extensions which can occur especially with schemes for land re-allocation.

The watercourses through which the run-off has to be discharged are provided with a symbol at the upstream end as well as the downstream end, providing there is space available on the drawing. It is recommended that a different denotation should be used for watercourses which are used exclusively for water-supply. Watercourses which are used both for drainage and supply receive the denotation applicable to the drainage system, but in addition, provided there is space available on the drawing, the denotation for the supply system is also represented. This denotation with symbols proceeds right up to the last subdivision which is supplied with water.

The reach of channel immediately upstream of the outfall is considered as the lowest reach of the main channel. All adjacent reaches of channel up to an upstream end point are also taken as belonging to the main water channel. The greatest possible length of channel should be chosen unless the special situation of a certain watercourse or a channel having its own particular name makes it preferable to consider such a watercourse as the main water channel. Only one main water channel is designated even if several watercourses meet at the outfall.

The various symbols for the subdivisions of the area and the watercourses, like those for the structures and cross-sections to be mentioned later, always follow a downstream sequence.

The denotation is continued down to where a tributary channel flows into the main channel. Upstream of the confluence the same rules hold for the tributary as have been described for the main water channel. Every branch is treated in the same fashion. When the main water channel has been reached again, the denotation is continued in the same sequence.

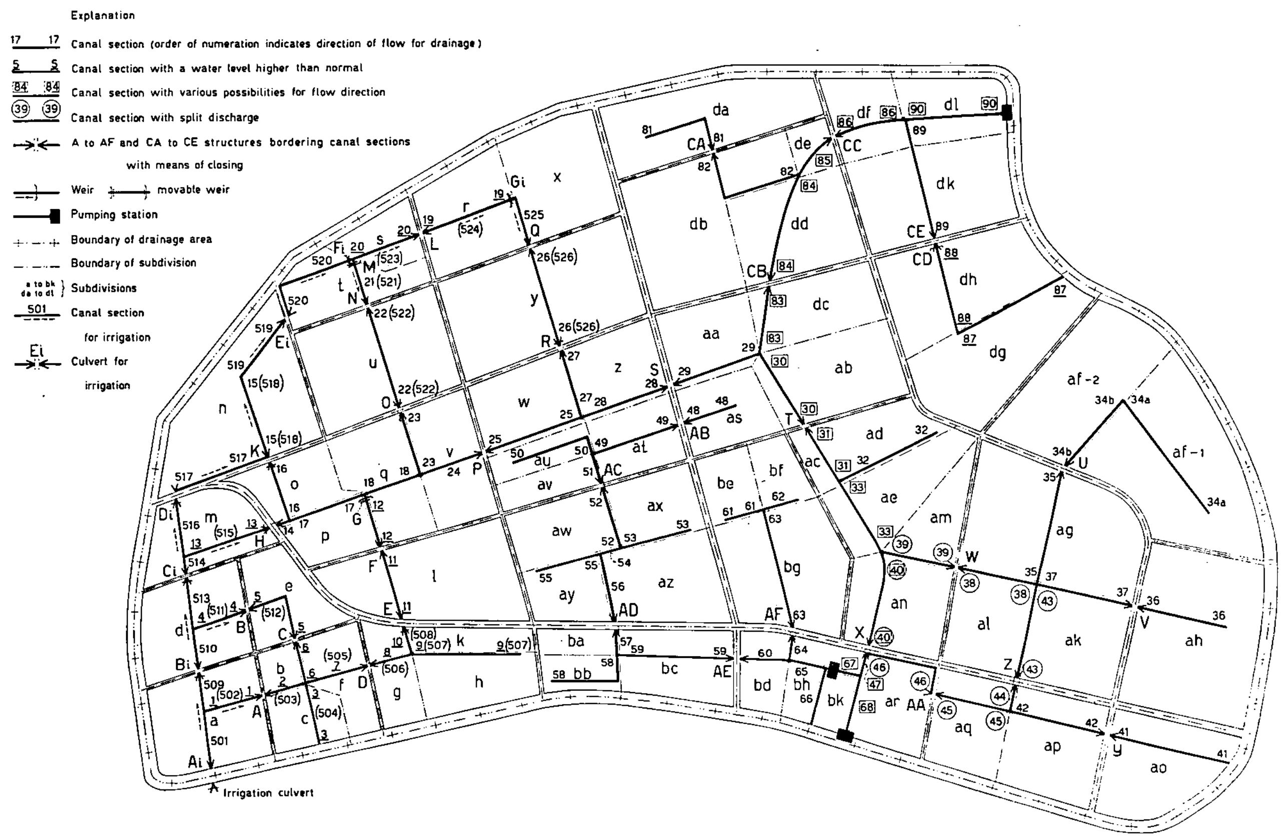
As a rule it is advisable to supply structures which bound upon the channel with a designation different from those which lie in the channel.

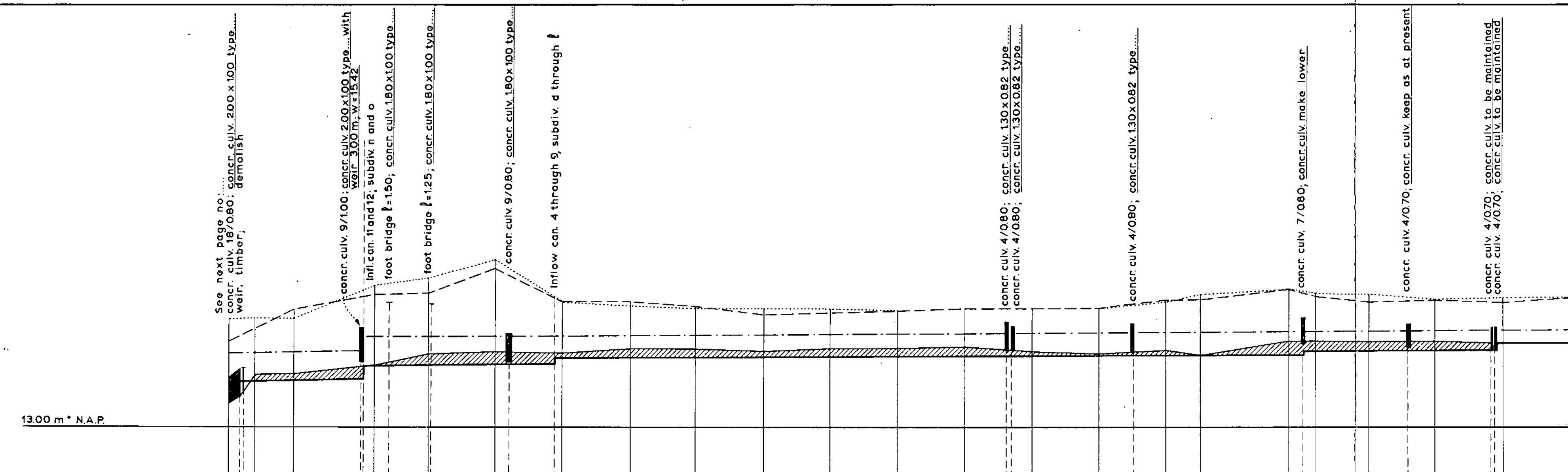
The points in the watercourses at which cross-sections have been drawn are also denoted. A cross-section is drawn such that the righthand side of the drawing represents the right bank, looking downstream.

Especially in areas with irregular contours it may be advisable to use longitudinal sections, provided that here too the system of denotation which has been adopted is continued. These sections are especially useful in areas where the required drainage-depth is obtained by backing-up the water profile by means of weirs. In Appendix 2 is to be found a worked example of an arbitrary longitudinal section in which an attempt has been made to present all known data.

With respect to the data for the calculation of the dimensions of watercourses and structures, it is recommended that these should be presented in a separate table.

Appendix 1. Plan for a drainage scheme





Distances	2020	2002	1997	1980	1920	1821	1817	1800	1780	1720	1716	1620	1600	1532	1520	1420	1320
Number of the cross-section				2	1			3		2		1			12	11	10
Number of the structure	3s	2s				1s			3s		2s		1s				
Left bank				16.24	16.24			17.24		17.46		18.00			16.77	16.65	16.55
Right bank				15.94	16.51			16.92		17.00		17.76			16.75	16.77	16.62
Existing bed level of the channel reach				14.54	14.59			14.83		15.19		15.24			15.20	15.34	15.33
Existing invert level of the structure or soffit level of bridge	13.74 / 13.92	14.78				14.99			16.73		16.68		14.99				
Condition: g – good f – fair p – poor	p	p				g			p		p		f				

Distances	1220	1120	1020	920	859	848	820	720	672	620	570	440	417	400	320	267	220	137	132	120	20	0
Number of the cross-section	9	8	7	6			5	4		3	2	1		3	2		1			2		1
Number of the structure					3s	2s			1s				B			1s		A	1s			
Left bank	16.54	16.54	16.46	16.58			16.69	16.58		16.77	16.98	17.11		16.98	16.98		16.82			16.86		16.90
Right bank	16.37	16.46	16.50	16.59			16.57	16.58		16.80	16.83	17.10		16.92	16.77		16.80			16.78		16.89
Existing bed level of the channel reach	15.26	15.38	15.37	15.42			15.25	15.19		15.32	15.16	15.56		15.62	15.55		15.51			15.52		15.45
Existing invert level of the structure or soffit level of bridge					15.30	15.24			15.25				15.48			15.33		15.29	15.23			
Condition: g – good f – fair p – poor					g	g			g				g			g		f	g			

Subdivisions: Symbol	p	m	c	b	a
Subdivisions: Area in ha	41	34	89	77	50
Subdivisions: Specific run-off in m³/sec/100 ha	0.133	0.133	0.133	0.133	0.133
Subdivisions: Discharge in m³/sec.	0.055	0.045	0.108	0.101	0.067
Number of the channel reach	13	10	3	2	1
Total area draining to the channel reach in ha	496	370	216	127	50
Total discharge of the subdivisions	0.810	0.492	0.274	0.168	0.067
Coefficient K_M	–	–	–	–	–
Hydraulic gradient in m/km	0.20	0.10	0.10	0.10	0.10
Side-slopes (vertical: horizontal)	1:2	1:1.5	1:1.5	1:1.5	1:1.5
Water depth in m	0.85	0.85	0.70	0.60	0.45
Bed width in m	1.40	1.35	1.10	0.80	0.60
Average velocity in m/sec.	0.30	0.22	0.18	0.17	0.12
Design water level	15.20+ 15.24	15.70 15.73	15.84	15.89	15.90
Bed level of channel reach	14.35+ 14.39	14.85 14.88	15.03 15.14	15.24 15.29	15.44 15.45
Invert level of structure	14.30+ 14.35	14.80 14.81 14.82	15.05/15.05 15.07	15.34 15.33 15.29	15.23
x)					

x) To be filled in as necessary: soffit level of bridge; width of weir; level of the crest with respect to Datum (write w for winter level and s for summer level); water level on the crest with respect to Datum etc.

In general the same rules hold when drawing up a plan for water-supply although the sequence of symbols is reversed. Thus it is necessary in designing an irrigation scheme first to determine the area of land requiring water. From the size of this area and the amount of water required in mm/day can be found the total flow of water required in m^3/sec.

APPENDIX 1 shows a water-management scheme based on a system usual in the Netherlands. APPENDIX 2 shows how the system is represented in a longitudinal section.

NOTE 2

THE CHOICE OF HYDRAULIC GRADIENT

In any design it first has to be established what drainage-depth is normally desirable.

This is determined by agricultural factors (choice of crop, soil profile etc.).

This means that in a flat area a horizontal polder level is the most desirable. In order for water to flow however, a certain gradient is required.

In the design, the design discharge is fixed for each reach of the channel and then a certain gradient and a value for the bed-roughness is assumed.

From this a certain cross-section for the conduit may be deduced.

How big should this hydraulic gradient be?

In choosing the hydraulic gradient consideration has to be given to the following:

1. the permissible velocities of flow;
2. the permissible decrease in the drainage-depth at the upstream end of the main channel;
3. the permissible increase in the drainage-depth at the outfall of the main channel;

4. the variations in the water levels;
5. the earth-moving and the area of land taken up;
6. the cross-sections of the existing channels.

1. *The permissible velocities of flow*

Proceeding from the critical velocities, the permissible hydraulic gradient decreases with increasing discharge.

In sloping areas the critical velocity can be decisive in the choice of the gradient. In flat areas it will only be in exceptional cases that consideration has to be given to the critical velocities. In particular these cases will occur when discharges are high and when the soils are non-cohesive.

2. *The permissible decrease in the drainage-depth*

The drainage-depth is usually determined for normal winter or summer conditions.

In the case of drainage, the design discharge is usually a discharge which occurs infrequently and only for short periods.

For these short periods a certain decrease in the drainage-depth is permissible. This decrease is dependent upon several factors which are not yet accurately known. There are indications that it may be allowed to reach 10-20% of the optimum drainage-depth.

It is evident that the hydraulic gradient increases with increasing water level chosen at the upstream end.

In the case of water supply a decrease occurs for longer periods but then the drainage depth is less so that in this case 10-20% can also be allowed.

In connection with the amount of earth-moving, the greatest possible gradient is desirable (see Section 5).

3. *The permissible increase in the drainage-depth*

At the outfall a certain increase is permissible in the chosen drainage-depth.

The size of this increase is dependent, among other things, upon the construction and running costs of the outfall structure (e.g. running costs of the pumping station).

In the winter the increase will hardly be determined by agricultural factors. Caution should however be exercised in peaty and sandy areas.

In cases where agricultural factors are of no importance, the increase is determined by the construction and maintenance costs of the channels compared with those of the outfall structure. A greater drainage-depth makes greater hydraulic gradients possible and this almost always leads to a smaller amount of earth-moving and thus lower construction costs (see also Section 5).

In addition to the costs of construction, maintenance costs should also be taken into consideration.

4. *The variations in the water levels*

When there is no discharge, the water surface is in theory horizontal. The amount of variation in the water levels increases with increasing value chosen for the hydraulic gradient corresponding to design flow.

In supply channels, agricultural interests are served by small variations and therefore by a low hydraulic gradient. In discharge channels, special attention must be paid to effect of variations in water level on the side-slopes.

5. *The amount of earth-moving and the area taken up*

Theoretically there exists a certain hydraulic gradient for which the amount of earth-moving is a minimum.

If the critical velocities and the permissible departures from the drainage-depth are taken as design criteria a gradient is usually arrived at which is smaller than that corresponding to the minimum amount of earth-moving.

In practice it may therefore be said that the total gradient should be chosen as great as possible. Moreover a channel with a high gradient occupies less surface area than one with low gradient.

6. *The cross-sections of the existing channels*

If an existing conduit already has a big cross-section, the design will naturally be adapted to suit. A correct choice for the gradient will help to achieve this.

Conclusions

1. To limit earth-works the highest permissible value for the gradient should usually be chosen.
2. In hilly areas this gradient is limited by the critical velocities. In flat areas the velocity of flow is hardly ever a design factor in the choice of gradient.

3. In connection with Section 1 it is advisable to proceed from the highest permissible departures from the optimum drainage-depth.
4. The permissible departures are usually absolute values so that long channels need a flatter gradient and in short channels a steeper gradient is permissible.

NOTE 3

THEORETICAL BASIS

1. Introduction

In order to satisfy the conditions of uniform flow, the forces acting on the flowing water must be in equilibrium.

Gravity supplies a component force acting in the direction of flow. If the slope of the bed and of the water surface which runs parallel is S, the component of gravity is $G \sin S \approx G.S$.

Friction acts in a direction opposite to that of flow. The frictional resistance depends on the roughness and the size of the surface over which the water flows.

The size of the surface is dependent on the shape of the cross-section and the occurrence of obstacles in the watercourse; the roughness is dependent on the smoothness of the bed and the nature of the obstacles. Here of course there is a big difference between a vegetated and non-vegetated cross-section. If vegetation is present, the leaves and stems are in fact additional surfaces over which the water sweeps. The continual variation in the vegetation makes it impossible to express this surface in the dimensions of area, so that here too the surface of the bed and banks is used but with a suitably estimated roughness coefficient.

In channels with no vegetation the bed-roughness may be equated to that of a bed covered with spheres of a certain diameter. This diameter is then the quantity in which the roughness of a non-vegetated cross-section is expressed.

The starting point for the theoretical approach to the problem is the condition for equilibrium in a 1 m long element of the flow considered.

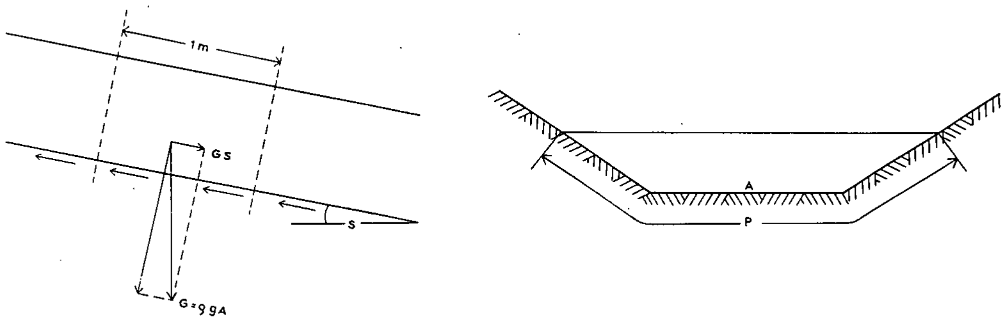


Fig. 1

From equilibrium we have $\varrho g A S = \tau P$

$$\tau = \varrho g \frac{A}{P} S = \varrho g R S$$

where ϱ = density of the water
g = acceleration due to gravity
τ = shear stress

In the following treatment the quantity $\sqrt{\tau/\varrho}$ will frequently occur. This quantity has the dimensions of a velocity and is therefore known as the shear velocity v_f. [1]).

Thus $v_f = \sqrt{\tau/\varrho} = \sqrt{gRS}$

In what follows we shall treat successively the velocity distribution in the cross-section, the formulae for discharge and average velocity, the calculation of channels with vegetation, the maximum permissible velocity in watercourses with no bed transport and the bends in a watercourse.

[1]) For an elementary description of this theory, see for example W. Kaufmann, Fluid mechanics. McGraw-Hill, 2nd. Ed., 1963.

2. The distribution of velocity over the cross-section

In laminar flow the velocity gradient dv/dz is directly proportional to the magnitude of the shear stress. The coefficient of the proportionality is the dynamic viscosity.

In turbulent flow the relation between velocity gradient and shear stress is less simple. Physical considerations lead to the relation:

$$\tau = \varrho\ (\varkappa z)^2\ (dv/dz)^2$$

So here the value of τ is dependent on the distance z from the nearest boundary and the density, ϱ, of the fluid. The viscosity is now of no consequence. The value of the number $\varkappa$ does not depend on the properties of the fluid; it is approximately equal to 0.4.

By integration an expression can be found for the velocity distribution over the cross-section. The calculation proceeds as follows:

$$\frac{dv}{dz} = \frac{1}{\varkappa z} \sqrt{\frac{\tau}{\varrho}} = \frac{v_f}{\varkappa z}$$

$$v = \frac{v_f}{\varkappa}\ ln\ z + constant$$

The velocity distribution in the section appears logarithmic. In experiments too, a logarithmic curve is always found, but it is also found that this curve does not intercept the line $v = o$ at the boundary but at a small distance z_0 from the boundary.

In the immediate proximity of the bed and banks the velocity distribution is no longer logarithmic. There, it depends on the nature of the surface of bed and banks.

By substituting $z = z_0$ for $v = o$, the integration constant can be eliminated from the last equation.

The velocity distribution then becomes: $v_z = \frac{v_f}{\varkappa}\ ln\ \frac{z}{z_0}$

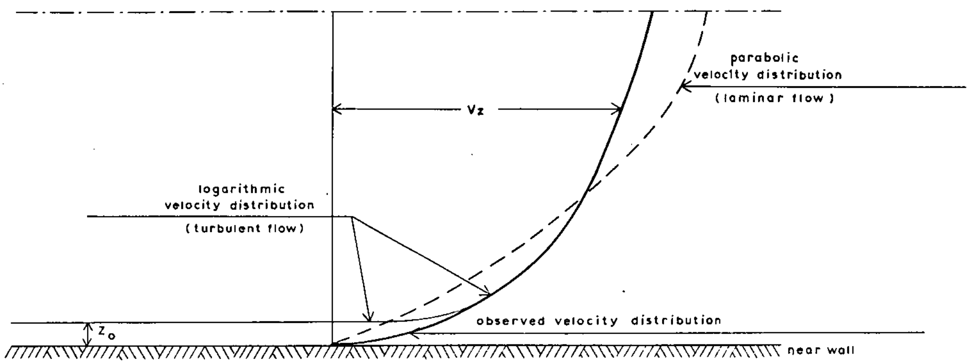


Fig. 2

3. FORMULAE FOR DISCHARGE AND AVERAGE VELOCITY

By integration of the latter formulae over the cross-section of a channel a formula for the magnitude of the discharge is obtained. In this way a different formula is obtained for every shape of cross-section. From the above diagram it appears that in turbulent flow the velocity is more evenly distributed over the cross-section than in laminar flow. It might therefore be expected that the influence of the shape of the cross-section on the magnitude of the discharge is less in the turbulent condition.

In order to verify this, the integration is carried out for the two most extreme possibilities, that is for the very wide channel with uniform depth and for the closed circular pipe.

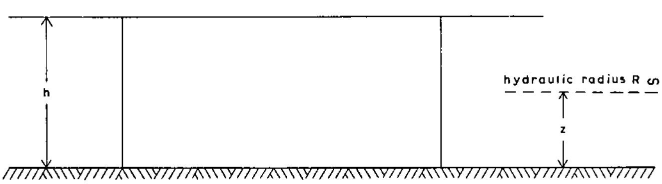


Fig. 3

a. *Very wide rectangular channel*

Consider a strip of unit width:

$$q = \int_0^h \frac{v_f}{\varkappa} \, ln \, \frac{z}{z_0} \, dz = \frac{v_f}{\varkappa} \left[h \, ln \, \frac{h}{z_0} - h \right] =$$

$$= \frac{v_f \, h}{\varkappa} \, ln \, \frac{h}{e \, z_0}$$

$$v = \frac{q}{h} = \frac{v_f}{\varkappa} \, ln \, \frac{h}{e z_0} = \frac{v_f}{\varkappa} \, ln \, \frac{0.37 \, R}{z_0} =$$

$$= \frac{\sqrt{g \, RS}}{\varkappa} \, ln \, \frac{0.37 \, R}{z_0} =$$

$$= \frac{\sqrt{g}}{\varkappa} \, 2.3 \log \left(\frac{0.37 \, R}{z_0} \right) \sqrt{RS} =$$

$$= 18 \log \left(\frac{0.37 \, R}{z_0} \right) \sqrt{RS}$$

So here we have the well known formula of De Chézy, $v = C \sqrt{RS}$, where $C = 18 \log \dfrac{0.37 \, R}{z_0}$

b. *Conduit with regular polygonal cross-section*

The circular pipe also comes under this heading, since a circle can be considered as a polygon with an infinite number of sides.

Consider one of the isosceles triangles of which the polygon consists:

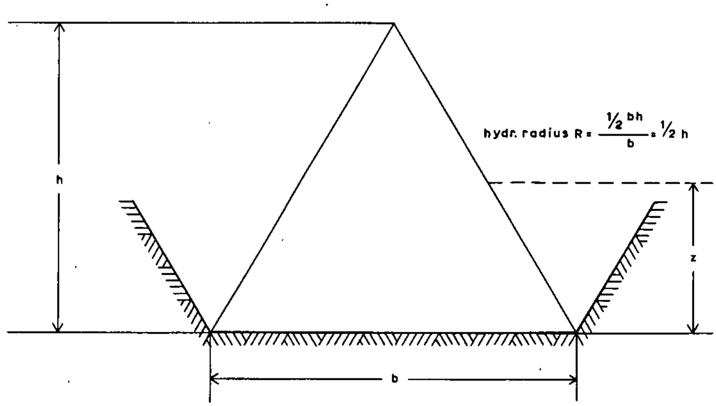


Fig. 4

$$Q = \int_o^h \frac{v_f}{\varkappa} \ln \frac{z}{z_o} \; \frac{h - z}{h} \, bdz =$$

$$= \frac{b\, v_f}{\varkappa} \left[\int_o^h \ln \frac{z}{z_o} \, dz - \frac{1}{h} \int_o^h z \ln \frac{z}{z_o} \, dz \right] =$$

$$= \frac{b\, v_f}{\varkappa} \left[h \left(\ln \frac{h}{z_o} - 1 \right) - 1/2\, h \left(\ln \frac{h}{z_o} - 1/2 \right) \right] =$$

$$= \frac{h\, b\, v_f}{2\varkappa} \left[\ln \frac{h}{z_o} - 3/2 \right] = \frac{h\, b\, v_f}{2\varkappa} \ln \frac{0.22\, h}{z_o}$$

$$v = \frac{Q}{1/2\, h\, b} = \frac{v_f}{\varkappa} \ln \frac{0.22\, h}{z_o} = \frac{v_f}{\varkappa} \ln \frac{0.44\, R}{z_o} =$$

$$= 2.3 \frac{\sqrt{g}}{\varkappa} \log \frac{0.44\, R}{z_o} \sqrt{RS} =$$

$$= 18 \log \frac{0.44\, R}{z_o} \sqrt{RS}$$

Here too therefore we find the formula of De Chézy:

$$v = C \sqrt{RS}$$

where now
$$C = 18 \log \frac{0.44\, R}{z_o}$$

Hence it appears that the shape of the cross-section has little influence on the value of C. The shapes of cross-sections occurring in practice lie in between the extremes calculated. When one formula is required which holds with reasonable accuracy for all types of cross-section then the best way is to take an average which lies between the two. The following is such a formula:

$$C = 18 \log \frac{0.40\, R}{z_o}$$

From experiments it is apparent that z_0 depends both on the size of the protuberances on the bed a and on the thickness of the laminar boundary layer δ.

Physical considerations give for the thickness of the boundary layer:

$$\delta = \frac{12\nu}{v_f} = \frac{12\nu}{\sqrt{g\,RS}}$$

According to White & Colebrook, the magnitude of z_0 is approximately equal to:

$$\frac{1}{15}\,a + \frac{1}{105}\,\delta$$

When this is substituted, we obtain the simplified formula:

$$C = 18 \log \frac{6\,R}{a + \delta/7}$$

Where $\delta/7$ is small in relation to a, the bed is described as relatively rough and the formula becomes:

$$C = 18 \log \frac{6\,R}{a}.$$

Where a is small in relation to $1/7\,\delta$, the bed is described as relatively smooth and the formula becomes:

$$C = 18 \log \frac{42\,R}{\delta}$$

In the transitional region, the logarithmic formula should be used in its complete form. The values of C can be read off from the accompanying Graph 1.

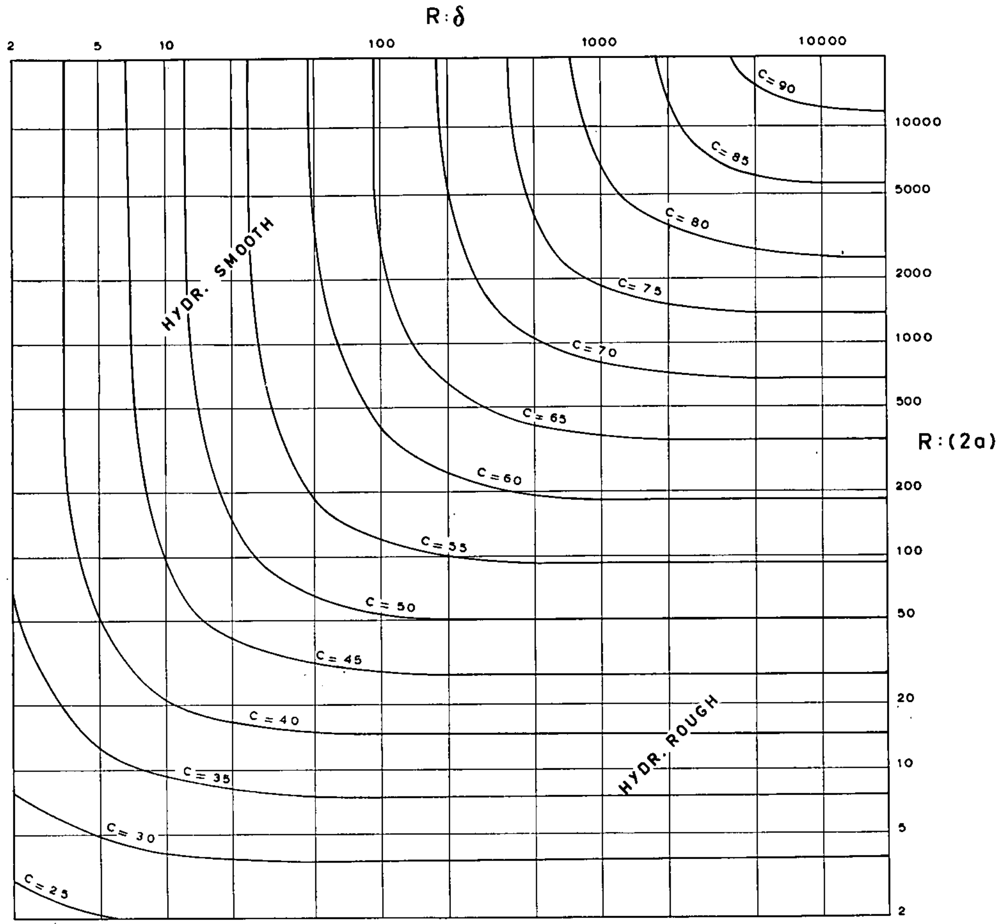
4. Calculation of vegetated channels

It was already noted in the introduction that in vegetated channels importance must be attached to phenomena quite different from that of friction along the bed particles.

The vegetation influences the roughness of the side-slopes and the cross-sectional area of flow to such an extent that exact calculations can hardly be put in hand.

The conclusions which are founded on a theoretical basis should therefore be considered very critically.

Every theory which is propounded for vegetated channels seems to founder on the irregular nature of the vegetation. Within a limited area the milieu of the plants is usually uniform. The ratio between the plant dimensions and the water depth


R : δ
2
5
10
100
1000
10000
C = 90
C = 85
C = 80
C = 75
C = 70
C = 65
C = 60
C = 55
C = 50
C = 45
C = 40
C = 35
C = 30
C = 25
HYDR. SMOOTH
HYDR. ROUGH
10000
5000
2000
1000
500
R : (2a)
200
100
50
20
10
5
2

Graph 1

will vary little for different watercourses in such an area, provided that these are relatively shallow.

The bed-roughness a in the logarithmic formula is directly proportional to the dimensions of the plant and in watercourses with varying depth it will thus have different values. The observations which have been carried out show that the ratio between the dimensions of the plants and the dimensions of the watercourse can be characterised better by the coefficient (k_M) in the formula of Manning mentioned below.

The observations mentioned were performed in several small vegetated channels under the auspices of the Civil Engineering Department of the Agricultural University at Wageningen, The Netherlands. The results of these measurements clearly showed that the relation between the vegetation and the theoretically assumed particle diameter in the logarithmic formula was scarcely tenable. In view of these measurements and numerous investigations abroad, for vegetated channels we have to be content with the empirical formula of Manning.

Moreover in practice it appears necessary that nomographs should be available for the calculation of open watercourses. In such nomographs it is required that the discharge may be read off directly from the bed-slope and the dimensions of the channel, but it appears that it is impossible to construct such a graph for the logarithmic formula. One would have to work with two nomographs simultaneously.

For the use of the exponential formula of Manning it is found that the production of nomographs is quite straightforward. This is another reason for employing this formula.

In view of the above considerations the working party chose the Manning formula for the calculation of vegetated conduits:

$$v = k_M \, R^{2/3} \, S^{1/2}$$

so that the C in De Chézy's formula ($v = C\sqrt{RS}$) becomes:

$$C = k_M \, R^{1/6}$$

For further discussion of the value of k_M and the measurements performed, reference should be made to Section 4.4. in the Code.

5. Limitation of the maximum permissible water velocity in a channel with no bed transport

a. *Very wide channel with loose sand bed*

The particles lying on the bed must remain at rest. This is the case when the shear stress exerted by the water on the bed is less than the shear stress which is needed to put the particles in motion.

The shear stress exerted by the water on the bed is:

$$\tau = \varrho gRS = \frac{\varrho g}{C^2} v^2$$

The force necessary to move one spherical particle of diameter d is:

$$F = f \frac{\pi d^3}{6} (\varrho_z - \varrho)\, g$$

where f is a 'friction coefficient' and ϱ_z is the density of the material. Let p be the portion of the bed surface occupied by loose particles. Then the number of loose particles per unit area of the surface is:

$$\frac{4p}{\pi d^2}$$

The force needed per unit area of the bed surface to bring particles into motion is then

$$\tau_c = \frac{2}{3} fp\, (\varrho_z - \varrho)\, gd$$

The magnitude of the constant $2/3\, fp$ may be derived from the many measurements which have been carried out. According to LANE[1]) a value may be assumed (allowing for a coefficient of uncertainty) from 0.05 to 0.065 or an average of 0.06.

The formula then becomes:

$$\tau_c = 0.06\, (\varrho_z - \varrho)\, gd$$

The diameter d of the particles corresponds to a sieve aperture which retains 25 % of the material by weight.

b. *Channel with trapezoidal cross-section in coarse loose sand*

For the very wide channel we were able to suppose that the shear stress was uniformly distributed over the whole width of the bed.

[1]) E. W. LANE, Proceedings American Society Civil Engineers, Sept. 1953, Vol. 79, Separate no. 280 (Fig. 3).

For a narrower trapezium-shaped cross-section this is no longer the case. O. J. OLSEN and Q. L. FLOREY, collaborators with LANE, investigated the distribution of the shear stresses over the bed and side-slopes of trapezoidal cross-sections. Their results can be represented graphically in a simple manner.

The largest shear stresses along bed and side-slopes are given by:

$$\tau = \Gamma g d S$$

where d is the water depth and Γ is a constant which for a certain shape of cross-section may be found from Graph 2.

The force needed to move the particles along the bed again follows from the formula for τ_c. In loose coarse material the force needed to move the particles along the side-slopes is not determined only by the shear stress exerted by the flowing water. In addition, the component of gravity acting in the plane of the side-slopes now plays a part. Depending on the gradient α of the side-slopes and on the angle of internal friction of the soil (Θ), the particle on the side-slope will be set in motion by a smaller shear stress than would be needed for a bed-particle.

For the sake of simplicity, the gradient of the side-slopes is allowed for by introducing a coefficient k. This is equal to the ratio between the shear stress necessary to set in motion the particles of the side-slope and the shear stress necessary to move the bed-particles.

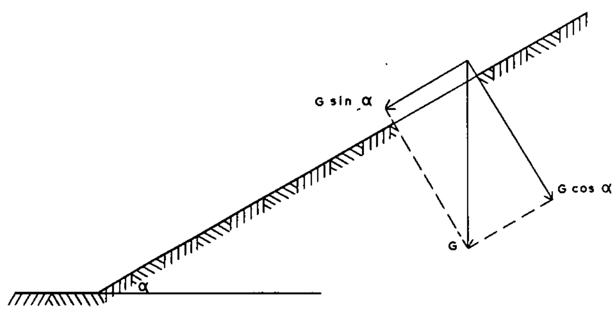


Fig. 5

The critical shear stress for the side-slopes thus becomes:

$$\tau_c = 0.06\ k\ (\varrho_z - \varrho)\ gd$$

In order to calculate k we proceed from an angle of internal friction Θ and side-slopes inclined at an angle α. The friction along the bed which is necessary to set the particle in motion is then:

$$F_b = G\ tg\ \Theta$$

On the side-slope is exerted a total frictional force: $F_t = G \cos \alpha \, tg\, \Theta$; this can be thought of as resolved into a gravity component which makes the particle roll downwards $F_g = G \sin \alpha$ and a frictional force F in the direction of flow which is determined by the formula:

$$F = \sqrt{F_t^2 - F_g^2}$$

$$F = k\, F_b = \sqrt{G^2 \cos^2 \alpha \, tg^2\, \Theta - G^2 \sin^2 \alpha}$$

$$k\, G\, tg\, \Theta = \sqrt{G^2 \cos^2 \alpha \, tg^2\, \Theta - G^2 \sin^2 \alpha}$$

$$k = \sqrt{\frac{\cos^2 \alpha \, tg^2 \Theta}{tg^2\, \Theta} - \frac{\sin^2 \alpha}{tg^2\, \Theta}}$$

$$k = \cos \alpha \sqrt{1 - \frac{tg^2\, \alpha}{tg^2\, \Theta}}$$

c. *Channel in cohesive soil*

The shear stress exerted by the water on the bed and side-slopes can for this case again be calculated from the formula $\tau = \Gamma g d S$. The graphs of OLSEN and FLOREY can prove very serviceable here (see Graph 2).

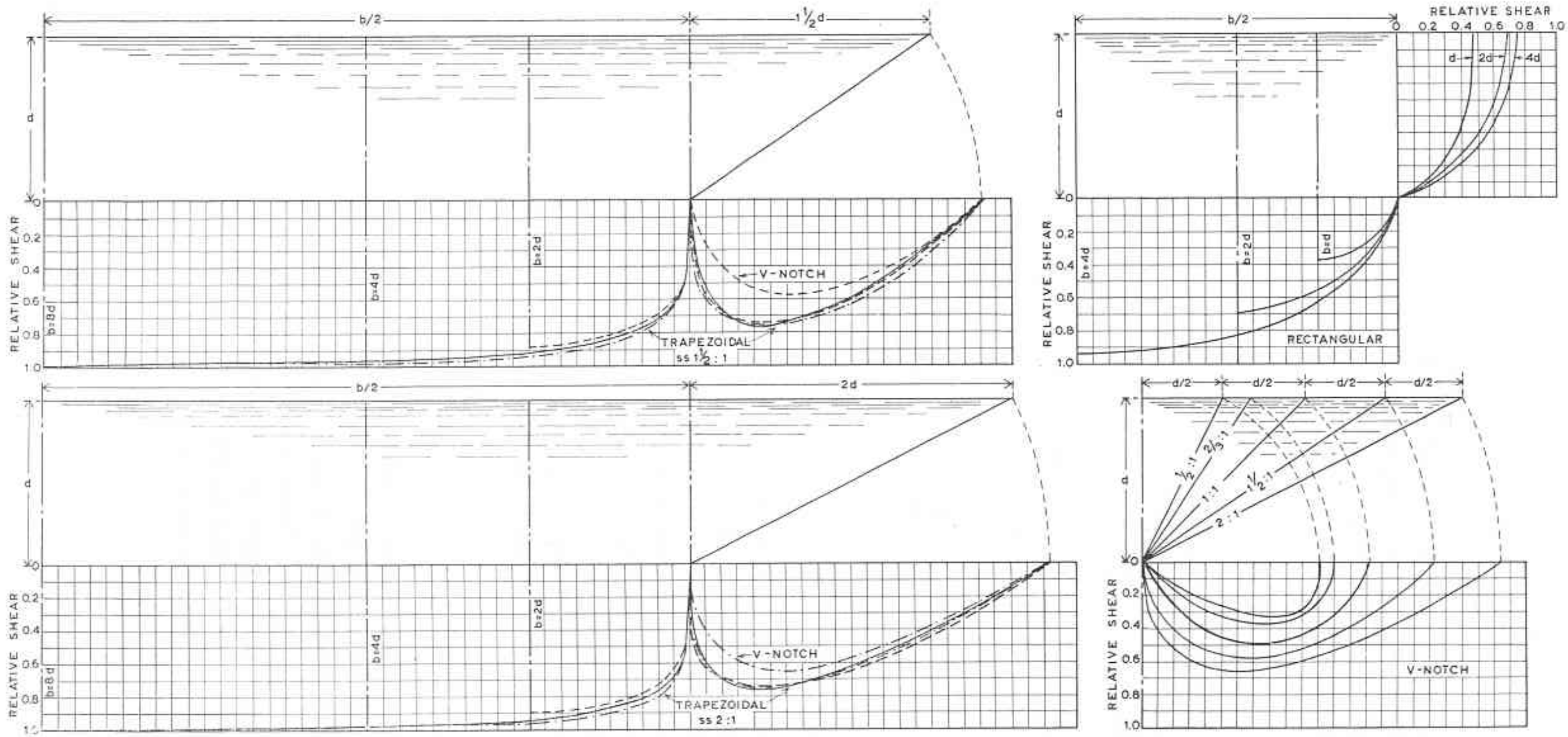
The critical shear stress which is necessary to set in motion the material of the sides and bed is however no longer given by the simple formula for τ_c. In this case we have to refer to empirical values for the critical shear stress.

Owing to the greater cohesion shown by the soil, it is here no longer necessary to introduce an extra coefficient to allow for the side-slopes.

d. *Channel in fine sandy bed*

This case stands more or less intermediate between the cases mentioned under *b* and *c*. If the channel has just been put into service case *b* will occur.

If water has previously been flowing through the channel, it is quite probable that the fine particles have already been bound together by transported silt. In this event, case *c* is present. Seepage out of the channel works conducive to this bonding whilst seepage towards the channel hinders it. Especially in this case a soil mechanics investigation will often be necessary.



Graph 2. Relative boundary shear distribution in channels of unit depth.
(Structural Laboratory Report, No. SP-34. Department of the Interior, Bureau of Reclamation, U.S.A.)

6. Bends in watercourses

Where bends are present in open watercourses, in addition to the velocity in the direction of flow, there is also a component perpendicular to this direction.

The bed friction will therefore also have a component perpendicular to the direction of flow.

In the bend therefore, the total shear stress will be greater than in a straight reach of the flow.

In the design of bends in open watercourses one of the following two principles may serve as basis:

a. In the bend a somewhat greater bed-shear may be accepted than in a straight part of the watercourse.
 The same cross-section is preserved in the bend as in a straight reach. The shear stress therefore increases and the consequent greater likelihood of material transport is tolerated to a certain degree.

b. The cross-section is increased in the bend by an amount such that the bed-shear in the bend is no greater than in a straight section of the watercourse.

For a further theoretical treatment of the above principles reference should be made to the article by Ir. L. van Bendegom in 'De Ingenieur' of 24 Jan. 1947 entitled: 'Enige beschouwingen over riviermorphologie en rivierverbetering' (Some considerations on river morphology and river improvement).

Proceeding from the equations derived in the above article, the cases a and b may be treated more fully.

a. If the cross-section in the bend is not increased, a minimum radius has to be calculated such that a permissible augmented shear stress is not exceeded.

For this we use the following symbols:

F = total shear stress
F_x = shear stress in the straight reach of channel
B = F/F_x
C = coefficient in De Chézy's formula
d = water depth
r = radius of the bend
g = acceleration due to gravity
p = percentage by which the shear stress increases in the bend

According to the treatment of VAN BENDEGOM:

$$B = \sqrt{1 + \frac{0.039^2\, C^4\, d^2}{r^2\, g^2}}$$

By equating B with $1 + p$ we get:

$$r = \frac{0.039\, C^2\, d}{g} \quad \frac{1}{\sqrt{p^2 + 2p}}$$

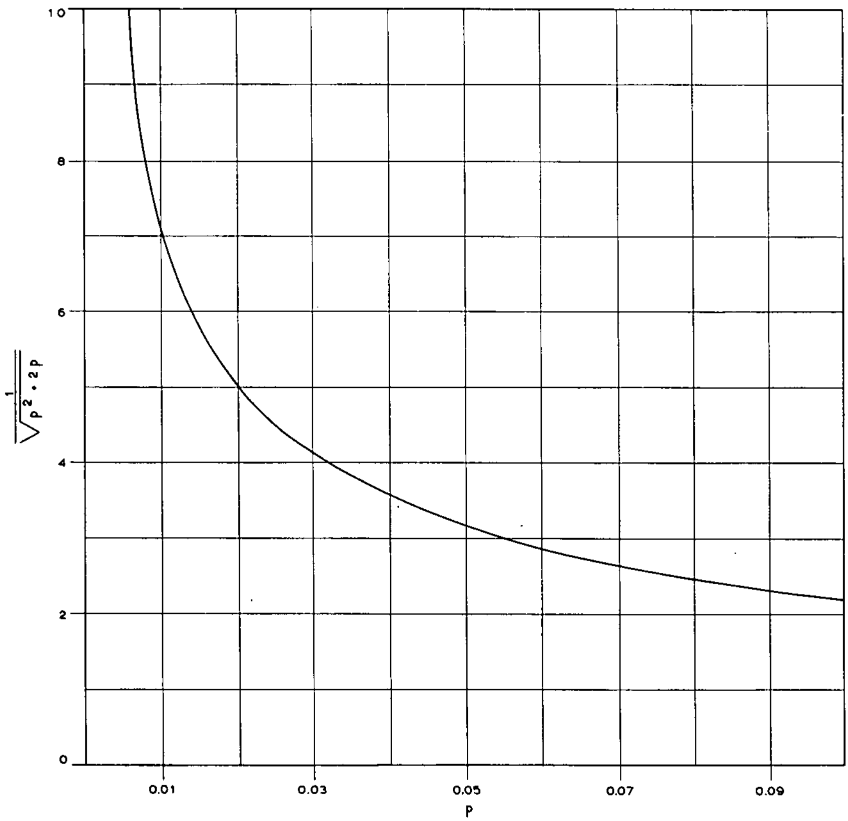


Fig. 6

From Fig. 6 it is seen that if the bed shear in the bend may exceed that in a straight reach by only a small amount, the minimum radius becomes quite large

(if $p \to o$, then $\dfrac{1}{\sqrt{p^2 + 2p}} \to \infty$).

Example: $d = 2$ m, $C = 40$ $m^{1/2}$/sec. A permissible shear stress increase of 1% ($p = 0.01$) gives a value of $r \approx 88$ m as a minimum.

b. If the shear stress is not allowed to increase in the bend, the increase in cross-section is calculated as follows:

The average velocity in the bend is v'.

If we assume that the increase in cross-section at the bend is not large, this allows the same value of C to be kept and then (τ is proportional to v^2):

$$\frac{gv'^2}{C^2} = \frac{gv^2}{BC^2}$$

or

$$v' = \frac{v}{\sqrt{B}}$$

Thus if care is taken that in the increased cross-section the average velocity is equal to or less than the normal average velocity divided by $\sqrt{B}$, then the bed-shear in the bend will not exceed that in the normal straight reach.

This is true only in so far as the increase in section is not so great that the value of C essentially alters.

In this treatment, wide watercourses have been assumed, so that R has been equated to d.

Attention must always be paid to the transition zone from curved to straight channel. Owing to the current, serious erosion of the bed and side-slopes can occur, so that extra provisions should always be considered here.

NOTE 4

MINIMUM CROSS-SECTIONS OF DITCHES AND SMALL WATERCOURSES

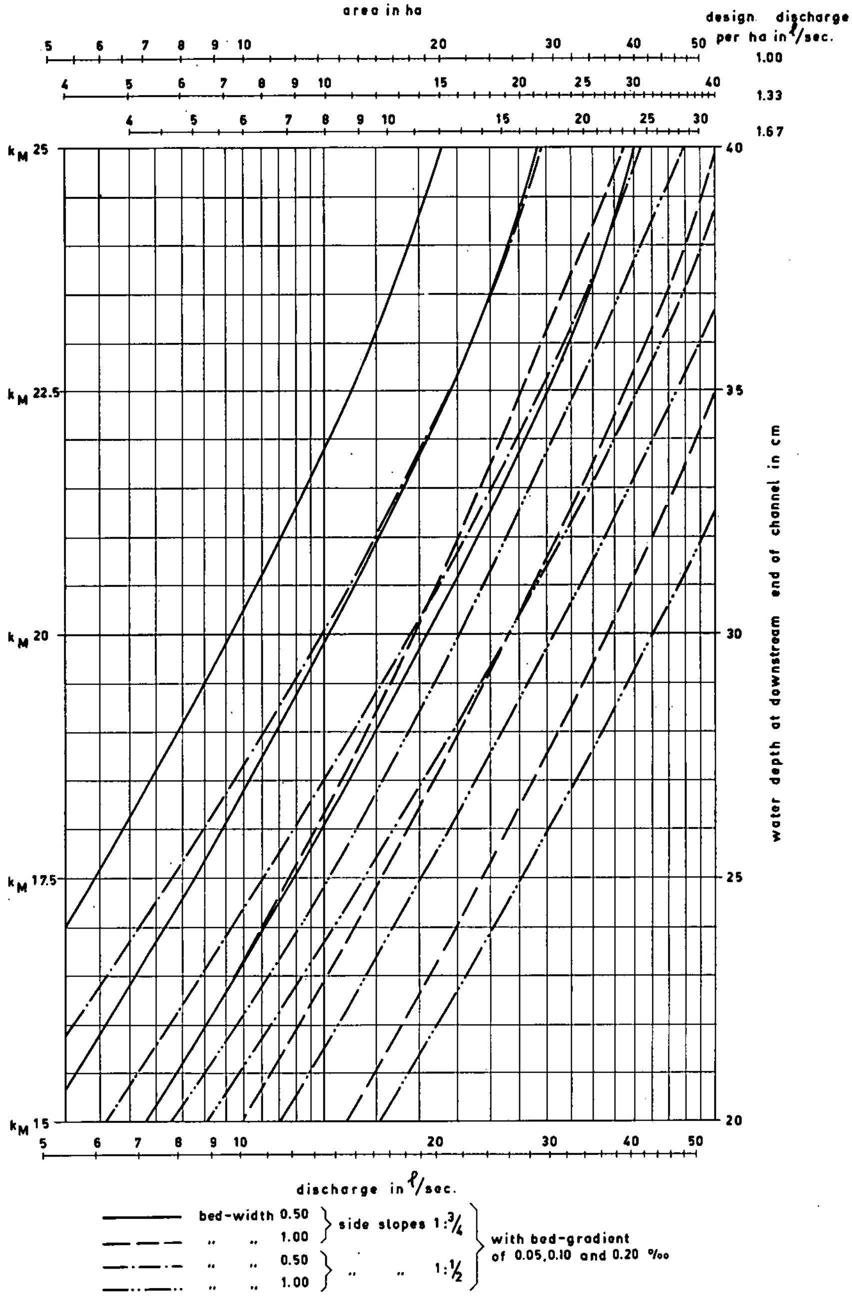
In designing drainage and irrigation schemes, not only does consideration have to be made of the channel dimensions which have been calculated, but also of those which have been determined on practical grounds. All parts of a certain area should have a connection with the main watercourse such that the discharge or supply of water will be satisfactory.

The dimensions of minimum cross-sections should be made dependent on the method of construction, weed-growth and consequent systems of cleansing. Once these dimensions have been fixed, it can be found what area may be drained or supplied by these minimum sections.

The ditches in catchments smaller than the area found do not then need to be calculated. The ditches in catchments bigger than this area should be calculated.

Ditches with no discharging function are not considered here because factors other than drainage or supply then play a part.

For circumstances in The Netherlands, Graph 3 shows the relation between the dimensions of minimum ditches and the drainage area.



Graph 3. Relation between the dimensions of small ditches and the area to be drained

NOTE 5

MEASUREMENTS FOR DETERMINING THE COEFFICIENT OF BED-ROUGHNESS

If, in a straight channel, a steady, uniform condition of flow is present, then by measurement of the gradient, the discharge and the wetted section, the C can be calculated and hence the coefficients of bed roughness which occur in the various formulae which exist for C.

Such measurements and calculations are available from various investigators e.g. BAZIN, RAMSER, SCOBEY, REE and PALMER. On this subject the following remarks may suffice.

The investigations of BAZIN[1] were chiefly related to non-vegetated channels; nevertheless his formula used to be employed quite generally in the Netherlands.

The great number of measurements performed under the direction of RAMSER[2]

[1]) H. BAZIN, Étude d'une nouvelle formule pour calculer le débit des canaux découverts. Annales des Ponts et Chaussées 1897.

[2]) C. E. RAMSER, Flow of water in drainage channels. Technical Bulletin No. 129, November 1929. United States Department of Agriculture.

still forms the most important source of information on the subject of the bed-roughness coefficient. These measurements, totalling something more than 560, were carried out in vegetated channels and by means of photographs and a description the conditions in each channel were defined.

Similarly, very many data can be taken from the 175 measurements in vegetated channels which were performed under the direction of SCOBEY[1]).

The tables of k_M values found in the various hydraulics books are founded for a large part on the above mentioned measurements of RAMSER and SCOBEY. All these k_M values must be regarded as a total resistance coefficient, since both investigators expressed in this coefficient not only the resistance due to bed-roughness but also all other resistances in the channels.

Further, the measurements of REE and PALMER[2]) may be mentioned, conducted in shallow ditches in which the bed and side-slopes were covered with different types of grass. These experiments were aimed at acquiring knowledge of the degree of erosion, the permissible velocity and the resistance for each separate type of grass.

The great difficulty in such measurements is to describe objectively the total resistance of the ditch concerned, which often varies from place to place.

In various countries investigations are still being performed into this question. As examples, the publications of Cowan and Ree may be mentioned.

COWAN[3]) attempted to resolve the total resistance into 6 components.

REE[4]) proposed that the height of the vegetation relative to the water depth should be taken into consideration.

There is no doubt that in the future many publications on this subject may still be expected.

In the period 1953-1956 the departments of Civil Engineering and Agronomy of the Agricultural University, Wageningen have conducted, under the leadership of J.M. GEENSE, a number of measurements in open earth channels. The aim of these measurements was to determine the total resistance in ditches with

[1]) F. C. SCOBEY, Flow of water in irrigation and similar canals. Technical Bulletin No. 652, February 1939. United States Department of Agriculture.

[2]) W. O. REE and V. J. PALMER, Flow of water in channels protected by vegetative linings. Technical Bulletin No. 967, February 1949. United States Department of Agriculture, Soil Conservation Service.

[3]) W. L. COWAN, Estimating hydraulic roughness coefficients. Agricultural Engineering, July 1956.

[4]) W. O. REE, Hydraulic characteristics of vegetation for vegetated waterways. Agricultural Engineering, April 1949.

various degrees of vegetation and the investigation of the increase in this resistance when a winter-vegetation changes to a summer-vegetation.

In total 49 small channels were measured, all with a hydraulic radius less than 0.4 m. The measurements were carried out principally during winter so that usually no heavy vegetation was present in the ditches. The ditches were straight with approximately parallel flow and usually had a fairly constant cross-section; the length of the experimental reach was as a rule 200 m. The discharge was measured at two points, the wetted section at three or more points, and the hydraulic gradient was determined every 20 m by double levelling of the water surface. Two photographs were taken of every ditch and all the factors which together give rise to the total resistance were noted. The most important measuring instruments were: a levelling instrument, a staff fitted with a water-level needle, a wooden and later an aluminium gauging bridge. The discharge was measured with the small Ott current meter.

From these data, C was determined for each channel and hence k_M, a and α were calculated using the formula of Manning, the logarithmic formula and the formula of Bazin.

In addition, the Department of the Waterstaat concerned with stream gauging supplied the results of 17 measurements for inspection by the working party. These were measurements in various small streams in The Netherlands with a hydraulic radius between 0.12 and 0.48 m, depths of water which mainly lay between 0.15 and 0.55 m, and the width of the water surface in the various streams (Geul, Gulp, Oostrumse Beek and Rode Beek) almost always lay between 3 and 8 m.

In general the k_M values recommended in the Code are based on the measurements of the Agricultural University; for a summary of the results of these measurements reference may be made to Section 4.4.3. of the Code.

From the measurements of the Waterstaat in the Geul and the Gulp, average k_M values were found which usually lay between $k_M = 22$ and $k_M = 38$. These however apply to very shallow cross-sections which were not constant, a winding course and bed and banks covered with very coarse gravel. Although the sections were clean, the above-mentioned factors naturally resulted in a fairly high resistance. In the Oostrumse Beek, as a result of scour channels and the very shallow cross-section, average k_M values were found of between 21 and 55.5.

Much less variation in resistance was met with in the Rode Beek and the Bornse Beek, entirely clean channels with a constant cross-section and with banks which consist wholly or partly of concrete. Here a values were determined which lay between 1.25 and 4 mm.

RAMSER recommended $k_M = 25$ for small channels with a fertile bed which leads to rapid growth of vegetation, with a very small dry-water flow and with annual cleansing of the channels. The Code gives the same value for k_M for winter-discharge in small channels with a fertile bed.

In irrigation channels the vegetation can increase very sharply in a short time; in an irrigation channel in the North East Polder 10 days after cleansing k_M had already fallen to 18.3.

In the accompanying photographs and descriptions, the results of a few measurements are reproduced; in the given values for a, k_M, and C are included all the resistances which occurred in the channels concerned.

Extending the number of measurements in small channels further has little to commend it. However, it is urgently required that a better insight is gained into the total resistance of medium and large channels in the Netherlands.

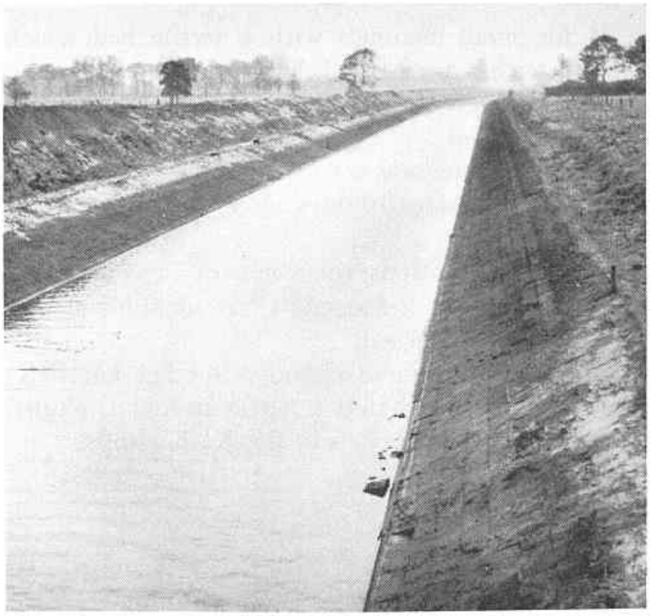


Fig. 7

$a = 0.0023$ m
$k_M = 45 \text{ m}^{1/3}/\text{sec}$
$C = 42 \text{ m}^{1/2}/\text{sec}$

Cross-section: completely regular and constant.
Vegetation: none. Side-slopes covered with concreter slabs. On bed a thin layer of mud. Extremely clean cross-section.
Soil type: humous sand.

Fig. 8

a = 0.018 m
k_M = 42 m $^{1/3}$/sec
C = 28.9 m $^{1/2}$/sec

Cross-section: regular and constant.
Vegetation: almost absent.
Very clean cross-section.
Soil type: sand.

Fig. 9

a = 0.06 m
k_M = 34 m $^{1/3}$/sec
C = 27.8 m $^{1/2}$/sec

Cross-section: regular and constant
Vegetation: bed and side-slopes here and there covered with algae.
Clean cross-section.
Soil type: sand.

Fig. 10

$a = 0.10$ m
$k_M = 24$ m $^{1/3}$/sec
$C = 16.8$ m $^{1/2}$/sec

Cross-section: approximately regular and constant.
Vegetation: moderate.
soil type: sand

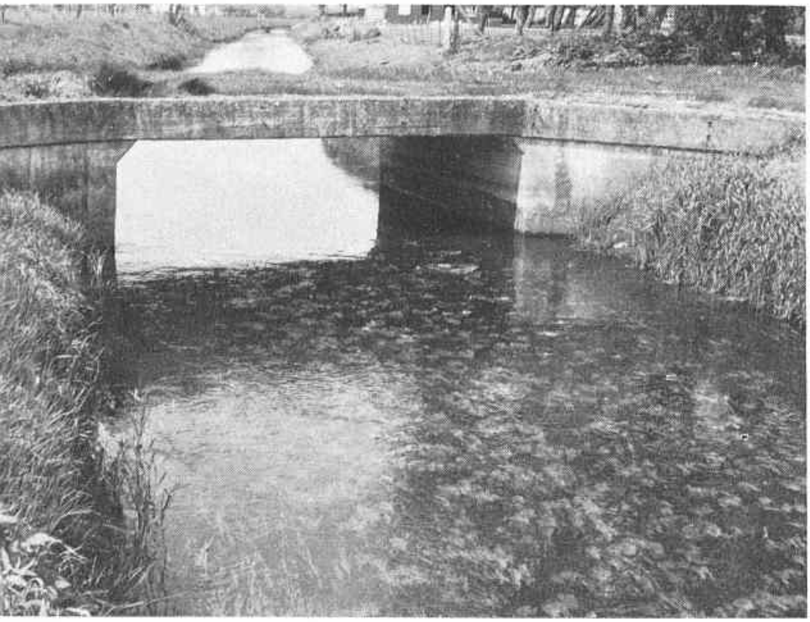


Fig. 11

$a = 0.25$ m
$k_M = 16$ m $^{1/3}$/sec
$C = 11.85$ m $^{1/2}$/sec

Cross-section: semi-irregular.
Vegetation: moderate to heavy.
Soil type: sand.

Fig. 12

$a = 0.40$ m
$k_M = 8$ m $^{1/3}$/sec
$C = 5.8$ m $^{1/2}$/sec

Cross-section: irregular.
Vegetation: very heavy.
Soil type: sand.

NOTE 6

THE INFLUENCE OF THE CHOICE OF THE COEFFICIENT OF BED-ROUGHNESS ON THE DESIGN AND THE WATER LEVELS SUBSEQUENTLY OCCURRING

In Manning's formula: $v = k_M R^{2/3} S^{1/2}$, which is used for the calculation of vegetated channels, the bed-roughness coefficient k_M occurs which represents the resistance which the flow experiences in the channel as a result of the roughness of bed and banks. It should be noted here that in the calculation of actual channels, the value chosen for k_M expresses not only the resistance due to the rugosity of the bed but also numerous other resistances due to differences between successive cross-sections, grass hanging in the water, reed growth (grid resistance) etc. In such cases therefore, k_M must be regarded more as a coefficient of total resistance.

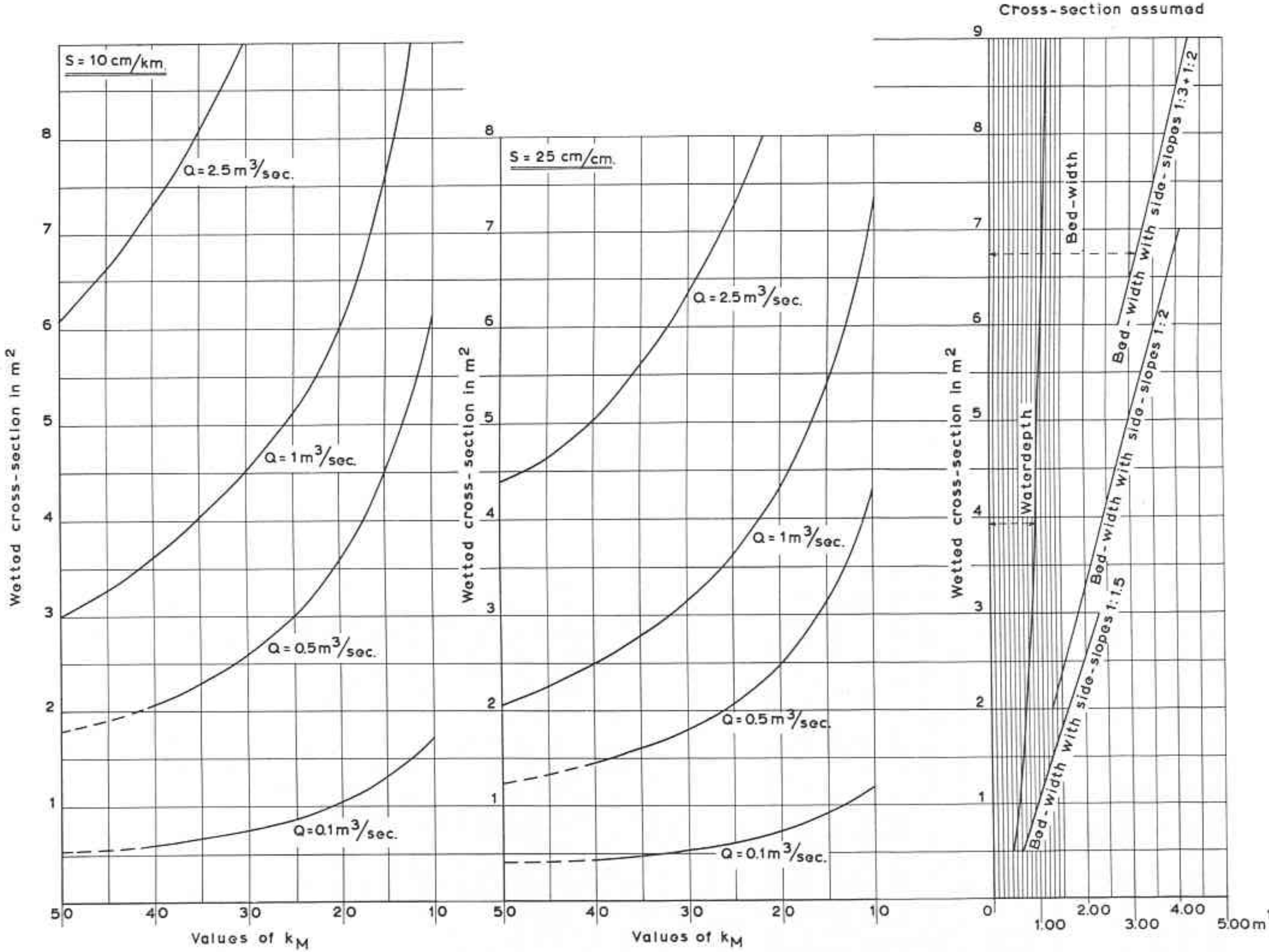
If Manning's formula is written in the form $Q/k_M = AR^{2/3} S^{1/2}$, it is seen that in a channel to which a certain gradient S has to be assigned, the assumed value of k_M exerts an influence on the design wetted cross-section equivalent to that exerted by the assumed discharge. In other words, if k_M is assumed $n \times$ too small a wetted cross-section is designed which is the same size as it would have been if the design discharge had been assumed $n \times$ too big.

Graph no. 4 shows the cross-sectional area of flow with various k_M values for the usual dimensions of various cross-sections and with gradients of 10 cm/km and 25 cm/km.

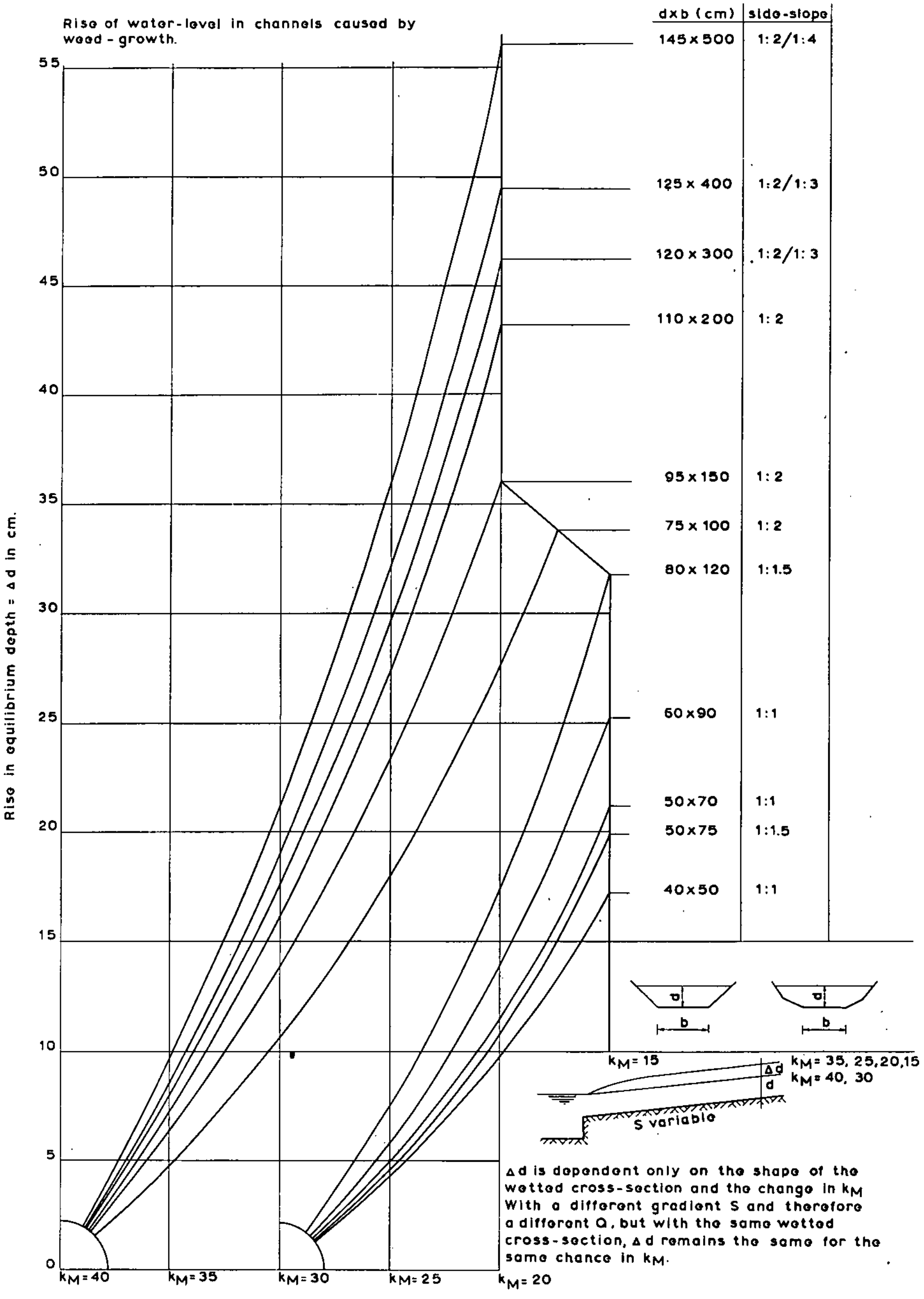
Graph no. 5 shows the rise in water level (equilibrium depth) for various cross-sections if the actual value of k_M subsequently falls below the assumed design values, which in this graph have been put at $k_M = 40$ for large sections and $k_M = 30$ for smaller sections. For example if a channel with a water depth of 0.80 m, a bed-width of 1.20 m, side-slopes 1:1.5 is assumed to have $k_M = 30$ in calculation, then Graph 5 indicates that the water level in the channel will rise 0.17 m if, owing to weeds etc., k_M has fallen to 20.

It can be proved that this rise is independent of the gradient of the channel.

Corresponding graphs can be drawn in which the k_M-scale is replaced by a scale for the design discharge.



Graph 4. Relation between A and k_M for various discharges and with S = 10 cm/km and S = 25 cm/km, determined by Manning's formula $V = k_M R^{2/3} S^{1/2}$


Rise of water-level in channels caused by weed-growth.
dxb (cm)
side-slope
145x500
1:2/1:4
125x400
1:2/1:3
120x300
1:2/1:3
110x200
1:2
95x150
1:2
75x100
1:2
80x120
1:1.5
60x90
1:1
50x70
1:1
50x75
1:1.5
40x50
1:1
Rise in equilibrium depth = Δd in cm.
0
5
10
15
20
25
30
35
40
45
50
55
kM=40
kM=35
kM=30
kM=25
kM=20
kM=15
kM=35, 25, 20, 15
kM=40, 30
Δd
d
b
S variable
Δd is dependent only on the shape of the wetted cross-section and the change in kM
With a different gradient S and therefore a different Q, but with the same wetted cross-section, Δd remains the same for the same chance in kM.

Graph 5

NOTE 7

CALCULATION FOR THE CONSTRUCTION OF THE BACKWATER CURVE FOR NON-UNIFORM FLOW

Differentiation is made between:

A. elevation of the normal flood-profile by an obstruction in the channel (e.g. culvert, weir);
B. lowering of the normal flood-profile by reduction of the resistance (e.g. outfall into a reservoir or watercourse with a lower water level.

A. Elevation of the normal flood-profile, known as the backwater curve

Fig. 13 shows a schematic example in which the line a-b would be the normal flood-profile if no weir is present (uniform flow which is defined by the normal depth y_n.).

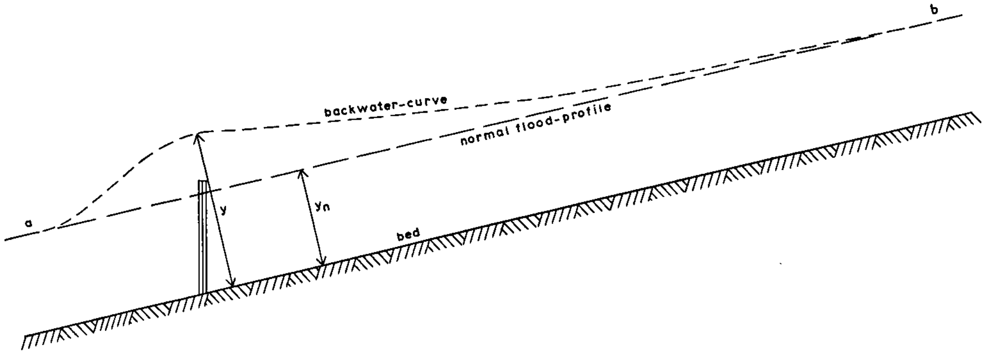


Fig. 13

The following calculation of prismatic watercourses has been performed according to the method given by BORIS A. BAKHMETEFF, for the theoretical basis reference should be made to this work[1]).

1. The data needed for the calculation with the usual symbols are:

   the discharge Q in m³/sec
   the bed-slope S
   the bed width b in m
   the gradient of the side-slopes (1 vertical to p horizontal)
   the Manning coefficient for bed-roughness k_M
   the width of the water surface W in m.

Initially a number of water depths are assumed, starting out with the water depth y_0 at the weir, see Fig. 14.

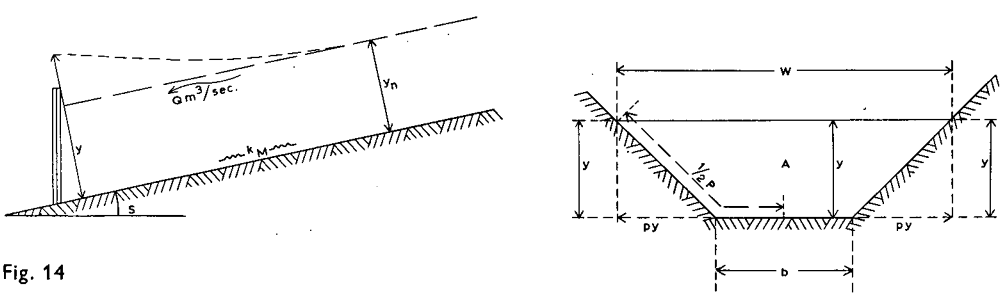


Fig. 14

[1]) BORIS A. BAKHMETEFF, Hydraulics of Open Channels. McGraw-Hill Book Co., New York - London, 1932.

2. We require to calculate

   a. the normal depth y_n at uniform flow.
   For this we use Manning's formula $Q = k_M \, A R^{2/3} S^{1/2}$.
   For rapid calculation reference should be made to the collection of graphs.
   b. the cross-sectional area of flow A for y_n and y_o;
   c. the wetted perimeter P, also for y_n and y_o
   d. the hydraulic radius $R = A/P$ for y_n and y_o and the average value
   e. $\eta = y/y_n = \dfrac{\text{local backed-up depth}}{\text{normal depth}}$

f. the hydraulic exponent n

$$Q = A \, k_M \, R^{2/3} S^{1/2}$$

$$S = \frac{Q^2}{k_M{}^2 \, A^2 \, R^{4/3}} = \frac{Q^2}{K^2}$$

$$K = \sqrt{\gamma y^n} \qquad (K = \text{conveyance})$$

$$\gamma = \text{a constant}$$

$$\frac{K_o{}^2}{K_n{}^2} = \frac{\gamma \, y_o{}^n}{\gamma \, y_n{}^n} = \left(\frac{y_o}{y_n}\right)^n = \frac{A_o{}^2 \, R_o{}^{4/3}}{A_n{}^2 \, R_n{}^{4/3}}$$

For several cross-sections, the value of n belonging to a certain b/y_n and η have been plotted in Graph no. 8 (it has been assumed that k_M is constant for the entire channel between y_o and y_n).

g. the average coefficient of resistance C from the formula $C = k_M \, R^{1/6}$ (see Graph no. 6 for this)

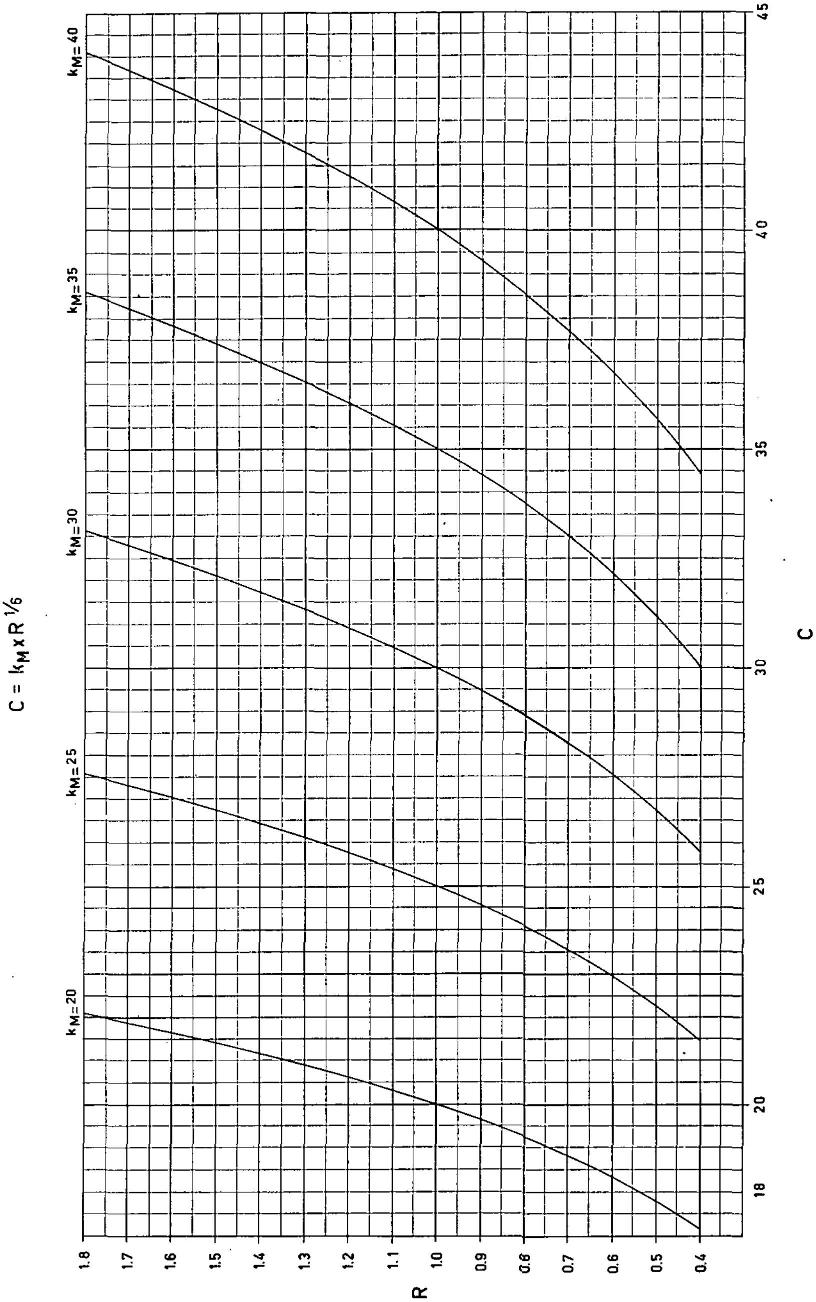
h. $B(\eta)$ for various values found for η and n; see Graph 7. In using this graph attention should be paid to the correct manner of reading off. With a backed-up flood-profile η is always greater than 1, whilst with a drawn-down flood-profile η is smaller than 1.

j. a factor β from the formula $\beta = \dfrac{C^2 \, WS}{gP}$.

All the data are now known which are needed for substitution in the formula

$$X = \eta - (1 - \beta) \, B(\eta).$$

For every water level a value for X may be calculated. If the water levels are



C = k_M x R 1/6
k_M=40
k_M=35
k_M=30
k_M=25
k_M=20
R
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
C
18
20
25
30
35
40
45

Graph 6

known for two points, the distance between them (in m) can be calculated from the formula:

$$L = (X_1 - X_2) \frac{y_n}{S}$$

$$X_2 = \eta_2 - (1 - \beta) B(\eta_2)$$

$$\eta_2 = \frac{y_2}{y_n}$$

$$X_1 = \eta_1 - (1 - \beta) B(\eta_1)$$

$$\eta_1 = \frac{y_1}{y_n}$$

$$\eta_2 < \eta_1$$

If the distances from the weir are known of the several points with given water depth, then the backwater curve can be constructed.

B. Drawn-down of the normal flood-profile due to reduction of the resistance

Naturally, the formulae and calculations of B. A. Bakhmeteff hold likewise for lowering of the water surface, but with the difference that $\eta = y/y_n$ is now smaller than 1, as shown schematically in Fig. 15.

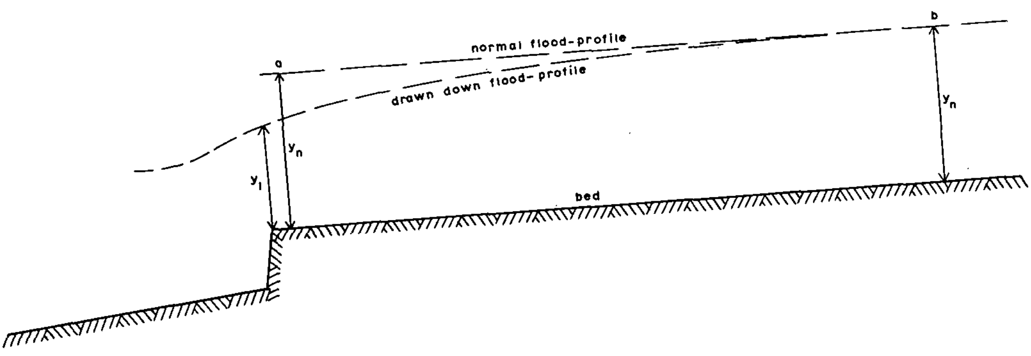
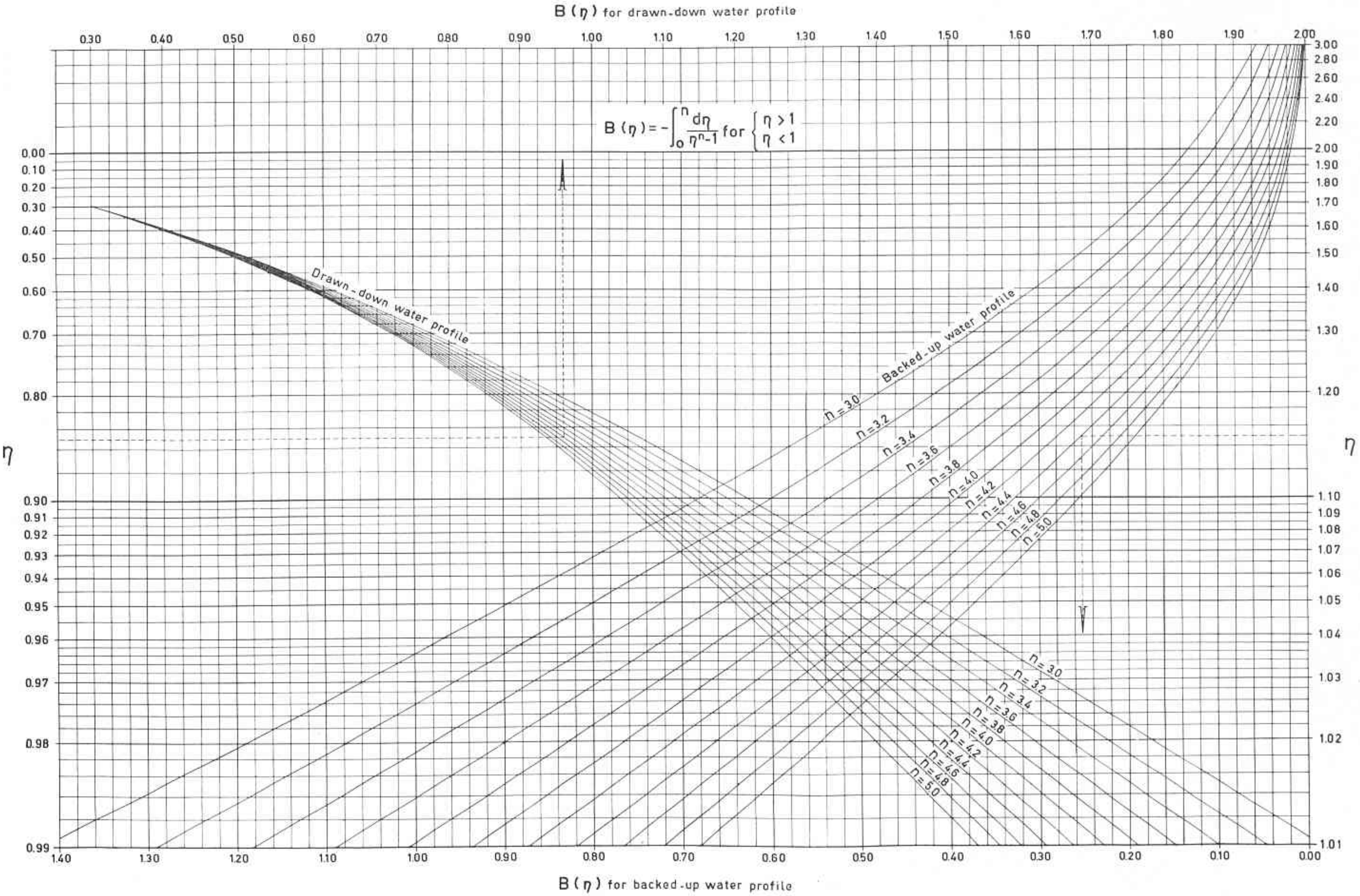


Fig. 15



Graph 7. Graph to assist the determination of the highwater profile for non-parallel flow

If in an actual case n and η have been calculated, the $B(\eta)$ is found by using Graph 7. Whereas A, $L = (X_2 - X_1)\frac{y_n}{S}$, with a lowering from the equilibrium depth $\eta < 1$, we now have:

$$L = (X_2 - X_1)\frac{y_n}{S}$$

$$X_2 = \eta_2 - (1 - \beta)\ B(\eta_2)$$

$$\eta_2 = \frac{y_2}{y_n}$$

$$X_1 = \eta_1 - (1 - \beta)\ B(\eta_1)$$

$$\eta_1 = \frac{y_1}{y_n}$$

$$\eta_2 > \eta_1$$

The starting point is the calculated water depth at the downstream end of the channel, and subsequently the distance to this point can be calculated for several different depths of water in the manner described above.

WORKED EXAMPLE OF THE CALCULATION OF A BACKWATER CURVE AS DESCRIBED UNDER A

Given is a weir immediately upstream of which the water depth y = 1.80 m (see Fig. 16, point A). We require to calculate the distances from the weir of the points at which the depths of water are 1.78, 1.76, 1.74, 1.72, 1.70, 1.65, 1.60, 1.55, 1.50, 1.40, 1.30 and 1.20.

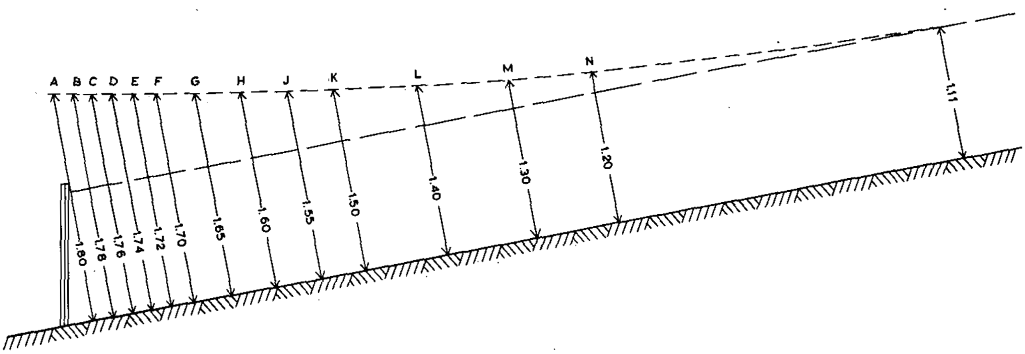


Fig. 16

The data to be used in the example are as follows:

$Q = 2.50\ \mathrm{m^3/sec}$
$S = 0.0005$ (i.e. 50 cm fall per kilometer)
$b = 3.00\ \mathrm{m}$
$p = 1$
$k_M = 30$
$y_n = 1.11\ \mathrm{m}$
$g = 9.81\ \mathrm{m/sec^2}$

Whatever the backwater curve to be constructed, every calculation of the distances commences with the calculation of the n-value.

If this n cannot be determined from Graph no. 8, it must be calculated for y_n and y_o from A and R.

For:

$$y_n = 1.11\ \mathrm{m}$$

$$A = (b + py)\, y = 4.562\ \mathrm{m^2}$$

$$P = b + 2y\sqrt{(1 + p^2)} = 6.139\ \mathrm{m}$$

$$R = \frac{A}{P} = \frac{4.562}{6.139} = 0.743\ \mathrm{m}$$

$$A^2 = 20.812$$

$$R^{4/3} = 0.673$$

$$A^2R^{4/3} = 14.006$$

For:

$$y_o = 1.80\ \mathrm{m}$$

$$A = 8.640\ \mathrm{m^2}$$

$$P = 3 + 2 \times 1.8\sqrt{2} = 8.092\ \mathrm{m}$$

$$R = \frac{8.640}{8.092} = 1.068\ \mathrm{m}$$

$$A^2 = 74.650$$

$$R^{4/3} = 1.091$$

$$A^2R^{4/3} = 81.443$$

$$\eta = \frac{y_o}{y_n} = 1.622$$

$$\eta^n = \frac{A^2R^{4/3}(y_o)}{A^2R^{4/3}(y_n)} = \frac{81.443}{14.006} = 5.815$$

$$\boxed{n = 3.6}$$

The reading from Graph 8 for $\eta = 1.6$ gives the same result.

Now C can be determined, using the average R value.

average $R = 0.91$

$C = k_M R^{1/6} = 29.5$ (take $C = 30$)

Reading from Graph 6 gives the same result.

The calculation of the required points can now proceed:

$\boxed{y_o = 1.80 \text{ m}}$ $B(\eta) = 0.119$ (Graph no. 7)

$$\beta = \frac{C^2SW}{gP} = \frac{30^2 \times 0.0005 \times 6.6}{9.81 \times 8.092} = 0.04$$

$$X_1 = \eta - (1 - \beta) \times B(\eta)$$

$$X_1 = 1.62 - (1 - 0.04)\ 0.119 = 1.506 \text{ m}$$

$$L = 0$$

$$y = 1.78 \text{ m}$$

$$\eta = \frac{1.78}{1.11} = 1.604$$

$$B(\eta) = 0.122$$

$\beta = 0.04$ (W and P have been altered here, $P = 8.034$)

$$X_2 = 1.60 - (1 - 0.04)\ 0.122 = 1.483 \text{ m}$$

$$L = (X_1 - X_2)\frac{y_n}{S} = 0.023\ \frac{1.11}{0.0005} = \boxed{51 \text{ m}}$$

Obviously, for every given water level an X value can be calculated. Deducted from the X-value at the weir and multiplied by the constant factor (see p. 76)

y_n/S $(= \frac{1.11}{0.0005} = 22.209)$, this yields the distance from the weir to the point with the given water level. In the following summary, all the results of calculation applicable to the several water depths have been tabulated.

y	A	P	R	C	n	η	$B(\eta)$	β	X	L
m	m^2	m	m							
1.80	8.640	8.092	1.068	30	3.6	1.622	0.119	0.04	1.506	0
1.78						1.604	0.122		1.483	51
1.76						1.586	0.126		1.465	99
1.74						1.568	0.130		1.443	148
1.72						1.550	0.135		1.420	198
1.70						1.532	0.140		1.400	249
1.65						1.487	0.153		1.340	376
1.60						1.441	0.169		1.279	513
1.55						1.396	0.187		1.216	649
1.50						1.351	0.207		1.152	793
1.40						1.261	0.264		1.008	1115
1.30						1.171	0.357		0.828	1513
1.20						1.081	0.535	0.04	0.567	2092
1.11	4.562	6.139	0.743	30	3.6	1				

– *The backwater curve*

Now that it has been calculated at what distance from the weir each of the given water depths is attained, the profile of the backwater curve is also known. Setting out the water depths against the corresponding distances yields the backwater curve, as shown in Fig. 17.

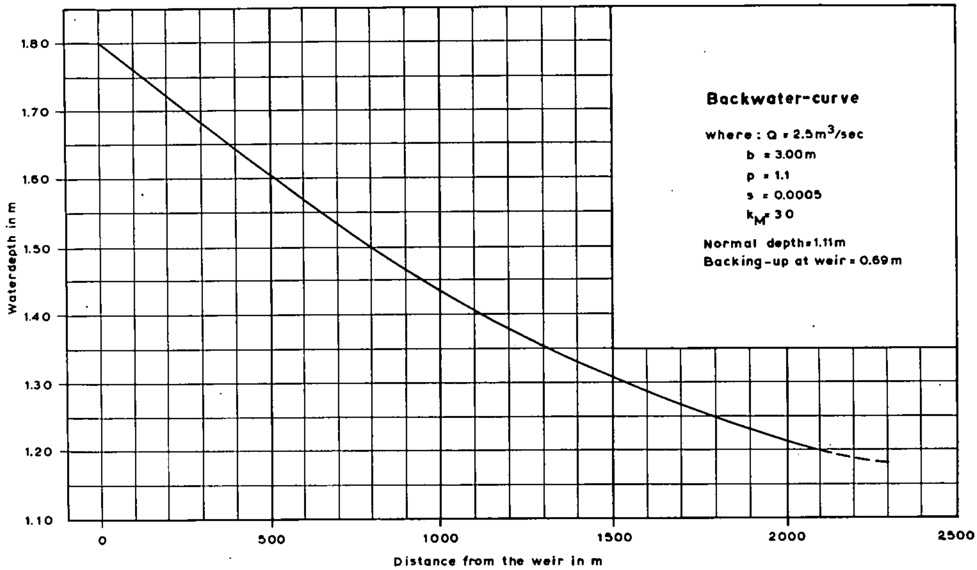


Fig. 17

SIMPLIFICATION

The above worked example demonstrates that the calculation of a backwater curve in a prismatic channel demands rather laborious calculation.

Fortunately, further investigation of this method of calculation shows that it is possible to introduce a considerable simplification without seriously detracting from the accuracy.

The value of β varies with the bed-gradient S. In regions with moderate bed-gradients, the value of β is frequently small. The inaccuracy then introduced by neglecting β is smaller than the tolerances that are generally allowed in calculating open watercourses.

It is advisable for greater bed-gradients to assess the order of magnitude of the error introduced by this neglect.

BAKHMETEFF gives in his book the values for β — B that can be used for the simpler calculation where β is neglected. VEN TE CHOW has in his more recent book[1]) given a simplified method of calculation that can often be used with advantage.

An approximation to the nearest 1% is permissible in most cases.

[1]) VEN TE CHOW, Open channel hydraulics. McGraw-Hill, London, 1959.

STEP-BY-STEP APPROACH

This method has the advantage that those acquainted with the technique of backwater calculation can see clearly what they are doing. The disadvantage is however that a continuous step-by-step operation is necessary for which more time is required when calculating a certain water level at a given distance from the weir. This calculation is given in Fig. 18.

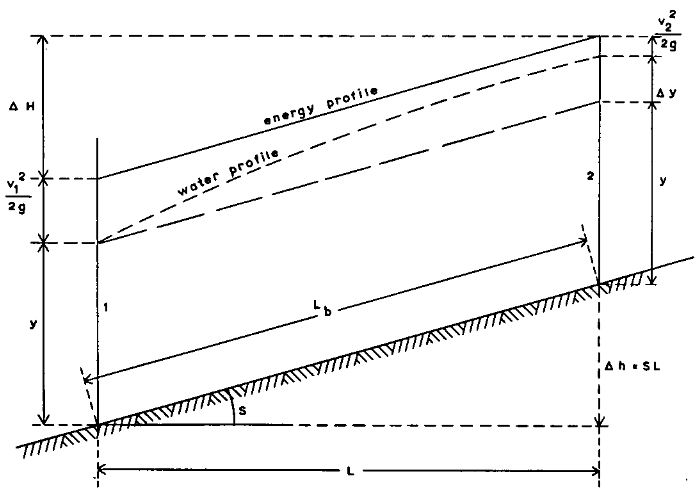


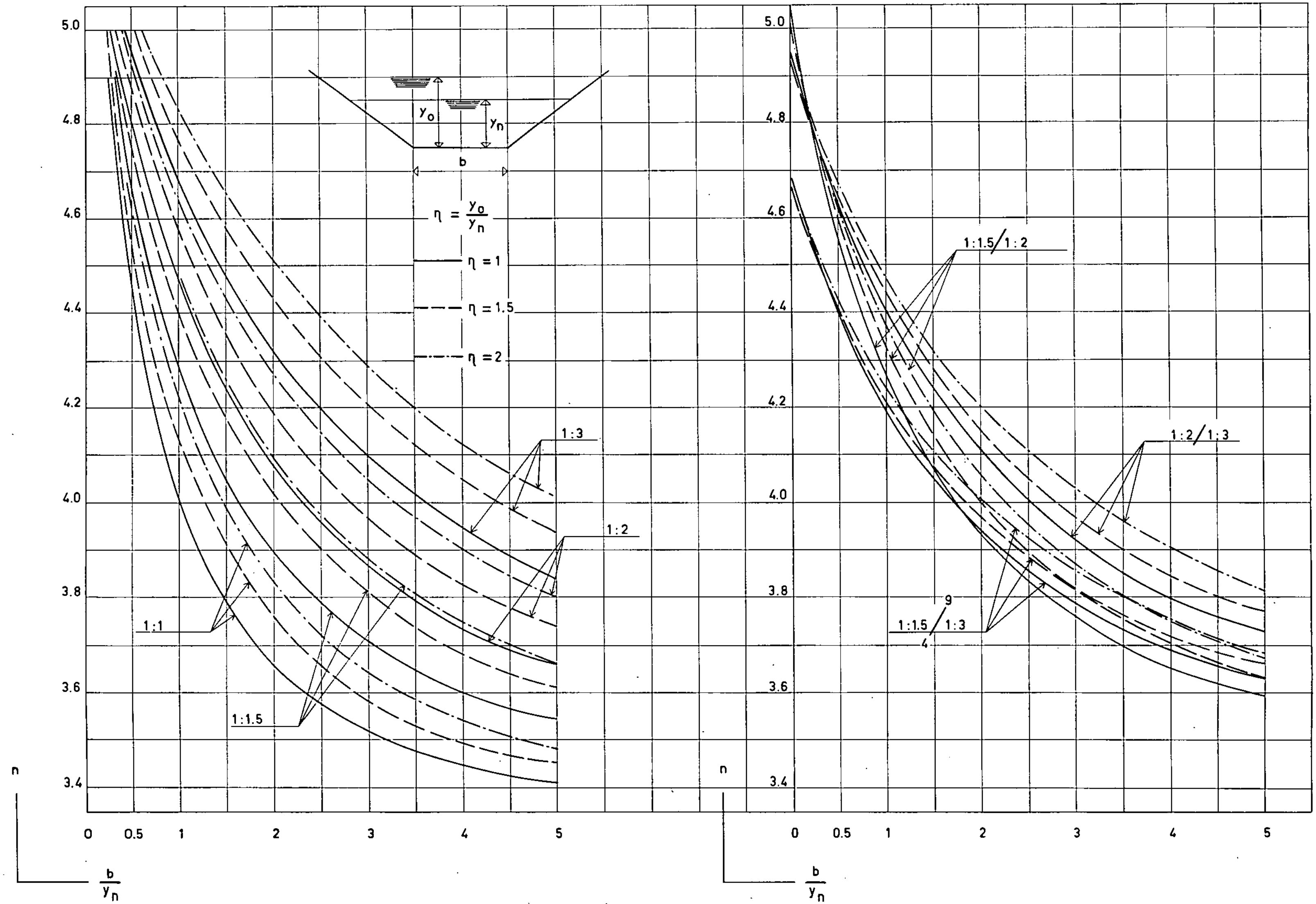
Fig. 18

S is small, so we can put $L_b = L$

$$y + \frac{v_1^2}{2g} + \Delta H = SL + y + \Delta y + \frac{v_2^2}{2g}$$

$$\Delta H - SL = \Delta y + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \Delta y + \Delta \frac{v^2}{2g}$$

$$\frac{\Delta H}{L} - S = \frac{\Delta y}{L} + \frac{\frac{v^2}{2g}}{L}$$



n
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
0
0.5
1
2
3
4
5
b/yn
yo
yn
b
η = yo/yn
η = 1
η = 1.5
η = 2
1:1
1:1.5
1:2
1:3
1:1.5/1:2
1:2/1:3
1:1.5 9/4 1:3

Graph 8

$\frac{\Delta H}{L}$ = energy gradient; this is caused only by frictional losses so that we can call it the friction-gradient:

$$S_w = \frac{\Delta H}{L} = \frac{v^2}{C^2 R}$$

The average values of v, C and R have now to be calculated, reach for reach, in a table for example, as performed above:

$$S_w - S = \frac{\Delta y + \Delta \frac{v^2}{2g}}{L}$$

Δy can be assumed and L then calculated from the equation:

$$L = \frac{\Delta y + \Delta \frac{v^2}{2g}}{S_w - S}$$

For S_w, the average value of $\frac{v^2}{C^2 R}$ over the reach may be assumed.

With low gradients, velocities are often low also and in certain cases $\Delta \frac{v^2}{2g}$ can be neglected. In most cases an investigation will be needed to check whether this neglect is permissible.