GROUNDWATER HYDRAULICS OF EXTENSIVE AQUIFERS

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INTERNATIONAL INSTITUTE FOR LAND RECLAMATION AND IMPROVEMENT ILRI WAGENINGEN THE NETHERLANDS 1972

Bulletin 13

CONTENTS[1])

[1]) The meaning of the symbols used in the titles are, D: thickness of the aquifer, n: recharge of the aquifer, φ: potential in the aquifer, φ': potential defining the water table in the low-permeability top layer, q: rate of flow per unit width over the height D of the aquifer. For further details about the meaning of symbols see 'Notes and list of symbols'.

INTRODUCTION

The term extensive aquifers is used to denote aquifers whose horizontal dimensions are much larger than their thicknesses, so that the losses of head due to the vertical velocity components may be neglected. The term groundwater hydraulics is used in the sense of deductive theory.
A series of strongly schematized problems is analysed with a view to applying the results in groundwater engineering. The publication is a text-book, not a manual, stress being laid on didactics, not on completeness or detail. The mathematical derivations are given in full, starting from the fundamental physical laws; mathematical methods, however, are not explained. As a rule, a problem is discussed in four stages: posing the problem, formulating the solution, deriving the formulae and analysing the result. The mathematical derivations are marked by a disjoined vertical line; they should be omitted at first reading, when the reader's attention should remain concentrated on the main issue of the theory.
The basic laws and assumptions adopted are more or less consecrated by tradition. They have, however, a limited range of validity. This range has been established for some laws (e.g. the law of linear resistance). In other instances it forms the subject of modern investigations (e.g. the law relating the stored or released quantities of water to the rise or fall of the water table, where the notion of effective porosity is only an approximation). This physical side of the problem is not treated. It is thought too important to be discussed in complementary remarks to an essentially deductive study. If it were to be treated comprehensively, it should be made the subject of a separate study.

The use to be made of the solutions of schematized problems may be summarized in the following points:

1. Since flow of groundwater is hidden to the eye, we have no everyday experience with it, as we have with mechanical phenomena; the best way to get acquainted with the nature of the phenomena is to solve, as an exercise, a series of elementary problems. We are also blind to the magnitude of quantities involved in problems of groundwater flow; to estimate them properly we need orientating calculations on strongly schematized models.
2. Physical formulas have a limited range of validity. When, for example, a phenomenon depends on three factors, A, B, and C, it depends on A alone when A is predominant, on A and B when C is negligible. Posing the problem requires an appreciation of the relative magnitude of the relevant quantities. This is another reason for starting with orientating calculations.
3. Engineering calculations generally cover two phases. First the hydraulic characteristics of the aquifer are determined on the basis of observations and tests; then the dimensions and flow rates of the design are determined on the basis of these characteristics. In both phases it is recommended that rough, orientating calculations be used to start with, and that they be repeated once or several times on a more refined basis. The reality should be compared with standard flow systems, preferably chosen so as to comprise the reality between conditions that are too favourable and too unfavourable. This is the principle usually adopted for the calculation of steel and concrete constructions.
4. Groundwater calculations are generally rough, because the underground is irregular, because tests are costly, and because some basic quantities, such as evaporation, are only approximately known. This stresses the importance of elementary calculations above refined ones. The use of computers is justified only when the observations and tests have been adequate to obtain precise results, and this precision is needed for the design. Another use of computers is to solve standard problems that are too difficult for mathematical analysis.
5. The basic laws of groundwater hydraulics are linear. Thus, in problems depending on several factors, the influence of each factor may be calculated apart, and the results added. This principle, that of superposition, will be the guideline throughout the theory. It makes understanding easy, and allows complicated calculations to be split up into elementary ones.

Part of the theory has been acquired from literature; part is the result of my own studies. Foreign elements have not been presented in the form chosen by the authors. All elements have been merged into greater units of thought, in which process they have lost their individuality. Each chapter forms a unit; the chapters form a sequence: the publication should be read as a whole.
Since foreign elements have not been given in their original form, and all derivations

are given in full, no reference is made to the original publications. A list of comprehensive modern books is added, to be used for further study as well as for detailed reference. However, the names of G. J. de Glee, J. P. Mazure, J. van Oldenborgh, and J. H. Steggewentz should be mentioned, since their work has been fundamental for the present studies.

Thanks are due to G. de Josselin de Jong, and A. Verruijt for critical remarks, as well as to N. A. de Ridder for his critical reading of the manuscript and to Mrs. M. F. L. Wiersma-Roche for linguistic corrections.

Wageningen, October 1968 J. H. E.

NOTES AND LIST OF SYMBOLS

1. A disjoined vertical line marks the derivations.
2. The text alway refers to the figure indicated at the beginning of the paragraph.
3. The following conventions are used in the figures
shaded: impermeable
dotted: low permeability
blank: permeable
4. Throughout this study the potential is defined as a pressure, while most engineers are accustomed to defining it as a height (length dimension). As long as one-fluid is concerned, they may read the formulas in their own interpretation, considering the potential φ as a height, the permeability k as a velocity, and reading for μ the effective pore space, a dimensionless quantity. In the theory of two-fluid systems, however, all symbols must be read according to the definitions of this study.
5. All formulas are dimensionless. They apply to any consistent system of units (founded on one unit for the mass, one unit for the length, and one unit for the time).
6. Where references are made to other parts of the text, the main units, 1 to 7, are called chapters; the smaller units, indicated by one or two decimal figures are called sections.
7. Most symbols are used throughout the text, often without explanation. Their meaning and dimension are listed below, and reference is made to the section in which they are introduced. As a rule the following distinctions are made:
Without prime: related to the fresh water in the aquifer.
With prime: related to the low-permeability top layer.
With double prime: related to the salt water in the aquifer.

Aquifer in one-fluid problems

RL	Reference level.
x and *y*	Horizontal coordinates.
z	Vertical coordinate.
t	Time.
T	Period of periodic movements.
ω	Equal to $\frac{2\pi}{T}$, used in sin ωt, defining sinusoidal variations. $[t^{-1}]$ (Section 5.2).
φ	Potential, defined as a pressure. $[ml^{-1}t^{-2}]$ (Section 1.1.1).
γ	Specific weight of water. $[ml^{-2}t^{-2}]$
h	Piezometric height. $[l]$ (Section 1.1.1).
k	Permeability, generally in a horizontal direction. $[m^{-1}l^{3}t]$ (Section 1.1.1).
D	Thickness of the water body contained in the aquifer. Either constant or variable. $[l]$.
kD	Transmissivity of the aquifer for horizontal flow. $[m^{-1}l^{4}t]$ (Section 1.1.2).
m	Effective porosity. Volume of water released from or taken into storage per unit area of the aquifer due to variation of the phreatic level by unit height. Dimensionless. (Sections 1.2 and 6.3.1).
μ	Volume of water released from or taken into storage per unit area of the aquifer, due to a change of the water level corresponding to unit potential. $\mu = \frac{m}{\gamma}$. $[m^{-1}l^{2}t^{2}]$ (Section 1.2).
v	Velocity in the sense of the quantity of water passing per unit time through a unit area including the section over the grains. $[lt^{-1}]$ (Section 1.1.1).
q	Quantity of flow through unit width of an aquifer with thickness *D*. $q = vD$.$[l^{2}t^{-1}]$ (Section 1.1.2).
Q	Quantity of flow through an arbitrary cross-section, e.g. the flow towards a well. $[l^{3}t^{-1}]$ (Section 1.1.2).
n	Recharge of the aquifer from the upper, nonsaturated soil layers, as a volume of water per unit time and per unit area of the aquifer. $[lt^{-1}]$ (Section 1.2).
N	Mathematical concept, introduced to make a general formulation of the law of continuity possible: the volume of water joining the groundwater flow in the aquifer per unit time per unit area of the aquifer, as a consequence of both recharge and fall of the piezometric level. $[lt^{-1}]$ (Section 1.2).

Low-permeability top layer

φ'	Potential (Section 1.1.2).
h'	Piezometric height (function of z) (Section 1.1.2).
k'	Permeability in vertical direction (Section 1.1.2).
D'	Thickness of the water layer contained in the top layer, always considered constant over the area of the aquifer (Section 1.1.2).
k'/D'	Transmissivity of the top layer for vertical flow.
m'	Equivalent of m for the top layer (Section 1.2).
μ'	Equivalent of μ for the top layer (Section 1.2).

Aquifer in two-fluid systems

RL	Reference level.
SL	Sea level.
γ and γ''	Specific weight of fresh and salt water respectively. (Section 6.1.1).
φ and φ''	Potentials of fresh and salt water respectively (Section 6.1.3).
h and h''	Piezometric height of fresh and salt water respectively (The piezometer tube filled with fresh and salt water respectively) (Section 6.1.3).
D and D''	Thickness of fresh and salt water body respectively (Section 6.2.1).
D_t	Sum of D and D''
Z	Elevation of the interface above reference level (negative when reference level coincides with sea level) (Section 6.1.3).
m	Effective porosity. Volume of water released from or taken into storage per unit area of the aquifer, due to variation by unit height of the phreatic surface or the interface. Dimensionless. (Sections 1.2 and 6.3.1).
μ	Same as for one-fluid system. Related to φ. For the water surface only; not for the interface.
v and v''	Horizontal velocities, or velocities parallel to the interface in fresh and salt water (volume of water displaced per unit time per unit section, including the section over the grains) (Section 6.1.3).
q and q''	Quantities of flow per unit width over the thickness of the fresh and salt water body respectively. $q = vD$; $q'' = v''D''$ (Section 6.2.1).
Q and Q''	Quantities of flow through an arbitrary section in fresh and salt water respectively. $[l^3t^{-1}]$.

1. FUNDAMENTALS

1.1. THE LAW OF LINEAR RESISTANCE

1.1.1. *Formulation of the law*

Water flowing through a porous medium loses energy. The quantity of energy per unit volume of water is called the potential φ, for reasons to be specified below. Its dimension $[ml^{-1}t^{-2}]$ is that of a pressure.

There is no uniformity in the definition of the potential. In engineering practice it is more common to define φ as the quantity of energy per unit weight of the water. The potential thus defined has the dimension of a length, and can be shown graphically as an elevation above a plane of reference. This definition, however, cannot be admitted in the following studies, as it would complicate the formulas of two-fluid problems. For fresh water problems all formulas describing the flow systems are identical in both practices, but with a different meaning of the symbols.

Figure 1. – The potential φ at a certain point P of the aquifer is given by

$$\varphi = p + \gamma z$$

where p is the pressure and γ the specific weight of the water at P; z is the elevation of P above reference level RL. This expression is taken from the theory of general hydraulics: it will not be derived here. In its general form it contains still a third term $(\rho v^2)/2$, depending on the velocity of the water, where ρ is the density and v the velocity of the water at P. This term can be ignored in groundwater hydraulics, where velocities are always low.

A piezometer is a simple tube, placed in the ground and screened over a certain

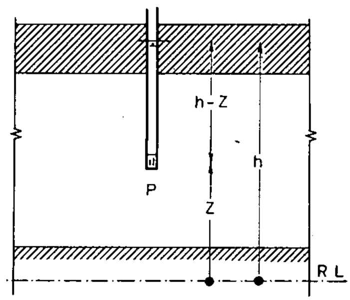


Fig. 1

length at its bottom end. For theoretical considerations the screen may conveniently be reduced to a point. The piezometric height h at this point is the elevation of the water level in the tube above the reference level RL. The potential φ is related to the piezometric height h by

$$\varphi = \gamma h$$

The pressure at P corresponds to the water column $h - z$ inside the tube; hence $p = \gamma(h - z)$. Substitution of this value for p in the expression $\varphi = p + \gamma z$ gives $\varphi = \gamma h$.

The discharge Q is the quantity of water flowing per unit time through a small section S perpendicular to the flow. It is customary to take for S the rough area over both pores and grains. The quantity $v = Q/S$, therefore, does not correspond to the average velocity of the water particles. Yet it is customary to call v the velocity of the water.
Throughout the study laminar flow will be assumed. In nature this condition is usually satisfied. Exceptions may exist locally where the velocities are particularly high, such as near pumped wells or where fresh groundwater flows into the sea. In laminar flow the losses of energy are proportional to the velocities. This law of linear resistance is known as Darcy's law, when applied to groundwater. It may be written as:

$$v_x = -k\frac{\partial\varphi}{\partial x}, \quad v_y = -k\frac{\partial\varphi}{\partial y}, \quad v_z = -k\frac{\partial\varphi}{\partial z}$$

where k is a constant, and v_x, v_y and v_z are the components of the velocity in the directions of the coördinate axes x, y and z. For an arbitrary direction s

$$v_s = -k\frac{\partial\varphi}{\partial s}$$

which will not be proved here.
Formulas of this type are well known in physics. Any quantity φ satisfying them is

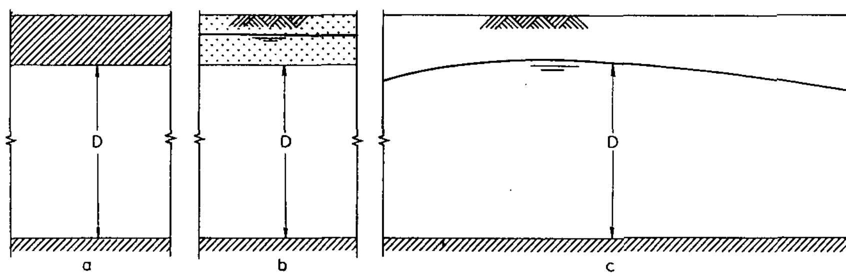
called a potential. If it defines a velocity, as in the present case, it is called a velocity potential; if a force, a force potential, etc. Consequently the theory of groundwater flow appears as a particular case of potential theory.
The constant k, as defined by these formulas, has the dimension $[m^{-1}tl^3]$. In current engineering practice, where the potential φ is defined as a length, and k is determined by the same formulas, k takes the dimension of a velocity. If values of k are given as velocities, they must be divided by the specific weight of the water, to obtain the corresponding values in the practice to be followed here.
The permeability k depends on the characteristics of the soil, and the viscosity η of the water. Strictly speaking, it is improper to call it the permeability of the soil, as it depends also on the properties of the water. Other names have been proposed, but since there is no uniformity on this point, simply the most current term, although improper, will be used.
As can be shown from the theory of Dimensional Analysis or from more thorough theoretical considerations, the following relationship exists:

$$k = k_0 \frac{d^2}{\eta}$$

This formula should be read with the idea of similitude in mind. If two ideal scale models are imagined, composed of grains of the same form and arrangement, but of different size, the geometrical scale of either model may be determined by any length dimension d of the grains, e.g. the average grain size, defined in any conventional way. The models may be filled with fluids of different viscosity η. The formula then indicates the relationship between k, d and η in each model. The coefficient k_0 is a dimensionless constant, depending on the form of the grains and the definition of d, and is the same in both models. It can be seen from this relationship that k is equal for both salt and fresh water filling the same medium, if the difference in viscosity between the fluids is neglected.

In the following studies homogeneous soil and water will be assumed. Where in Chapters 6 and 7 two-fluid systems are described, the property of homogeneity will apply to each of the fluids separately. Strictly speaking, granular material is not homogeneous. The term will be used with respect to the average values of the soil characteristics (pore space, permeability, etc.) in units of volume, large compared with the dimensions of the grains, and small compared with those of the aquifer. Used in this sense, the word homogeneity expresses that these average values of the soil characteristics are the same throughout the aquifer. As a consequence, the hydraulic quantities (pressure, velocity, etc.) are continuous functions of the coördinates. In nature the condition of homogeneity is in general not fully satisfied. The main exceptions are:

Fig. 2



- Sandy aquifers are usually made up of alternating layers of sediments having different properties. They may contain layers of fine material, or even lenses of silt or clay, which impede the vertical water movement. Natural aquifers usually have a greater permeability in horizontal than in vertical direction.
- Limestone, if finely fissured, has the characteristics of a permeable medium, but its degree of fissuring is seldom uniform throughout the aquifer.
- The density and viscosity of water vary with temperature. In thick aquifers the increase in temperature with depth plays a role.
- The viscosities of fresh and salt water are slightly different. This factor will be ignored in Chapters 6 and 7.

1.1.2. *Extensive aquifers*

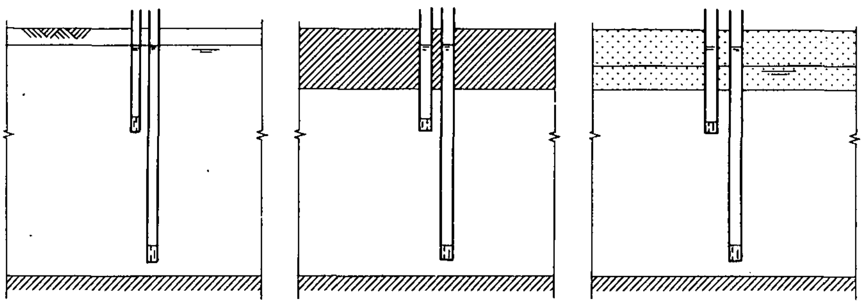
The following studies will deal alternately with three types of aquifers: confined, partly confined, and phreatic, to be described below. As a general assumption they rest on a horizontal, impermeable base. Some special cases will be considered where the aquifer dips slightly.

Figure 2a represents a confined aquifer, covered with a horizontal impermeable layer, and saturated with water under pressure. The thickness D of the water layer is constant and equal to that of the aquifer.

Figure 2b represents a partly confined aquifer, i.e. covered by a layer of low-permeability, and saturated with water under pressure, the phreatic level being in the top layer. The term low-permeability will be used in the sense of low compared with the permeability of the aquifer, but not zero. Since the horizontal flow in the top layer will be neglected, as will be explained below, the lateral water movement depends on the thickness D of the aquifer, which is a constant, as in the previous case.

In Figure 2c the groundwater has a free surface. For the sake of simplicity no capillary fringe is considered. Groundwater having a free surface is called phreatic water; the surface, the phreatic surface. The thickness D of the water body is variable from one point to another. For exact calculations the variations of D are considered. For approximate calculations the variations in water height are neglected in comparison

Fig. 3



with the total thickness of the water-layer. The variable thickness is then replaced by its average value D. This assumption is frequently made in engineering practice.

In all cases the thickness of the aquifer is assumed to be small compared with its horizontal dimensions. This property is indicated by the term extensive aquifers. The water transport through such aquifers is mainly horizontal. The horizontal velocity components are generally greater than the vertical components and since, moreover, the water moves horizontally over much greater distances than it does vertically, the losses of energy in a horizontal direction are much greater than in a vertical direction, so that as a basic assumption for all following studies, the vertical energy losses are neglected. This assumption may be considered either as an approximation, applicable to an aquifer composed of isotropic material, or as an exact characteristic of an aquifer composed of anisotropic material, having a permeability k in all horizontal directions and an infinitely great permeability in vertical direction.

Figure 3. – When the vertical losses of energy are neglected, the potential φ is a constant in a vertical. Mathematically φ is a function of x and y only, and not of the vertical coördinate z. The same is true for the piezometric head h, which differs from φ only by a factor γ. Hence the water rises to the same level in two piezometers placed in the same vertical at different depths. This is shown for a phreatic, a confined and a partly confined aquifer respectively. In phreatic water the piezometric level corresponds to the water table. In a confined aquifer it rises above the top of the aquifer. The same holds good for a partly confined aquifer, where the piezometric level is generally different from the water level in the top layer.

Figure 4. – The left-hand side shows an aquifer bounded by a river or a lake. It follows from the above that the potential φ is equal at all points to the right of A, regardless of whether these points are chosen in the lake or in the aquifer underneath. In all models, therefore, canals, lakes or the sea will be assumed to be in contact with the aquifer along a vertical plane down to the impermeable bottom, as indicated on

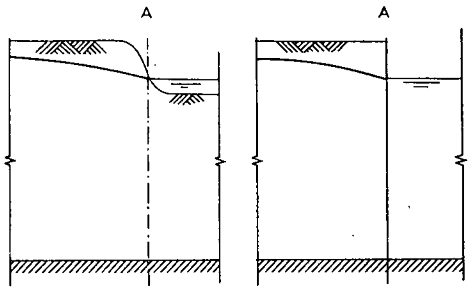


Fig. 4

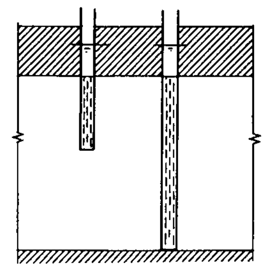


Fig. 5

the right-hand side of the figure. The mathematical expression for the boundary condition is a given value of φ in the vertical plane passing through A.

Figure 5. – Similar considerations can be applied to wells. On the left-hand side a partially penetrating well is shown. Since losses of head inside the wells will be neglected in all studies, mathematically the well face is a boundary of the aquifer, characterized by a constant value of φ. But since vertical losses of energy in the aquifer are neglected as well, the cylindrical part of the aquifer below the bottom of the well has the same characteristic. Therefore, only completely penetrating wells will be considered, as shown on the right-hand side of the figure.

If φ is independent of z, it follows that the same is true for $\partial\varphi/\partial x$ and $\partial\varphi/\partial y$, hence for v_x and v_y. In other words, the water flows at the same rate at all levels. Summation of the discharge over the height of the aquifer is then easy. The symbol q will be used to denote the quantities flowing per unit width over the entire thickness of the aquifer. The notations q_x, q_y and q_s apply to the discharges in the direction of the x and y axes, or in an arbitrary direction s. It follows from the law of linear resistance that

$$q_x = -kD\frac{\partial\varphi}{\partial x}; \quad q_y = -kD\frac{\partial\varphi}{\partial y}$$

which formulas will be taken as a starting point in the following chapters. The product kD is called the transmissivity of the aquifer for horizontal groundwater flow.

The symbol Q will be used to denote the rate of flow through an arbitrary cross-section. In case of radial flow for example, it denotes the flow through a cylinder with radius r and a height equal to the thickness of the aquifer. The quantity Q is then related to q by the elementary relationship $Q = 2\pi rq$, hence

$$Q = -2\pi rkD\frac{\partial\varphi}{\partial x}$$

It should be noted that neglecting the losses of energy due to the vertical components of the velocity does not imply that these components would not exist in the schemes to be examined. Water supplied by infiltrating rain, and reaching the top of the aquifer, is distributed over at least a part of the thickness of the aquifer by vertical velocity components. Or, to give another example, the interface between fresh and salt groundwater may, in case of nonsteady flow, move upwards or downwards, and these displacements can even be calculated when the vertical energy losses are neglected.

In the case of a partly confined aquifer, the flow through the top layer of low permeability has to be considered. The physical quantities of this layer will be indicated with primes to distinguish them from the corresponding quantities of the aquifer. It will be assumed that the thickness D' of the waterbody in the top layer is less than the thickness D of the aquifer; moreover, that the permeability k' is low compared with the permeability k of the aquifer, though not zero. It follows then that the horizontal flow through the top layer can be neglected in comparison with that through the aquifer, because it depends on the product $k'D'$, which is small compared with the product kD.
This assumption can be considered either as an approximation when the top layer is composed of isotropic material, or as an exact formulation when it consists of anisotropic material with a permeability k' in the vertical direction and zero permeability in all horizontal directions. Consequently, if a canal, a lake, or the sea borders the top layer, it will be assumed that no lateral exchange of water takes place, although the potential of groundwater and free water on either side of the boundary might be different.

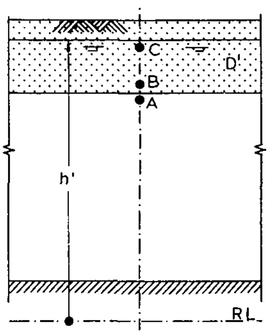


Fig. 6

Figure 6. – Since no horizontal flow is considered here, the rate of vertical flow per unit area, N, is equal at all points of a vertical. It follows from the law of linear resistance that the gradient $\partial\varphi'/\partial z$ is a constant, which means that φ' varies as a linear